Property Value Models

by

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I. INTRODUCTION

Valuing environmental goods is often difficult because they are usually non-market goods. One of the only places where environmental quality is traded on explicit markets is real estate. Yet even here, extracting information on environmental values is often quite complex. Housing and other types of real property are differentiated products, and virtually every unit is different. This results in the complexity, but it also is the reason we can observe environmental quality being sold. Property value studies are one of the most frequently applied techniques for benefit measurement.[1]

There are several techniques that can be used to study the effects of environmental quality on property values and infer willingness to pay for improvements. The spatial nature of almost all environmental problems is crucial for each of the techniques. The most commonly used method is the hedonic model. The somewhat misleading name was coined by Court (1939) in a study of automobiles, and the technique was popularized by Griliches (1971) with the same product. One of the earliest applications was Waugh (1928) on vegetables, although Colwell and Dilmore (1999) uncovered a 1922 Masters thesis by G.C. Haas that used similar techniques on farmland. The technique has been applied to a wide variety of goods and used in many different fields since those early studies. However, in environmental economics the hedonic model has mainly been applied to the prices of real property and to wages. The hedonic model assumes that there is a schedule of prices for the differentiated product (i.e., houses) that can be estimated. Estimation of this hedonic price schedule can provide some information on valuing public environmental goods. However, often it is necessary to go further and learn about individual behavior in the market, the second stage of the hedonic model. An alternative set of models postulates that consumers' choices are discrete between houses rather than continuous in characteristics as in the hedonic model. Discrete choice models are applied to estimate consumer preferences. Finally, recently a model has been developed that mixes discrete and continuous decisions and emphasizes the locational equilibrium.

This chapter is organized as follows. The next section describes the theoretical models that underlie these techniques. The theoretical hedonic model is developed first, and then the theoretical modifications that are necessary for the discrete choice models are described. This section also has a detailed theoretical discussion of welfare measurement in these models under the different circumstances that arise. The main models are developed for residential properties, but differentiated factors of production are discussed briefly at the end of the section. Section III is devoted to the empirical issues involved in estimating a hedonic price schedule. This is the most common type of estimation in property value models, and there is a wide range of issues that must be considered. Section IV discusses the empirical application of the second stage of the hedonic model, the estimation of the underlying preferences. Section V covers the two types of discrete choice models that are used in environmental economics, random utility models and random bidding models. Section VI briefly discusses the new locational equilibrium models, and the final section is devoted to conclusions and directions for further research.

There have been a number of thorough reviews of property value models previously (see Bartik and Smith, 1987; Palmquist, 1991; and Freeman, 1993). While the current chapter attempts to provide a fairly complete overview, the emphasis has been on more recent developments. Those interested in more details on the earlier literature will find these other surveys complementary. The emphasis here is on methodological issues and innovations. There is a vast literature of applications of the techniques to a wide range of issues. Examples of the empirical applications are discussed Smith and Huang (1993, 1995) and Palmquist and Smith (2002).

II. THEORETICAL MODELS AND WELFARE MEASUREMENT

The Theory of Consumer Behavior in Markets for Differentiated Products

Most property value models deal with residential housing. Housing is a differentiated product. A differentiated product is one where there are obvious differences between units of the product, yet the various types of units are traded in a single market.[2] Since each unit differs, there will not be a single, uniform price in the market even if the market is competitive. The price for which a house sells will depend on consumers' preferences for the characteristics that house embodies. If one is modeling new construction, the price would also depend on the cost of producing a house with those characteristics and firms' profit maximizing decisions. However, the stock of existing houses dominates the market in most areas. Researchers generally assume that the supply of houses is fixed in the short-run. The prices of existing houses are demand determined. For that reason, the models discussed here will concentrate on the market equilibrium and the consumer side of the market and take the supply of houses to be fixed.

Following Rosen (1974), let $\mathbf{z} = (z_1, \ldots, z_n)$ represent the n characteristics of houses. All of the models discussed here assume that the price of a house is related to the characteristics it contains. The hedonic models are the most explicit about this relationship and will be followed here initially. The equilibrium price of a house is a function of the characteristics it contains, $P = P(\mathbf{z}) = P(z_1, \ldots, z_n)$. This function is the hedonic price function or hedonic price schedule. It is the equilibrium price schedule in a given market and holds for all the houses in that market. Individual consumers can affect the price they pay for a house by the characteristics they choose.

However, they cannot affect the equilibrium price schedule in a competitive market. They are price-schedule-takers.

The hedonic price schedule may or may not be linear. If it were possible to move characteristics between houses easily (costless repackaging in Rosen's terminology), arbitrage would force a linear price schedule. Since it would be very costly or impossible to move characteristics between houses, the hedonic price schedule need not be linear, although it can be. The functional form is determined in the market with few theoretical restrictions.[3]

The consumers and their preferences and the variety of houses that are available underlie the equilibrium price schedule. Consumers are assumed to purchase one house. If a consumer purchases more than one house, the houses are assumed to be purchased for different purposes (i.e., a primary residence and a vacation home) and to enter the utility function separately. Following Rosen (1974) consumer *j*'s decision involves maximizing a utility function,

$$u^j = U^j(z_1, \ldots, z_n, x, \boldsymbol{\alpha}^j), \tag{1}$$

subject to a budget constraint,

$$m^j = P(\mathbf{z}) + x. \tag{2}$$

The z_i ($i=1,\ldots,n$) are the characteristics of the house as before, x is the non-housing numeraire good, and m^j is consumer *j*'s normalized income. All prices and income have been normalized by dividing by the price of x, and x is a Hicksian composite good representing all the other goods. The utility function is assumed to be strictly concave as well as conforming to the usual assumptions. Socio-economic variables that vary across individuals are included in the vector $\boldsymbol{\alpha}^j$. The first-order conditions are:

$$U_{z_i}{}^j = \lambda^j p_i \qquad i = 1,\ldots,n \tag{3}$$

$$U_x{}^j = \lambda^j \tag{4}$$

$$m^j = P(z) + x \tag{5}$$

where subscripts on functions denote partial derivatives, p_i is the marginal price of characteristic *i* (the partial derivative of P(**z**) with respect to z_i), and λ^j is the Lagrange multiplier. Thus, the marginal rate of substitution between a characteristic and the numeraire is equated to the marginal price of the characteristic:

$$\frac{U_{z_i}{}^j}{U_x{}^j} = p_i \tag{6}$$

It is important to remember that the *i* equations in equation (6) are simply the result of manipulating equations (3) and (4) and do not incorporate the budget constraint.[4] If one minimized expenditure subject to achieving a given level of utility, one would also derive equation (6). Which constraint is used determines the nature of the demand equations derived.

It is useful to consider how much an individual would be willing to pay for a house with a particular set of characteristics. This would depend on the characteristics of the house, the income the individual had, the preferences of the individual, and the level of utility attained. This is Rosen's bid function, θ^j, which is defined implicitly by

$$U^j\left(z, m^j - \theta^j, \alpha^j\right) = u^j \tag{7}$$

where $\theta^j = \theta^j(\mathbf{z}, u^j, m^j, \boldsymbol{\alpha}^j)$. Implicitly differentiating equation (7) (dropping, for the moment, the *j* superscript and the α vector) yields insights into the function.

$$\theta_{z_i} = \frac{U_{z_i}}{U_x} > 0 \tag{8}$$

$$\theta_u = -\frac{1}{U_x} < 0 \tag{9}$$

$$\theta_m = \frac{U_x}{U_x} = 1 \tag{10}$$

$$\theta_{z_i z_i} = U_{z_i z_i} U_x^{\ 2} - 2U_{z_i x} U_{z_i} U_x + U_{xx} U_{z_i}^{\ 2} < 0 \tag{11}$$

Most of these make intuitive sense. If the quantity of one of the characteristics increases, holding constant income and satisfaction, the bid increases (equation 8). If the characteristics and income are held constant, an increase in utility is obtained by consuming more x which means the bid has to be lower (equation 9). Equation 11 shows that the marginal bid for z_i is less as the quantity of z_i increases. The partial derivative with respect to income (equation 10) is slightly less intuitive. However, holding constant the characteristics and utility also means that x is being held constant. This means that all additional income must be used in the bid for the house. Since $\theta_m = 1$, we can rewrite θ as

$$\theta(z,u,m) = \theta^*(z,u) + m \tag{12}$$

$\theta^*(\mathbf{z},u)$ will be negative, so it is not a bid function. It is introduced to provide intuition for some of the results to come. For example, the marginal bid function, $\partial\theta/\partial z_i$, depends only on **z**, u, and α, but not m. Also, if characteristics change, the difference in the bids does not depend on income (as long as income does not change).

Rosen's familiar diagram is shown in the upper part of Figure 1. The diagram shows one characteristic, z_i,[5] and holds the other characteristics constant. The equilibrium hedonic price schedule is represented by the darker line and could also be convex or linear. For the moment, the small circles on the price schedule can be ignored. Level curves for the bid function (or bid contours) for one individual are shown. The contours differ by the level of utility, with increasing subscripts on θ representing higher levels of satisfaction. Other individuals would have other families of bid contours (and tangencies) depending on their income and socio-economic attributes. The individual shown would choose to locate where z_0 units of the characteristic were available and pay $P(z_0)$ for the house.

Since the utility function is monotonically increasing in x, $u = U(z_1, \ldots, z_n, x)$ can be inverted to get

$$x = W(z_1, \ldots, z_n, u). \tag{13}$$

This gives the quantity of the numeraire necessary to reach u for given quantities of the characteristics, z. Thus, the numeraire is a function of the characteristics and the level of utility. For a given level of utility, this is the equation for an indifference surface in terms of the numeraire. It is also true that

$$x = W(\mathbf{z},u) = m - \theta(\mathbf{z},u,m) = -\theta^*(\mathbf{z},u). \tag{14}$$

Thus, there is a close relationship between changes in the bid function and changes in the numeraire that will hold the level of utility constant. This will be useful in developing welfare measures.

The derivative of the bid function with respect to a characteristic, $\theta_{z_i}(z,u)$, is called the marginal bid function. The first-order conditions for the utility maximization problem imply that $\theta_{z_i}(z,u) = p_i$ for all i and $P(\mathbf{z}) = \theta(\mathbf{z},u,m)$ from the budget constraint. Each individual's utility function depends on the vector $\boldsymbol{\alpha}^j$ of that individual's socio-economic variables. As long as there are these differences in the individuals' utility functions and income, the bid functions will also differ between individuals. This results in the house characteristics, house prices, and marginal characteristic prices differing between individuals.

The consumers bidding for the houses establish a short-run equilibrium in a market. In equilibrium, each individual has the winning bid for one house[6]. No one could improve their level of satisfaction by increasing their bid for some other house above the current resident's bid. The hedonic model assumes that the resulting winning bids can be modeled as a continuous equilibrium hedonic price schedule depending on the characteristics. On the other hand, the discrete choice models assume the distributions of winning bids and the underlying characteristics are discontinuous. Such models will be discussed subsequently.

Because of the potential non-linearity of the hedonic price schedule, the consumers may face nonlinear budget constraints. In this case the marginal prices of the characteristics are not parameters, but rather are determined by the choices consumers make. The theory of the comparative statics in the presence of nonlinear budget constraints has been developed by Blomquist (1989) and Edlefsen (1981). A few of the results (Engel and Cournot aggregation, Slutsky decomposition) for demand systems with linear budget constraints continue to hold with nonlinear budget constraints after natural generalizations. However, this is not true for homogeneity, negative semidefiniteness, and symmetry. In addition, the wealth of econometric tools that have been developed with linear constraints may not be useful with nonlinear constraints.

Fortunately, the complexities of nonlinear budget constraints can be avoided in most environmental property value studies. With knowledge of the nonlinear hedonic price function and the house where an individual chooses to locate, one can infer the marginal prices that an individual faces. We know from equation (6) that the individual's marginal rate of substitution between a characteristic and the numeraire will be equal to that marginal price:

$$p_i^{\,j} = \frac{U_{z_i}^{\,j}(z,x)}{U_x^{\,j}(z,x)} \tag{15}$$

By linearizing the budget constraint, an uncompensated inverse demand for characteristic *i*, $P_i^j(\mathbf{z},x)$, can be derived in the following way.

In environmental economics we are interested in learning about the underlying utility function. An individual would choose the same house with a linearized budget constraint where the prices for the characteristics are parameters equal to the marginal prices at the chosen house and income has been adjusted to allow the same bundle of goods to be purchased (Palmquist, 1988). Income has to be adjusted because the actual expenditure on housing with the true equilibrium price schedule will differ from the expenditure if the price of each unit of a characteristic were equal to the marginal price:

$$m_a = m - P(\mathbf{z}) + \Sigma p_i z_i \tag{16}$$

where m_a is adjusted income.

These parametric prices and adjusted income are closely related to the concepts of virtual prices and virtual income used in the rationing literature. By using virtual prices and income, much of the existing literature on demand systems can be used. For welfare measurement when there are changes in quantities of characteristics, knowledge of the direct utility function is quite useful even without forecasting how the equilibrium price schedule will change.

The information in the upper part of Figure 1 can also be shown in a typical indifference curve diagram that will be useful later. The lower part of Figure 1 again features characteristic z_i and holds the other characteristics constant, but now the numeraire is measured on the vertical axis. The true budget constraint (equation 2) is shown with the darker line. An indifference curve with the usual properties represents the maximum attained level of satisfaction, u_1, at (z_0,x_0). The marginal price or virtual price, p_z is the slope of the indifference curve at that point. The virtual income, m_a, is also shown, as is the linearized budget constraint.

In estimating inverse demands, prices are usually normalized by income because a linear budget constraint is unchanged if all prices and income are multiplied by a constant. The Hotelling-Wold Theorem yields uncompensated inverse demands. In contrast, for the hedonic model prices and income are normalized by the price of the numeraire x. Nevertheless, equation

(15) and the linearized budget constraint can yield an analogous Hotelling-Wold identity. The linearized budget constraint is

$$m_a = \Sigma p_i z_i + x \tag{17}$$

where m_a is the virtual income. Substituting equation (15) in the budget constraint yields

$$m_a = \sum \frac{U_{z_i}}{U_x} z_i + \frac{U_x}{U_x} x = \frac{\sum U_{z_i} z_i + U_x x}{U_x}, \tag{18}$$

so
$$U_x = \frac{\sum U_{z_i} z_i + U_x x}{m_a}. \tag{19}$$

Substituting equation (15) and reorganizing yields

$$P_i^j(z, x) = \frac{m_a \cdot U_i}{\sum U_{z_i} z_i + U_x x} \tag{20}$$

the uncompensated inverse demand for characteristic z_i. The researcher could specify the utility function and derive the estimating equations using equation (20). Because of the linearization of the budget constraint the estimated demands would have limited usefulness on their own, but they could be used to recover the parameters of the utility function and evaluate changes in characteristics.

Empirically, first the equilibrium price schedule is estimated using data on the prices of houses and their characteristics.[7] From this, the marginal prices of the characteristics are calculated for each individual and the relevant characteristics. Then the quantities of the characteristics and the marginal prices of the characteristics are combined with data on income and other socio-economic variables to estimate the inverse demands for the characteristics generated from equation (20). Ideally, the inverse demands for all characteristics would be estimated simultaneously as a system. However, because of the large number of characteristics this usually is not possible, and separability assumptions may have to be invoked to reduce the dimensionality of the problem. System methods such as the direct translog (e.g., Palmquist and Israngkura, 1999) and single equation estimation (e.g., Boyle, et al., 1999, 2000) have been used.

Discrete Choice Models

The random utility models view the consumer's decision in a somewhat different light. The consumer still cares about the characteristics of the houses but makes a discrete choice between houses rather than a continuous choice about the levels of the characteristics. The utility function and the bid function are identical to those in the hedonic model. The difference is in the choices the market presents to the consumer and the way in which the stochastic elements are assumed to enter.

The hedonic model seeks to predict house prices and assumes that there is a stochastic element that the researcher cannot observe. This may be due to omitted characteristics (which might introduce bias in the coefficient estimates), or it might be due to aspects of the negotiations between the buyer and seller that diverge from the competitive model because of search costs, etc. (which might be uncorrelated with the observed characteristics). The hedonic regression reveals the expected value of the house given the characteristics and the expected contribution of each of the characteristics to that value. These expected marginal prices are used in the second stage.

The random utility models, on the other hand, take the sales prices as representative of market prices available to all consumers. This makes sense if the error in the hedonic equation is due to omitted characteristics, but may be less appropriate if search costs account for the unexplained part of prices. A given consumer makes a discrete choice among houses to maximize utility. The individual perceives the satisfaction he or she would receive from each of the N houses that is available. For example, house k would provide $u^k=U(\mathbf{z}^{*k},x^k) = U(\mathbf{z}^{*k},m-P^k)$, where U is the individual's true utility function, z* is a vector of all the characteristics of the house that are important to the consumer, and P^k is the market price of that house. The randomness is introduced because the researcher does not know the true utility function or all of the characteristics. The consumer selects the house that provides the highest level of satisfaction, and receives $u = \max(u^1,...,u^N)$.

This can be visualized in the context of the diagrams used earlier. In the top diagram of Figure 1, ignore P(z) and assume instead that the only houses available are at the little circles. The individual considers each of the houses before choosing the one at z_0. Each of the other houses lies on a higher bid contour, which implies a lower level of satisfaction. The same concept is shown by the little circles in the lower diagram in Figure 1. The budget constraint only exists at those circles, and one can see that all but the chosen house would be on lower indifference curves. In actuality, the price-characteristics combinations would not all be on the hedonic price schedule but rather would be scattered in the neighborhood of it because of the error term in the hedonic regression. The hedonic model assumes the utility function is tangent to the hedonic price schedule, while the random utility model uses the actual prices. If the prices are truly available to all consumers, the prices that lie below the hedonic price schedule provide information on the shape of the bid contours in the random utility model that would not be available in the hedonic model. On the other hand, if transactions and search costs mean that each potential buyer only considers a subset of the houses used in the hedonic, then the curvature of the bid functions may be exaggerated in the random utility model if all the houses are consider to be alternative choices.[8]

The theory of the random bidding model, which was developed by Elickson (1981) and modified by Lerman and Kern (1983), is quite different from the models discussed so far. It uses econometric techniques that are very similar to random utility models, but the theory is not based on an individual maximizing utility by choosing the best house as the previous models are. Rather it models the equilibrium in the housing market. It seeks to predict the type of individual who will have the winning bid for a house with given characteristics in an equilibrium allocation. In fact, as originally proposed by Elickson, the model could not be used for welfare measurement. Lerman and Kern modified the model to utilize the information in the observed price (which would be equal to the winning bid), so that welfare measurement would be possible.

Theory of Hedonic Welfare Measurement

The goal of most hedonic estimation in environmental economics is welfare measurement when there are changes in the quantities of some of the environmental characteristics. However, the techniques that must be used differ greatly depending on the type of environmental change, the transaction costs in the housing market, and the time period considered. The environmental change might affect only a small number of properties relative to the size of the market. This is the case of a localized externality (Palmquist, 1992a), and the hedonic price schedule will not be changed. On the other hand, the environmental change may affect a large part of the market, resulting in a change in the price schedule. The other dimension in which the welfare measurement techniques will differ is whether or not households move in response to the environmental change. They may not move because of the costs involved in moving or the short time period considered. Depending on the assumptions, the effect on consumers may be a change in quantities or prices.[9]

Localized externalities

If the hedonic price schedule does not change because of the limited scope of the environmental change and there are no transactions or moving costs, the consumers of housing services are able to get exactly the same bundle of characteristics in a house for the same price as before. Thus, their level of utility will be unchanged, and their willingness to pay or accept will be zero. However, the owners of the houses with the environmental changes will experience capital gains or losses, and this would represent the amount the owners would be willing to pay or accept for the change. The capital gains or losses can be forecast from the unchanging hedonic price schedule. This is the simplest case for hedonic welfare measurement.

The analysis can also be done for a localized externality when there are transactions and moving costs without estimating the bid functions in some cases (Palmquist, 1992b). If those costs are low enough, the consumer will move to a house providing the original characteristics, the consumer's welfare cost will be equal to the transactions and moving costs. This will partially offset the gain to the owner. If the transactions and moving costs are high enough that the resident chooses not to move, these costs still provide an upper-bound to the welfare cost. If the costs cannot be quantified, then one must estimate the underlying bid functions (Bartik, 1986).

Non-localized externalities without moving

If there is a major environmental change, the localized techniques above are not appropriate. However, there may be reasons that the consumers do not move in response to the environmental change. For example, the transactions and moving costs may be substantial compared to the benefits of relocating because of the environmental change. Under such circumstances, the welfare measure must be based on the preference structure of the consumers and deal with quantity changes. A vast majority of the research on welfare measurement has dealt the welfare effects of changes in prices. In that case there is an intuitive metric for the gains or losses, the change in income that would leave the individual at a given utility level.

These measures can be defined implicitly using the indirect utility function or explicitly using the expenditure function. For example, if a price falls, income can be reduced to maintain the original level of satisfaction. If prices are normalized by income, then effectively all normalized prices are scaled up to maintain the utility level. This maintains the new relative prices, changes income, and allows the consumer to optimize.

In quantity space, the analogous operations would involve the direct utility function and the distance function.[10] An increase in the quantity of one or more of the characteristics would allow the consumer to reach a higher level of satisfaction. This consumption bundle could be scaled back to return the individual to the initial level of satisfaction. This is the welfare measure for quantity changes proposed in Palmquist (1988) and Kim (1997). However, compensating by scaling the consumption bundle is only one way of returning to a given utility level. The duality between the expenditure function and the distance function is mathematically elegant, but it doesn't lead to the most useful welfare measures. In the short-run, if one or more of the environmental characteristics of a house change, the resident will not move. The bundle of characteristics cannot be scaled back proportionally to return the individual to a level of satisfaction the way normalized prices can be if income is changed.

The compensation returning the individual to a level of utility can be done in an infinite number of ways. Luenberger (1992) introduced a new dual form for the utility function, the benefit function. The benefit function selects a reference bundle of goods, **g**. Any vector of goods **X** (not to be confused with the numeraire, x, used here) could be increased or decreased by some multiple of **g** to reach a reference level of utility. Chambers, Chung, and Färe (1996) have extended Luenberger's benefit function to a production context and introduced an alternative name for it, a directional distance function. Formally, the benefit function b(X,u;g) is defined as

$$\mathrm{b(X,u;g)} = \sup_{\beta}\left\{\beta \in R^1 : (X - \beta g) \in R^n{}_+, U(X - \beta g) \geq u\right\} \tag{20}$$

While this has been called a directional distance function, it should be remembered that it is based on a directional translation **X**, whereas the usual distance function uses scaling to the origin. If **g** is taken to be equal to **X**, then

$$\mathrm{b(X,u;X) = 1\text{-}1/D(X,u)} \tag{21}$$

where D(X,u) is the usual distance function, but for other vectors **g** the two distance functions are very different concepts.

While Luenberger's benefit function achieves generality by not restricting **g**, the benefit function also has little policy relevance unless **g** is selected appropriately. In the hedonic model, the numeraire good x provides the natural choice for **g**: (0,...,0,1) where the last element is the x direction. The compensation should be measured in units of x. Because of the normalization, this provides a money measure, and it holds constant the quantities of the characteristics, which is appropriate for the policy changes being analyzed. [11] In what follows **g** always will be taken to be this vector.

Suppose that an initial vector of characteristics, $\mathbf{z}^0$, has one or more of its elements changed to $\mathbf{z}^1$. The initial level of utility is $U(\mathbf{z}^0,x^0) = u^0$, which can be expressed in terms of the bid function as $U(\mathbf{z}^0,m\text{-}\theta(\mathbf{z}^0,u^0;m)) = u^0$, and in terms of the indifference surface function as $U(\mathbf{z}^0,W(\mathbf{z}^0,u^0)) = u^0$. The benefit function initially is $b(\mathbf{z}^0,x^0,u^0) = 0$, where **g** is omitted and is understood to be (0,...,0,1). After the characteristics change, the utility in terms of the bid function is $U(\mathbf{z}^1, m\text{-}\theta(\mathbf{z}^1,u^0;m)) = u^0$, in terms of the indifference surface function it is $U(\mathbf{z}^1, W(\mathbf{z}^1,u^0)) = u^0$, or in terms of the benefit function it is, $U(\mathbf{z}^1, x^0\text{-}b(\mathbf{z}^1,x^0;u^0)) = u^0$. As long as actual income does not change, the monetary welfare measure for the change in the quantity of the characteristics can be expressed in three alternative ways:

$$b(z^1,x^0,u^0) = \theta(\mathbf{z}^1,u^0;m) - \theta(\mathbf{z}^0,u^0;m) = W(\mathbf{z}^0,u^0) - W(\mathbf{z}^1,u^0) \qquad (22)$$

The use of the benefit function for welfare measurement in a non-hedonic framework is discussed in Luenberger (1996). The use of the bid function is discussed diagramatically in Palmquist (1991). The use of $W(\mathbf{z},u)$ has not been previously suggested.

Equation (22) holds utility at its initial level. If one were to follow the nomenclature of Hicks (1956), this would be called "compensating valuation." If utility were held at its new level, this would be "equivalent valuation." In the literature today, the commonly used terms for these quantity welfare measures are compensating and equivalent surplus. However, when Hicks introduced these terms, he was referring to a distinctly different concept that was a welfare measure for changes in prices. Because the use of the terms compensating and equivalent surplus for quantity changes is so widespread, the valuation and the surplus terms will be taken to be interchangeable. Compensating valuation and equivalent valuation will be denoted by CV_q and EV_q.

A diagram of an indirect utility function provides additional insights into these welfare measures. Figure 2 highlights one of the characteristics, holding the other characteristics constant in the background. The budget constraint has been linearized by using marginal prices and virtual income, m_a. Typically prices are normalized by income, so $u = V(r_1,r_2)$ where r_i represents a price normalized by income. In that case, level curves of the indirect utility function would be shown in a diagram with normalized prices on both axes. In the hedonic model we have normalized by the price of the numeraire. The indirect utility is $u = V(p_z,m_a)$. Therefore, Figure 2 has p_z on the horizontal axis and m_a on the vertical axis. The level curves are upward sloping because higher prices would require higher incomes to maintain the level of satisfaction. The level curves are increasing at a decreasing rate because the consumer can substitute away from the characteristic as the price rises. Higher level curves represent higher levels of satisfaction ($u_1 > u_0$ in the diagram).

Linearized budget constraints, $m_a = p_z z + x$, can also be shown in the diagram. For a given level of z and x, as p_z increases, m_a must increase linearly to satisfy the budget constraint, and the slope is equal to z. If $p_z = 0$, then $m_a = x$. Typically, the dual problem is to select income-normalized prices to minimize the indirect utility function subject to the budget constraint. For the hedonic problem, one selects p_z and m_a to minimize $V(p_z, m_a)$ subject to the constraint. Thus given z_0 and x_0 the optimum is at *a*, the marginal price is p_0, the virtual income is m_a, and the level of utility is u_0. If there is an increase in z to z_1, the budget constraints rotates up, the optimum shifts to *b*, and the minimum value of the indirect utility function increases to u_1. In order to return to u_0, the quantity of the numeraire must be decreased from x_0 to x_1, and the tangency is at *c*. This maintains the new level of z and the original level of satisfaction. The amount by which x can be reduced, $W(\mathbf{z}_0, u_0) - W(\mathbf{z}_1, u_0)$, is the monetary welfare measure we are seeking. In terms of the bid function, $m_a = m - \theta + \Sigma p_i z_i$, so $\theta = m - m_a + \Sigma p_i z_i$. Since m and m_a don't change, the difference in bids, $\Delta\theta = \theta_1 - \theta_0$, can also be seen in the diagram.

Welfare measurement for changes in prices is often done by integrating to the left of compensated demands. There is no ambiguity because compensation in terms of income is the natural choice. Unfortunately, welfare measurement for changes in quantities using compensated inverse demands is not as clear. In fact, there are a variety of compensated inverse demands that could be used. It depends on what type of compensation is used to return the individual to a given level of satisfaction.

This can be clarified using the diagrams in Figure 3. The top diagram is an indifference curve diagram showing the initial position of the individual at z_0, x_0. The virtual price of z or the marginal willingness to pay for z is represented by the slope of the indifference curve at that point (the slope of the line tangent to the indifference curve at z_0,x_0). In the upper diagram, the various points z_i,x_i are labeled with the number i and the tangent lines are labeled with p_i at each end of the line. In the lower diagram p_i represents the price itself. Now assume that the level of the characteristic changes exogenously to z_1. The virtual price changes to p_1, the slope at the new tangency. The lower diagram shows the uncompensated inverse demand (labeled $p_z(z,x)$). It goes through the two points (z_0,p_0) and (z_1,p_1) where the prices are the slopes of the tangents. If x is normal, it will be downward sloping.

What sort of compensation should be used to return the individual to u_0? The most common compensation leads to Antonelli inverse demands (see Deaton, 1979). However, note that the compensation using the distance function uniformly scales the commodity bundle to the reference level of utility, moving back towards the origin to point 3. The virtual price there is p_3. However, this associates z_1 with a virtual price at a different level of z. A more intuitive form of compensation is in terms of the numeraire, holding z constant. This takes us to the tangency at point 2 with virtual price p_2. This is exactly the type of compensation in the bid function when z changes and u and m are held constant. Thus, the lower diagram also shows this compensated inverse demand or marginal bid function.[12] It is labeled θ_z. If z is normal, $\theta_z(\mathbf{z},u)$ will be more steeply sloped than $p_z(\mathbf{z},x)$. The desired welfare measure can be obtained by integrating under this compensated inverse demand between z_0 and z_1 to obtain the compensating valuation, CV_q.[13] Now consider the Antonelli compensated demand. The virtual price, p_3, is less than p_0 and greater than p_2 if z and x are normal, but we cannot say its relation to p_1. Integrating this demand will yield a higher willingness to pay, but it is based on an odd type of compensation.

The specific techniques for welfare measurement that are appropriate depend on the estimation that is done. One can start with an explicit form for the direct utility or subutility function and derive the estimating equations for a system of uncompensated inverse demands. The specified utility function can be inverted to get W(**z**,u) which can be used in developing welfare measurements.[14] An alternative would be to estimate a system of inverse demands that could have been derived from some unspecified direct utility function and use numerical methods to solve for the welfare measures.[15] The third alternative is welfare measurement when only the inverse demand for a single environmental variable is estimated. The techniques for an inverse demand and a quality change are similar to those developed by Hanemann (1980), Hausman (1981), and Bockstael, Hanemann and Strand (1984) for an ordinary demand and a price change.[16]

For example, suppose one estimates an uncompensated inverse demand using marginal prices from hedonic regressions and a log-linear functional form,

$$\frac{\partial P}{\partial z_1} = e^{\alpha} z_1^{\beta_1} z_2^{\beta_2} x^{\gamma} , \tag{23}$$

where z_1 is the environmental variable of interest, z_2 represents all other housing characteristics, x is the numeraire (income minus the house price)[17], and the Greek letters are parameters. A standard duality result converts Marshallian demands to Hicksian demands by substituting the expenditure function for income. A similar change is possible here. Using the fact that $P_z = \theta_z$ and by substituting $W(z_1,z_2,u)$ for x and writing this in terms of the bid function, equation (23) becomes an ordinary differential equation in θ,

$$\frac{\partial \theta}{\partial z_1} = e^{\alpha} z_1^{\beta_1} z_2^{\beta_2} (m-\theta)^{\gamma} \tag{24}$$

This differential equation is separable, so it can be solved analytically for θ,

$$\theta = m - \left[-(1-\gamma)\frac{e^{\alpha} z_1^{\beta_1+1} z_2^{\beta_2}}{\beta_1+1} + C \right]^{\frac{1}{1-\gamma}}, \tag{25}$$

where C is the constant of integration that will depend on u but not z_1. Setting C equal to u, and inverting yields

$$u = (m-\theta)^{1-\gamma} + (1-\gamma)\frac{e^{\alpha} z_1^{\beta_1+1} z_2^{\beta 2}}{\beta_1+1}. \tag{26}$$

The condition for economic integrability is

$$\frac{\beta_1}{z_1} - \frac{\gamma}{m-\theta} P_z < 0. \tag{27}$$

If β_1 is negative and γ is positive, as would be expected, the integrability condition is fulfilled, although this is not necessary. This provides sufficient information to calculate the compensating valuation and equivalent valuation (compensating and equivalent surplus),

$$\begin{aligned} CV_q &= \theta(z_1,u_0) - \theta(z_0,u_0) \\ &= m - \left[-\frac{1-\gamma}{\beta_1+1}\left[\left(e^{\alpha} z_{11}^{\beta_1+1} z_2^{\beta_2}\right) - \left(e^{\alpha} z_{10}^{\beta_1+1} z_2^{\beta_2}\right)\right] + (m-P_0)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} - P_0 \end{aligned} \tag{28}$$

$$\begin{aligned} EV_q &= \theta(z_1,u_1) - \theta(z_0,u_1) \\ &= P_0 - m + \left[-\frac{1-\gamma}{\beta_1+1}\left[\left(e^{\alpha} z_{10}^{\beta_1+1} z_2^{\beta_2}\right) - \left(e^{\alpha} z_{11}^{\beta_1+1} z_2^{\beta_2}\right)\right] + (m-P_0)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \end{aligned} \tag{29}$$

where z_{10} is the level of the environmental characteristic before the improvement and z_{11} is its level after the change, and P_0 is the initial price of the house. P_0 appears in both equations (28) and (29) because the uncompensated change holds x constant, which implies the m - $\theta(z_1,u_1)$ equals m - $\theta(z_0,u_0)$. Since income does not change, this means $\theta(z_1,u_1)$ equals $\theta(z_0,u_0)$ which, in turn, equals P_0 from the consumer's optimization.

The cases of a semi-log and linear functional form are presented in the appendix to this chapter.

In the case of environmental changes where residents do not move, it is also possible to derive welfare measures from discrete choice models. Random utility models estimate the parameters of the utility function, which can be inverted to derive the bid function. Random bidding models derive the parameters of the bid function directly. In either case, once the bid function is estimated, the techniques discussed above can be used to develop the welfare measures.[18]

Non-localized externalities with moving

The most difficult situation for welfare measurement using property values is when there is a major environmental change, the hedonic price schedule changes, and residents move. Most property value studies are used to develop welfare estimates before the environmental change takes place.[19] This means the researcher must forecast the change in the hedonic price schedule that will result from the environmental change. It is difficult to forecast a change in a scalar price for a homogeneous product, let alone forecasting the change in the entire price schedule for housing. Because of this difficulty, one alternative is to use the measures discussed in the last section as lower bounds to the true welfare measure (Palmquist, 1988). How tight this bound will be depends on many factors and cannot be known *ex ante*.

Bartik (1988) has shown that the hedonic price schedule might provide an upper bound on the value of an environmental change, although this will not always be true. Because the bid contour is tangent to the hedonic price schedule from below, it is always true that the willingness to pay along that bid contour is necessarily less than the price change predicted by the initial hedonic. However, Bartik is considering the more general case where individuals move and the hedonic price schedule changes. He shows that the initial hedonic schedule may still provide some information about benefits. However, since this is not always an upper bound and we don't know how good an approximation it is, it is important that simulations be done to learn under what circumstances the approximation is accurate. If close approximations are possible using only the initial hedonic schedule, it will greatly reduce data requirements and further enhance the usefulness of hedonic techniques.

There is a promising alternative that has not yet been implemented in a hedonic framework.[20] As discussed earlier, the real estate market price equilibrium is established by a process that allocates each house to the highest bidder. The houses differ in their characteristics, and the consumers differ in their socio-economic attributes. Matching individuals to houses and determining the equilibrium price is a typical assignment problem. Environmental economists do not typically worry about this assignment problem because the market solves it for us. However, in a valuable pair of articles, Cropper, et al (1988, 1993) have used an assignment model developed by Wheaton (1974) to simulate a hedonic market in order to better understand estimation issues. A similar assignment model could be used in a different framework to develop welfare measures.

We can observe the real estate market before the environmental change. Typical techniques can be used to estimate the hedonic equation and, at the second stage, the preferences of the consumers. Once the *ex ante* distribution of houses and individuals is known, a hypothetical environmental change can be introduced. The assignment model can then be used to allocate the consumers to the houses with the new characteristics and estimate the new hedonic price schedule. Now the change in the quantity of the environmental characteristic has been transformed into a change in the prices of houses. With this forecast of the *ex post* hedonic price schedule, techniques such as those discussed in Palmquist (1988) can be used to forecast welfare changes.

If one were using discrete choice models, one would also have to forecast the new prices of houses using the assignment model in order to introduce both price and characteristics changes into the welfare measures.

Differentiated Factors of Production and Land Markets

The models discussed so far deal with residential properties, and most empirical studies have used data on residential properties. However, only a small fraction of land is used for residential purposes, and there is the potential to use other types of land values to reveal some types of environmental benefits or costs. However, this land or property is a differentiated factor of production in this case, and the bids are from firms rather than consumers. The Rosen model can be modified to deal with this case, and the welfare measures are somewhat different (see Palmquist, 1989).

To date most such studies have dealt with agricultural land values.[21] Some of these studies have considered characteristics that have environmental implications, such as erosivity or the drainage of wetlands. Examples include Miranowski and Hammes (1984), Ervin and Mill (1985), Gardner and Barrows (1985), and Palmquist and Danielson (1989). There have been more recent hedonic studies of agricultural lands, but they have followed similar methodologies. An exception is a recent article by Bastian, et al. (2002) that has used variables derived from a geographical information system to improve the spatial specification of the characteristics of agricultural lands. They find that non-agricultural characteristics can have a significant effect on rural land prices.

III. ESTIMATING THE HEDONIC PRICE SCHEDULE

The most widely used of the property value models is the first stage of the hedonic model. This methodology uses data on sales prices for properties and the characteristics of the properties. The type of data necessary for these studies is often readily available. Although some of the initial hedonic studies used census data, which are aggregated spatially and use owner-estimates of prices, today almost all studies use micro data. These are available from a variety of sources, including multiple listing services, assessor's offices, and companies specializing in real estate data. The first-stage does not require information on the individuals living in the houses, which greatly ease the data collection problems. The econometric issues are also simpler for the first stage, although there are some important estimation issues that must be addressed.[22]

Extent of the Market

The hedonic price schedule is the equilibrium price schedule in a market. The researcher must be confident that the observations come from a single market. Early hedonic studies used market areas that varied from separate markets every few square blocks to nationwide.[23] It is difficult to determine the appropriate size of the market using statistical or econometric tests. The usual F-tests assume the equations estimated are correctly specified. As was pointed out earlier, theory provides almost no guidance as to the specification of the hedonic equation. If separate hedonic equations were run for two locations in a city and an F-test rejected the hypothesis that the coefficients were equal, this could be because there were two separate markets. However, it also might be because the functional forms were not appropriate or the equations were otherwise misspecified. Such misspecifications need not be significant economically, even if they are significant statistically.[24] Given the large sample sizes that are typical in hedonic studies today, F-tests will almost always reject combining areas in hedonic regressions.

For this reason, it is more appropriate to think about the types of transactions that are taking place in the area. As discussed in footnote 2, if there are a reasonable number of consumers who would consider the alternative areas, then those areas can be treated as a single market, even if many people only consider one or the other. If there are almost no consumers who would consider the two areas viable substitutes, they can be treated as separate markets. Most researchers today take an urban area to be a single market, although in large, consolidated areas such as Los Angeles one might choose to treat this as several markets. Even if one considers an urban area to be a single market, this does not preclude studying just a subset of that market if that is appropriate for the research question being asked. Problems only arise when separate markets are treated as one. In fact, there are often advantages to using smaller areas if the environmental issue can be addressed within that area (e.g., hazardous waste sites, highway noise, etc.). With more limited areas, one can avoid the complexity of fully specifying all the important characteristics that vary over an urban area but not within a neighborhood.[25]

Stability over time

Frequently a researcher would like to combine data over time. This may be because there has been an environmental change, or it may be to increase the number of useable observations. Whatever the reason, this is only valid if the contributions of the various characteristics to the value of a house have been relatively stable over that period. Intuitively, one would expect that consumers' tastes for the various characteristics relative to one another would change more slowly than changes in the overall housing price level, which may vary within and between years. It is possible to do statistical tests of aggregation over time, while allowing for changes in the real estate price index. For example, dummy variables for the year of the sale can be included in the hedonic regression where the dependent variable is the natural log of price. Frequently, though, F-tests reject aggregation over more than a very few years. Ohta and Griliches (1975) have suggested a less stringent guide to aggregation over time in hedonic regressions. As with aggregation over space, one would reject most aggregation over time if one used F-tests. An important consideration is the usefulness of the approximations such aggregation allows. They base their decision on comparing the standard errors of the constrained and unconstrained regressions. If the standard error increases by more than ten percent, they reject aggregation. Based on this guideline, aggregation over longer periods is often acceptable. This issue of aggregation over time is relevant both to hedonic applications and to the repeat sale model to be discussed later.

Functional Form

The functional form of the hedonic equation has to be determined from the data. Early hedonic studies sometimes chose among simple functional forms such as linear, semi-log, and log-linear. As computing costs came down, greater flexibility was introduced. In an influential article, Halvorsen and Pollakowski (1981) suggested the use of the quadratic Box-Cox functional form. This allowed considerable flexibility, albeit local flexibility. Most of the commonly used functional forms were nested within the quadratic Box-Cox, so testing the restrictions was possible. However, computer limitations usually forced the researcher to have the same Box-Cox transformation applied to all the characteristics. If the interest was in a minor characteristic such as air quality, the transformation might be determined largely by other more important characteristics. The wrong transformation of the environmental variable could have a large impact on the environmental welfare measures. Because of this concern, Cassel and Mendelsohn (1985) advocated the use of simpler functional forms. A more flexible alternative was implemented in Israngkura (1994) where a separate transformation parameter for the environmental variables was introduced.[26]

Another potential problem with the quadratic Box-Cox is the possibility that omitted or misspecified variables in the hedonic equation might reduce the desirability of introducing greater local flexibility. This is exactly what Cropper, et al. (1988) found in simulation studies on the accuracy of various functional forms in predicting marginal prices. The quadratic Box-Cox form did well when the estimating equation contained all the characteristics, although not particularly better than the linear Box-Cox. However, when there were omitted or incorrectly measured variables, the quadratic Box-Cox performed poorly. The linear Box-Cox and even simpler forms (linear, semi-log) did best. For the moment, the linear Box-Cox seems to be the best compromise, and having different Box-Cox parameters for the various characteristics is a promising next step.

Nonparametric and Semiparametric Estimation

Since neither the functional form of the hedonic equation nor the distribution of the error term can be known *a priori*, there has been some work on using nonparametric methods in the estimation. With nonparametric regression, the researcher need not specify parametric forms, but there are tradeoffs. The robustness of the estimators comes at the cost of a reduction in the rate of convergence of the estimators. Larger sample sizes are important with nonparametric regression. This problem is aggravated as the number of independent variables increases. As Yatchew (1998) points out, "100 observations in one dimension would yield the same accuracy as 10,000 observations would in two dimensions and 10 billion would in five dimensions."

Obviously, some restrictions are necessary. We will highlight smoothing with kernel estimators and semiparametric regressions.[27]

Suppose the price of a house, P, is hypothesized to be influenced by a vector **z** of *n* characteristics. For a given set of observations of houses, there is an *n+1* dimensional joint density f(P,**z**). We are interested in an unspecified function M(**z**) such that P = M(**z**) + u or E(P|**z**) = M(**z**). This is equal to

$$M(z) = E(P \mid z) = \int \left[\frac{Pf(P,z)}{f_z(z)} \right] dP \tag{42}$$

Nonparametric estimation of this regression can utilize techniques for nonparametric estimation of a density function. The most commonly used method in the housing literature has been kernel estimation. Given *T* observations, one wishes to estimate the underlying density function. The estimated density at a point is a function of the observations near that point, with the weight given to an observation declining with distance from the point of interest. The kernel function assigns non-negative weight to each observation, is usually symmetric and centered on zero, and the weights sum to one. A density function is often used as a kernel function. The normal is a common choice, although there are other possibilities that are used. The particular choice is usually not significant.[28] However, what is important is the dispersion of the kernel, which is referred to as bandwidth or window width.[29] For a univariate distribution let $K(w_j)$ represent the kernel function where $w_j = (x_j - x_0)/h$, x_0 is the point for which the density is to be determined, x_j is a data point (j = 1,...J where J is the number of observations), and h is the bandwidth. The density function evaluated at x_0 is

$$\hat{f}(x_0) = \frac{1}{hJ} \sum_{j=1}^{J} K(w_j) \ . \tag{43}$$

For a multivariate distribution let $w_{jk} = (x_{jk} - x_{0k})$, and

$$\mathrm{K}(\mathrm{w}_{\mathrm{j}}) = \prod_{k=1}^{K} K(w_{jk}) \ , \tag{44}$$

where k = 1,...,K is the index of jointly distributed variables, and K = n + 1 in our case.

Returning to the regression,

$$\hat{M}(z) = \int \frac{P\hat{f}(P,z)}{\hat{f}_{z}(z)} = \sum_{j=1}^{J} P_j r_j(z) \tag{45}$$

where

$$r_j(z) = \frac{K(w_j)}{\sum_{j'=1}^{J} K(w_{j'})} \tag{46}$$

Determining the bandwidth is crucial. There is a tradeoff between bias (which decreases with reductions in bandwidth) and variance (which increases with reductions in bandwidth). Usually a bandwidth proportional to $J^{-1/(4+K)}$ is chosen if a uniform bandwidth is used. A variable bandwidth that is wider in the tails of the observed distribution is sometimes used.[30]

Not surprisingly, nonparametric regressions do not generate the usual parameter estimates obtained with parametric regressions. One can define $\beta_{ji} = \partial M/\partial z_i$ at point *j* as

$$\beta_{ji} = \lim_{h\to 0} \{[M(z+h/2) - M(z-h/2)]/h\} \tag{47}$$

or with an alternative proposed by Ullah (1988) that utilizes the kernels from the estimation. Either of these yields a regression coefficient that varies with the values of the explanatory variables. One can report a single coefficient estimator by evaluating β at the mean of the explanatory variables or by calculating β at each observation and averaging. It is also equivalent to the semiparametric estimates of a single index model, where $\mathbf{z}\boldsymbol{\beta}$ is a linear index that is then entered nonparametrically in the regression (see Pace, 1995, and the references therein).

Semiparametric estimators like the single linear index model improve convergence by imposing additional structure. One can use a partially linear model where, for example, $P = \mathbf{z_a}\boldsymbol{\beta} + M(\mathbf{z_b}) + u$, where $\mathbf{z_a}$ is the vector of characteristics entered parametrically and $\mathbf{z_b}$ is the vector of characteristics entered nonparametrically. This substantially reduces the dimensionality problems. One can also assume additive separability between groups of explanatory variables to reduce the interactions.

There have been several applications of nonparametric and semiparametric estimation to housing. Pace (1993, 1995) has demonstrated nonparametric estimation with three different data sets and also applied a semiparametric estimator (the single index model) to one of them. Anglin and Gencay (1996) use a partially linear model where the dummy variables are entered parametrically and the continuous or count variables are entered nonparametrically. Stock (1989, 1991) also used a partially linear model with the town dummy variables entered linearly. Of these studies, two consider environmental conditions. Pace (1993) used the Harrison and Rubinfeld (1978) data from their study of air pollution. Using the census tract data he found that the nonparametric estimator was better able to deal with some erroneous observations. The averaged coefficient on the air pollution variable was reduced greatly in magnitude, although it was still significant. Stock (1991) studied hazardous waste sites using indexes based on weighted distances to the sites, size of the sites, and publicity about the sites. He considered the benefits from cleanups and found that the choice of the kernel affects the results.

Environmental hedonic regressions must include a substantial number of characteristics of the houses. The environmental variables of interest generally play a minor role in the determination of price, and the equation must be well specified to obtain reliable estimates. This means that completely nonparametric estimation may prove difficult. Fortunately, many years of experience with appraisal techniques provides guidance on the parametric specification with respect to the major characteristics. It is with the environmental variables that such guidance is not available. Entering the environmental variables nonparametrically in a semiparametric specification may yield valuable insights.

Measurement of the Environmental Variables

An important issue for all characteristics is the appropriate measure of the characteristic that will capture the way consumers view the characteristic. This is particularly relevant for the environmental variables since we are attempting to learn exactly that. Typically objective measures (micrograms per cubic, parts per million, etc) are the easiest to obtain. However, if property values are to be affected, the pollutants have to be perceived by the residents. In some cases (for example, certain air pollutants), the objective measures may be highly correlated with what is perceived. In other cases, some transformation of the objective measure may be a better proxy for perceived levels. Often these transformations would suggest increasing marginal damages from the pollutant. However, if residents can take steps to mitigate the damages, marginal damages may not be monotonically increasing. As the level of the pollutant increases, at first the damages would increase at an increasing rate, but once the pollutant levels were high enough to induce averting behavior total damages might increase at a decreasing rate. Transformation that allow such inflection points are sometimes useful but have not often been applied to date. Another technique that may be appropriate in some cases is combining readings on several pollutants into a single index.

A promising alternative to using objective measures is to use surveys to get at the subjective perceptions of the levels of environmental quality. The price of an individual house would not depend very much on the specific perceptions of the resident of that house because the price is the result of a market equilibrium determined by the interactions of all potential purchasers, not just the winning bidder.[31] However, knowledge of the relationship between the average perception of residents and the objective measures could be quite useful. The survey does not have to be of the same individuals or even the same area as the hedonic study.

The usual pollution measures focus on a single measure for each pollutant (annual arithmetic or geometric mean, annual second highest reading, etc.). However, the variability of the pollutant may be as important in some cases. Murdoch and Thayer (1988) study visibility considering not only annual mean but also variance and also the probability that visual range on a given day will be in each of four categories. While the variance has no explanatory power, using the probabilities instead of the mean is desirable. Further investigation of the distribution of pollution readings over time is desirable for fluctuating pollutants.

Other Specification Issues

The issue of multicollinearity is frequently raised in hedonic studies and is often used to explain problems encountered. Concern about these issues led Atkinson and Crocker (1987) to use Bayesian techniques to consider collinearity. They speculate (as do many others) that "(n)umber of rooms, number of bathrooms, house size, and lot size, for example, are highly collinear due to their nearly fixed proportions in housing construction." They are concerned that with collinearity the bias-variance tradeoff can lead to substantial coefficient instability and that mismeasurement bias in some variables will be transferred to the variables of interest. They are really concerned about issues of specification and measurement that would be important even in the absence of degrading collinearity. Graves, et al. (1988) consider these same issues using a different data set and emphasizing environmental variables.

Both articles use 'focus', 'free', and 'doubtful' variables, where focus variables are the variables of interest (environmental variables), free variables are well specified but not of interest (structural variables), and doubtful variables may or may not belong in the equation. They find that altering the specification of the doubtful variables can affect the magnitude and even sign of some focus variables. For example, Graves, et al. (1988) find that the coefficient on visibility changed markedly when different combinations of doubtful variables were included, while the coefficient on total suspended particulates did not. Both articles also considered measurement errors in the focus and doubtful variables. An interesting result in Graves, et al. (1988) is that measurement error in the environmental variables created problems, while errors in the other variables were less of a problem.

These results on variable choice and measurement are potentially quite significant and deserve further research. These issues would be exacerbated by multicollinearity. However, the question remains, is degrading multicollinearity a significant problem in estimating hedonic regressions. There has been some research on this topic. Using the diagnostic techniques developed by Belsley, Kuh, and Welsch (1980), Palmquist (1983) estimated hedonic regressions for 14 cities with an extensive set of variables including four air pollutants. Only in one of the cities was there collinearity between a pair of the pollutants (one out of 39 possible pairs). There was collinearity between a pollutant and a non-pollutant variable in three cases (out of 53 possible cases). There was more frequent collinearity among the neighborhood variables, which were census tract data, but the environmental variables were not involved.

Spatial Econometric Techniques

In the past, almost all environmental property value studies assumed that the disturbances were spherical. The possibility of heteroskedasticity was occasionally considered, but the tools necessary to allow spatial autocorrelation were difficult to implement. Recently, though, exciting progress has been made in spatial econometrics, and there have been interesting implementations in urban and regional economics and, more recently, in environmental economics.

Allowing for serial correlation with time series data has been common for many years. This difference in evolution between autocorrelation techniques with time-series and cross-sectional econometrics is because of the difference in the complexity of the problem. With time-series data, the autocorrelation is one-dimensional (the time dimension) and it may be uni-directional (the past may influence the present but not vice versa). With cross-sectional data, the error term for a house can be correlated with the error terms for houses in any direction. The problem is at least two-dimensional (three-dimensional in the case of high-rises). To make matters worse, the relationships are bi-directional. The natural ordering provided by time is not available spatially.

A different econometric issue is also considered in spatial econometrics, autoregression. It is possible that the price for a given house is influenced by the prices of other houses nearby. If the prices of these other houses are included in the regression, they are correlated with the error term with all the attendant implications.

Spatial autocorrelation and spatial autoregression are distinct concepts, although they are often discussed together. Spatial autocorrelation is also referred to as spatial error dependence or a spatially dependent error term.[32] In contrast, spatial autoregression is also called spatial lag dependence, structural spatial dependence, or a spatial autoregressive dependent variable process.

With spatial errors there are two major approaches that have been followed. It is possible to specify the error covariance (or correlation) matrix directly. Anselin and Bera (1998) refer to this as "direct representation," following the usage in other fields. Obviously all elements of the correlation matrix cannot be estimated independently, so the correlation is generally hypothesized to be a function of the distance between the two observations. This function, the correlogram, is usually assumed to take one of three forms.[33] For early examples of this method applied to housing see Dubin (1988, 1992) (see also Dubin, 1998a,b). Unfortunately, this method cannot yet deal with the large data sets that are commonly used for property value studies.

The alternative to direct representation is the lattice approach. In this approach the error for each house is assumed to be influenced by the errors at some neighboring houses but not all houses in the sample (as is done with direct representation). This approach is preferable with data that are spatially aggregated (census tract, counties, etc.) because of the discrete observations, but it is also the most common technique used with disaggregated data. The method can be used with much larger data sets than direct representation. With the lattice approach, a spatial weights matrix is used to specify the neighbors and the weights they receive. An example is a spatial weights matrix where element w_{ij} is equal to 1 if observations i and j share a common border or are within a certain distance of each other ($i \neq j$), and 0 otherwise.[34] One can also have the weights decline with distance, reaching zero at a given distance.

More formally, spatial autocorrelation is modeled as

$$P = z\beta + \varepsilon \quad \text{where } \varepsilon = \lambda W\varepsilon + u. \tag{48}$$

In this equation, P is a vector of house prices, z is a matrix of characteristics, β is a vector of coefficients, and ε is vector of random error terms. W is the spatial weights matrix discussed above, λ is a scalar parameter to be estimated, and u is a vector of random error terms with mean equal 0 and variance-covariance matrix $\sigma^2 I$. Rewriting equation (48) yields

$$P = z\beta + (I - \lambda W)^{-1}u \tag{49}$$

and so

$$E[\varepsilon\varepsilon'] = \sigma^2[(I - \lambda W)' (I - \lambda W)]^{-1}. \tag{50}$$

Alternatively, the spatial autoregression model is

$$P = \rho WP + z\beta + u \tag{51}$$

where ρ is a parameter to be estimated. Here the prices of some neighboring houses influence the price of any given house. The spatial lag term WP will be correlated with the error term. Equation (51) can be rewritten as

$$P = (I - \rho W)^{-1}z\beta + (I - \rho W)^{-1}u. \tag{52}$$

Comparing equations (49) and (52) makes clear the similarities and differences in the two models.[35]

The spatial weights matrix must be specified by the researcher, and only λ or ρ is estimated. A good deal of judgment is involved in specifying W, and many articles estimate the models with several versions of W. The researcher must decide which covariances should be nonzero. Sometimes a specific number of nearest neighbors are given a nonzero covariance, while an alternative would be specify some distance where the covariance goes from nonzero to zero. Bell and Bockstael (2000) and Can (1992) use the inverse of distance with a cutoff distance (as well as some other specifications). The potential problem with this specification is the discontinuity in the weight at the cutoff distance. The inverse of the square of distance has also been used (Can, 1992), but with a cutoff distance the same problem arises. An alternative that avoids the discontinuity is used by Pace and Gilley (1997). They use the maximum of 1 -

(d_{ij}/d_{max}) and 0, where d_{ij} is the distance between house *i* and house *j* and d_{max} is the cutoff distance. Bell and Bockstael (2000) have recently introduced an interesting modification of weights that cutoff at a certain distance. Instead of estimating a single spatial autocorrelation parameter as in equation (48), they estimate three parameters by introducing three weights matrices. Each of the three weights matrices has nonzero elements for different ranges of distances between the houses. The three spatial correlation parameters decline with distance, as would be expected.

The estimation of these spatial models raises some interesting issues. Most researchers have used maximum likelihood methods. However, the likelihood function involves the Jacobian $ln||I - \lambda W||$. This involves repeatedly taking the determinant of an NxN matrix (where N is the number of observations) or finding the N eigenvalues of the spatial weights matrix (Ord, 1975). Either of these can be quite difficult or impossible as N becomes large, although this limit is being relaxed with increased computing power and sparce matrix techniques. Pace and Berry (1997a, 1997b) have been important contributors to the use of sparce matrices for spatial econometrics, and estimation with sample sizes over 100,000 is now feasible.[36]

There are alternatives to maximum likelihood estimation that are used. For the spatial autocorrelation case, if λ were known, familiar generalized least squares techniques could be used. However, the estimation of λ is more complex than with time-series (see Anselin and Bera, 1998). For spatial lag models, the endogeneity of the prices of neighboring properties can be considered using instrumental variables. Typically, the characteristics of the neighboring houses are used as instruments for their prices. Finally, Kelejian and Prucha (1999) have developed a generalized method of moments estimator for the spatial autocorrelation case. Bell and Bockstael (2000) have applied this estimator. This allows them to specify the three spatial weights matrices discussed above, something that currently would not be possible with maximum likelihood estimation.

There have now been a number of applications of spatial econometrics to urban property value studies, many of which have been cited above. Closer to environmental economics, a series of studies by Nancy Bockstael and her graduate students have used spatial econometrics to model land conversion and property values. Many of these studies have concentrated on open space and landscape indices (see Bockstael, 1996; Geoghegan, Wainger, and Bockstael, 1997; Geoghegan and Bockstael, 1995; and Bell and Bockstael, 2000).

There are a few studies that consider typical environmental variables like air and water pollution. Pace and Gilley (1997) took the Harrison and Rubinfeld (1978) data set and reestimated the model using spatial econometric techniques. These data are at the census tract level and have been used previously by Belsley, Kuh, and Welsch (1980). Pace and Gilley found that the spatial techniques reduced the sum-of-squared-errors by about 45 percent. The coefficient on the age of the house changed from positive to negative and significant, as would be expected. Finally the coefficient on the air pollution measure fell by about 38 percent but was still significant.

Kim, Phipps, and Anselin (1998) use spatial econometric techniques to study the effects of air pollution on property values in Seoul. They discuss both spatial lag and spatial error models, but the empirical results with the spatial lag model get the most attention. The houses were allocated to subdistricts and houses in the same and adjacent subdistricts were considered neighbors. They used both maximum likelihood and spatial two stage least squares. They prefer the latter because their maximum likelihood estimates impose normality. They also find that considering the spatial nature of the data reduced the coefficient on the air pollutant, SO_2.

However, because of the spatial lag, a spatial multiplier must be use in determining the marginal effect on a house price of a general marginal improvement in air quality. The marginal effect is $\beta_k[I - \rho W]^{-1}$. After this transformation, the estimated marginal effect with the spatial model was quite close to that with OLS. A final study by Leggett and Bockstael (2000) considers the effect of water quality in Chesapeake Bay on surrounding property values. They use both OLS and spatial error models. The coefficient on the water quality variable is relatively stable between the two models. Because of the importance of spatial considerations in both real estate and the environment, we should see increasing use of these tools.

Rental Prices vs. Asset Prices

Most of the theoretical hedonic models abstract from the role of houses as an asset with a relatively long lifetime. Yet a majority of the environmental hedonic studies that have been done have used sales prices rather than rental prices. It is important to take this into account when interpreting the meaning of the coefficients on environmental variables.

The sales price (asset price) is the capitalized value of the anticipated future services provided by the house. The rental price is the value of those services over the coming month or other rental contract period. However, the difference between the two can be more than simply a financial calculation. If it is expected that there will be a change in the environmental conditions in the future, this expectation will be reflected in the sales price but not the rental price. The most stark example would be beach front property that is so close to the ocean that it has a beautiful view but is also expected to be washed away in the next storm. Ocean proximity might increase the rental price and reduce the sales price. Less dramatic examples are common.

One must be careful in interpreting the results of a study of hedonic sales prices. The researcher must specify the commonly held view as to the course of environmental change before determining what is being measured. In many cases this is well known, but in others its determination may require a separate research effort.

Timing of Environmental Impacts

When there is an environmental change, an important question is when the change is reflected in the property values. This issue also depends on information and expectations. Property values, since they capitalize future rents, provide information on residents' expectations. Property values often react before the environmental change. Property values also will adjust if people revise their expectations. Finally, an effect on property values requires that consumers are aware of the environmental conditions. These considerations can be considered by conducting hedonic studies at different times in the evolution of the changing environmental conditions.

There have been a number of studies that considered the timing of impacts. Many, but not all, have studied hazardous waste sites and related operations. One example is Kohlhase (1991). She studied toxic waste dumps in the Houston area to see when residents became aware of problems and how significant the effect on property values was. Her findings indicated that property values were not affected when the sites were operating but not well publicized. It was only after a site was placed on the National Priorities List of Superfund that significant negative effects on property values were found. A tentative encouraging finding was that after a toxic site was cleaned up, there seemed to be no residual effect.

A second example is Kiel and McClain (1995) where the effects of the construction of a hazardous waste incinerator were studied. They considered five stages starting with pre-rumor and going through rumor, construction, bring on-line, and finally ongoing operation. They ran separate hedonics for each stage and found there was no effect in the first two stages. During construction there was a significant negative effect on property values that became larger when the incinerator started operation. However, after it had been operating a while, the effect, while still negative and significant, returned to the lower level it had had during construction. Presumably, this occurred as the residents gained more information about the operation.

Dale, Murdoch, Thayer, and Waddell (1999) did a similar study of the impacts in different time periods from a lead smelter that was eventually closed and the site cleaned up. They find that, while there was a significant negative effect on property values while the smelter was operating and immediately after it closed, once the site was cleaned up, property values rebounded. The stigma of the smelter disappeared.

Information may be as important as clean up. Gayer, et al. (1999) found that property values surrounding hazardous waste sites were significantly reduced before the U.S. Environmental Protection Agency released its assessment of the sites and this effect was mitigated by the information subsequently released.

Repeat Sales

A typical environmental property value study treats each observation as independent. However, frequently a data set will contain repeat sales of houses, and such data can provide additional information. Between sales, many of the characteristics remain unchanged. For observed characteristics this may or may not allow simplified estimation. For unobserved characteristics, repeat sales provide additional information that should be incorporated in the estimation. Environmental repeat sales studies have the most to contribute when there has been a change in the environmental quality.

Repeat sales real estate price indexes were developed by Bailey, Muth, and Nourse (1963), and frequently have been applied in that context. However, the method was modified by

Palmquist (1982) to estimate the value residents put on environmental changes. Certain assumptions are necessary for the simplest version. An unspecified general functional form for the hedonic equation is hypothesized except for three restrictions. Geometric depreciation is assumed. The environmental variable must be transformed so it has a linear effect on the natural log of the house price. Finally, the index of the change in real estate prices does not depend on the characteristics of the house. All three assumptions can be relaxed, although the last one is the only one that is very restrictive. With these assumptions the hedonic equation for house *i* at time *t* is

$$P_{it} = B_t g(z_i) \exp(\gamma E_{it}) \exp(-\delta A_{it}) \exp \varepsilon_{it} \qquad (53)$$

where B_t is the real estate price index at time *t*, E_{it} is the environmental variable at house *i* at time *t*, A_{it} is the age at the time of the sale, and $\varepsilon \sim N(0,\sigma^2 I)$. With repeat sales at *t* and *t′*, the ratio of the prices can be formed, and the function $g(z_i)$ will cancel out.[37] The price index and depreciation are perfectly collinear, so if one cares about the price index it is necessary to use external information on the geometric depreciation rate of houses. On the other hand, if the only interest is in the environmental effect, this is not necessary. Taking the natural log of the ratio yields

$$r_{itt'} = -\beta_t + \beta_{t'} + \gamma E_{itt'} + \nu_{itt'} \qquad (54)$$

where r is the natural log of the price ratio, the β's are the natural logs of the B's, and ν is the difference in the error terms in the two hedonic equations. The β's are the coefficients of a vector of dummy variables taking a value of 1 at *t′*, -1 at *t*, and 0 otherwise. The environmental effect can be estimated without a complete specification of the characteristics or the functional form. If there are more than two sales, the correlation of the error terms must be incorporated in the estimation. Examples of the application of this technique are Palmquist (1982), Parsons (1992), Mendelsohn, et al. (1992), and Poulos and Smith (2002)[38].

There are some drawbacks to this technique. First, it assumes the hedonic equation g(z) is stable over time. Second, it only uses data where there are two or more sales, which is usually a small fraction of the full data set. These problems have been relaxed recently in the real estate price index literature, and similar modifications will be possible in the environmental area. Case and Quigley (1991) used a stacked regression to combine a hedonic equation for houses that sold only once and resale equations for houses where the characteristics did and did not change between sales. They also allowed the parameters of the characteristics to vary over time. Estimated generalized least squares is used to allow for the three types equations. A later study by Hill, Knight, and Sirmans (1997) generalizes Case and Quigley by incorporating serial correlation and using maximum likelihood joint estimation.

The objective of both papers was to develop a real estate price index, not to measure the effects of changes in characteristics. However, similar efforts in the environmental area should be productive. A panel hedonic study will utilize all the data that are available. Since the hedonic equation often changes over time, it will be important to allow for changes in g(**z**). On the other hand, some of the sources of the error in the estimation relate to the specific house, so it also will be important to utilize this information in developing the error covariance matrix.

Search Costs and Time on the Market

The basic hedonic model assumes that there is a perfectly competitive market with no significant transactions costs. In actuality, real estate markets are subject to a variety of transactions and moving costs. Some of these were important in developing the welfare measures earlier. The search costs in real estate markets are particularly relevant in determining if environmental conditions are completely captured in real estate prices or if some of the impacts take the form of longer selling times.

In selling a house, the owner determines a reservation price, accepting offers that meet or exceed the reservation price and rejecting others. The reservation price is set to maximize the expected value of the sale to the owner when the costs of waiting are considered. If a house is subject to an adverse environmental effect this may have two effects. The probability of receiving an offer in a given time period may be reduced since there may be fewer potential customers. On the other hand, the reservation price probably will be lower, which will lead to shorter duration on the market. Which effect dominates is an empirical question. The issue is important because waiting costs can be significant.

Real estate search costs and marketing times have been considered by some researchers in real estate (e.g., Miller, 1978; Haurin, 1988; Kang and Gardner, 1989; Asabere and Huffman, 1993). However, in an environmental context the issue has not received much attention except with respect to highway noise (see Nelson, 1982, and the studies discussed there). Huang and Palmquist (2001) have proposed a model of the impact of environmental disamenities on the duration of sales and used an econometric model to simultaneously estimate the reservation price equation and the duration equation. The results suggest that while highway noise has a significant negative effect on reservation and sale prices, it does not have a significant effect on duration. There is evidence that the reservation price is reduced as duration increases, which is the offsetting effect described above.

IV. ESTIMATING THE DEMAND FOR ENVIRONMENTAL QUALITY

In the theoretical section we saw that some policy questions can be addressed using the information contained in the hedonic price schedule. One can establish whether or not there is a statistically significant marginal willingness to pay for an environmental improvement. For non-marginal environmental changes it is possible to estimate or at least bound the total willingness to pay if the effects are localized. However, there are important environmental issues that are not localized. A non-marginal change in environmental quality often will result in changes in the equilibrium price schedule for houses. Knowledge of the underlying preferences will be necessary to develop welfare measures. This is the reason that there has been considerable interest in using the hedonic model to estimate the underlying preferences, even though it is considerably more complex than estimating the hedonic equation.

Estimating preferences or demands for environmental quality in the hedonic model is usually done in two steps.[39] The estimation of the hedonic equation proceeds as described in section III. The parameter estimates from the hedonic equation are used to calculate marginal prices for the characteristics. These prices are combined with data on the quantities of the characteristics and socio-economic data on the purchasers to estimate the demands. This sounds like a straightforward procedure, but we now realize that the data and estimation requirements are quite stringent.

Identification

In his original article Rosen suggested that since a nonlinear hedonic price equation provided varying marginal prices within a single market, it would be possible to estimate the demand and supply functions for a characteristic.[40] However, for any one individual, we observe only one data point on his or her demand function, the chosen house with a given amount of the characteristic and a given marginal price.[41] There are an infinite number of demand functions that could go through that point. If the demand function is to be identified, additional information must be used. This issue is not just the usual identification problem with the demands and supplies of characteristics, which was discussed by Rosen (1974). It is present even with micro data that are commonly used today, where only the demands and the price schedule often are estimated. Several of the researchers who initially implemented the Rosen model were unaware of this issue and unintentionally achieved identification through functional form restrictions.

The identification issue was demonstrated by Brown and Rosen (1982). They used the simple case where the hedonic price schedule is quadratic in the characteristics and the uncompensated inverse demands are linear in the characteristics. If we estimate the quadratic hedonic, the calculated marginal prices will be linear in the characteristics. If we try to explain the marginal prices with quantities of characteristics and socio-economic data, there will be nothing for the socio-economic data to explain. We just get the same information we got from the hedonic equation and are unable to identify the demands. Brown and Rosen suggest that data from separate markets or restrictions on the functional forms could be used to achieve identification.[42]

In a single market, identification can be achieved in several ways such as by excluding some characteristics from the demand equations, imposing restriction on the parameters, or by nonlinearities in the equations. In a sense, all of these imply *a priori* restrictions on the functional forms of the hedonic equation and the demand equations.[43] If the restrictions are correct, this is quite useful, although one cannot test the restrictions. Quigley (1982) restricted individuals to have identical generalized constant elasticity of substitution utility functions. With a homothetic utility function the varying marginal prices allow identification of the parameters of the utility function. Similarly, Kanemoto and Nakamura (1986) impose the restrictions on the bid function to achieve identification. Recently Chattopadhyay (1999) has shown that the nonlinearities in his functional forms allow him to meet the rank conditions with a linear Box-Cox specification for the hedonic equation and restricted forms of either the Diewert (generalized Leontief) or translog utility functions.

An alternative way of achieving identification is by using data from several markets. The markets could be separate spatially or temporally. It is necessary to assume that consumers with the same socio-economic profile have the same preferences, regardless of the city in which they live.[44] First, a separate hedonic price schedule is estimated for each market, and marginal prices are calculated for each resident. Then the demand function or system is estimated by combining data across markets. In different markets a particular type of individual would face different prices and make different choices because of that. Thus, we effectively observe as many different points on a demand function as we have markets.[45] If the assumption that an individual would have the same tastes no matter where he or she lived is appropriate, this method of identification is more acceptable than depending on a chosen functional form. However, one must have good data on the individuals' socio-economic characteristics. In some cases, one might want to interact the socio-economic characteristics with an attribute of the city in cases where the demand for the characteristic is influenced by that attribute. A non-environmental example would be the demand for central air conditioning being influenced by summer temperatures, but the technique might also be useful with some environmental issues. Separate markets have been used for identification in a non-environmental context by Witte, Sumka, and Erekson (1979), Palmquist (1984), Parsons (1986), Bartik (1987), and Ohsfeldt (1988). The technique has been used by Palmquist (1983), Palmquist and Israngkura (1999), and Zabel and Kiel (2000) to study the demand for air quality and by Boyle, Poor, and Taylor (1999, 2000) to study lake water quality.

Endogeneity

If the hedonic price schedule is nonlinear, both prices and quantities will be endogenous in the demand equations.[46] For example, for inverse demands the error term represents unexplained variation in the marginal price. If there were different draws from the error distribution, different marginal prices would result. Different marginal prices are associated with different quantities of the characteristics. Thus, both price and quantity are correlated with the error term. Least squares estimation is no longer consistent. Murray (1983) first raised this problem of endogeneity in the hedonic second stage, and Epple (1987) has analyzed it in depth. In addition, while actual income is exogenous, adjusted income or the quantity of the numeraire non-housing good is also endogenous.

Given the endogeneity, the search for instruments begins.[47] A valid instrument must be uncorrelated with the error (orthogonality condition), correlated with the endogenous variable (relevance condition), and provide additional information or not be redundant (uniqueness condition). The orthogonality condition rules out using any of the other characteristics or marginal prices. If actual income and the demographic traits of the individual are not measured with error, they can serve as instruments.[48] When multiple markets are used, dummy variables for the different locations or times can be used. Finally, transformations of these potential instruments (e.g., powers of the variables or interaction terms) can be used.[49] While there are statistical tests of the orthogonality condition that can be done, one may create pre-test estimator problems.[50] Using theory to select instruments may be more appropriate.

It is not enough to have orthogonal instruments if the instruments explain little of the variation in the endogenous variable. While such weak instruments may provide an unbiased estimator asymptotically, there may be very large standard errors. In addition, for finite samples the estimator will be biased in the same direction as ordinary least squares. Bound, Jaeger, and Baker (1995) demonstrate these potential problems. It is important to have instruments that, *a priori*, one would expect to be highly correlated with the endogenous variables. Lacking that, some techniques such as the one in Hall, Rudebusch, and Wilcox (1996) have been developed to measure instrument relevance, but they may actually make the finite sample problems worse. However, Hall and Peixe (2000) have recently developed a method for selecting instruments based on their relevance that may avoid the problems of pre-testing. They suggest using Andrews' (1999) method to select orthogonal instruments and then their technique to select instruments that are relevant.

If only weak instruments are available, one must make a choice between using weak instruments and not using instruments. Nakamura and Nakamura (1998) have argued that a policy of 'always instrument' can be a mistake. Indeed, with weak instruments and finite samples, one may be accepting much higher variance for the estimates without eliminating bias. On a mean square error criterion, ordinary least squares may dominate. One must carefully consider the uses to which the results will be applied.

A closely related issue in estimating the second stage of the hedonic model is the fact that the socio-economic characteristics of the residents may be measured with error. This has not been an issue with the empirical applications mentioned above since they have used data on the individual purchasers. However, the difficulty of obtaining such data explains the small number of such studies. One could use aggregate data on socio-economic variables from census blocks or block groups, etc., to greatly expand the number of data sets available for this estimation. However, this measurement error will be correlated with the error term in the equation being estimated, and the same issues discussed above are present. However, the search for instruments will be that much more difficult because the socio-economic variables obviously cannot be used as instruments in this case.[51]

V. DISCRETE CHOICE MODELS

Discrete choice models represent an alternative approach to modeling consumers' preferences for the characteristics of heterogeneous housing. This topic has been introduced in earlier sections on the theoretical models and welfare measurement. This section provides more detail on the estimation techniques that are available and some of the applications that have been done. There are two types of qualitative models that have been used with property value models: random utility and random bidding models. Random utility models have had their greatest application with recreation demand, and the chapter of this volume on Recreational Demand Models provides much more detail on the technique. While there is a brief overview provided here, this section focuses on the issues that are most important with housing.

Random Utility Models

In random utility models each consumer is assumed to make a discrete choice between *K* houses.[52] The consumer knows all of the characteristics of each house that are relevant to him or her ($\mathbf{z}^{*\mathbf{k}}$ for house *k*) and the exact form of the utility function, $U^k(\mathbf{z}^{*\mathbf{k}},x^k)$. However, the researcher does not know the exact specification of the true utility function, so the perceived utility provided by house *k*, $V(\mathbf{z}^{\mathbf{k}},x^k)$ is measured with error. For example, for house *k*,

$$u_k = U(\mathbf{z}^{*\mathbf{k}},x^k) = V(\mathbf{z}^{\mathbf{k}},x^k) + \varepsilon_k \qquad (55)$$

The individual maximizes utility by selecting the one house that yields the highest level of satisfaction,

$$\max U = \max\ \{u_1,\ldots,u_K\} = \max\ \{V(\mathbf{z}^{\mathbf{1}},x^1) + \varepsilon_1,\ldots,\ V(\mathbf{z}^{\mathbf{K}},x^K) + \varepsilon_K\} \qquad (56)$$

The probability of a consumer selecting house k is

$$\text{Prob}(k) = \text{Prob}[\varepsilon_{k'} < V^{k} - V^{k'} + \varepsilon_{k} \text{ for all } k' \neq k] \quad (57)$$

The distributional assumptions made about ε determine the type of estimation done. The simplest estimation assumes the ε are independently and identically distributed as Type I Extreme Value, which leads to conditional logit. However, because this distributional assumption imposes independence from irrelevant alternatives (IIA), some studies have assumed a special case of the Generalized Extreme Value Distribution, which leads to nested logit.[53] This avoids IIA between nests, although IIA is assumed within nests.

The researcher must choose the functional form for the observed utility function, V(**z**,x). The numeraire, x, is replaced by income minus the price of the house, m - P. The same coefficient(s) applies to income and the negative of price. The consumer's income does not vary between houses, but price does. One can use the estimated coefficient(s) on price to derive the implicit coefficient(s) on income. If the numeraire enters linearly, the estimated coefficient on price is the negative of the (constant) marginal utility of income. Early studies used a linear form for the utility function because it was easy to estimate. Subsequently, the characteristics were entered nonlinearly, although the m - P term was usually entered linearly. Recently, researchers in the recreation literature have allowed for non-constant marginal utility income, although this has not yet been done in the property value literature. This is unfortunate because, for most people, housing makes up a large fraction of expenditure. Assuming a constant marginal utility of income may be inappropriate. Modifying the estimation to allow for a varying marginal utility of income will be relatively straight forward, but the welfare measurement in some cases will become considerably more complex (see McFadden, 1999, and Herriges and Kling, 1999).

The demographics of the individual can be expected to influence the level of utility provided by a house with a particular set of characteristics. However, we are modeling the discrete choice the individual makes, and that person's demographics do not vary with the different choices. This means that demographic variables can only be entered in the estimating equation if they are interacted with one of more of the characteristics of the houses, since these will vary over the choices. By analogy with estimation with panel data sets, one could refer to this as allowing for fixed effects of the demographics. Random effects are discussed below.

An important difference between most recreational site choice studies and housing choice studies is in number of alternatives available. In estimating a general random utility model, the researcher uses data on the house chosen and all of the houses that could have been chosen but were rejected. Thus, each observation may use data on a large number of houses, making estimation time consuming. Fortunately, with conditional logit and nested logit, McFadden (1978) has shown that one only needs to use the chosen house and a random sample of the houses not chosen. If the sampling gives equal probability to the selection of each house that was not purchased (within a given nest for nested logit), the sampling rule satisfies the uniform conditioning property. Consistent estimates can be obtained with the sample if the uniform conditioning property holds. This sampling is possible because IIA holds within nests.

A closely related issue is the relevant choice set from which to sample. This is a complex issue here, as it is with all random utility models. The researcher wants to know the alternatives available to the individual, and time plays a crucial role in real estate markets. If a house is sold before the purchaser enters the market or comes on the market after the purchaser has closed on a house, it was not in his or her choice set. However, the length of time between entering the market and purchasing a house varies greatly between purchasers. In addition, when a person enters the market, he or she only considers a subset of the houses that are available. Only houses in a certain price range, or with certain characteristics, or in a certain part of the city are considered. Unfortunately, the researcher almost never knows this type of information. Obtaining it would require extensive surveying. For this reason, most studies have sampled from all house sales.[54] These choice set issues deserve further consideration.

Welfare measurement within the random utility models for housing is either simpler or more complex than for recreation, depending on the assumptions made. With recreation, if there is a change in some characteristics, some recreationists will decide to visit a different site because there are no transactions costs to doing so. The welfare measures for the change will require predicting the new site. However, the travel costs of visiting the sites are unaffected by the change. With housing there are substantial transactions costs of moving to a new house. It may be reasonable to assume that an environmental change alone will not result in moving. Then one does not have to predict the new locations and can use the estimated utility function to calculate the willingness to pay for non-marginal changes. If there is movement, this still provides a lower-bound for willingness to pay for an improvement. However, if the environmental change is significant enough to induce moves, the equilibrium house prices will change, unlike travel costs. The random utility model alone is insufficient to predict those changes. However, the different utility functions estimated for different demographic types could be combined with information about the existing houses in an area using an assignment model similar to that discussed in the hedonic sections above to predict the new equilibrium prices. Then the welfare measurement techniques used for recreation could be applied. (See the chapter on recreation.)

Recently there have interesting developments in the estimation of discrete choice models that represent potential improvements over logit and nested logit. In the past the errors were assumed to have some type of extreme value distribution because this yielded models that were tractable to estimate. With the advent of greater computing power and techniques such as simulated maximum likelihood and simulated method of moments, using more general distributions became possible. Multinomial probit models were estimate using a Gibbs sampler (e.g., McCulloch and Rossi, 1994) and the GHK simulator (e.g., Chen, Lupi, and Hoehn, 1997). However, while our capabilities are expanding, the number of choices still is somewhat limited, at least by the standards of housing markets. One of the main advantages of using probit is to avoid the restrictive assumptions such as IIA, but it is exactly IIA that allows us to use sampling to reduce the number of choices not selected. Research movement is this direction will probably be slow.

Another recent generalization may be more promising: random parameters logit.[55] It is assumed that individuals differ in unobserved ways and that the parameters in the logit model differ by individual. Since the coefficients cannot be observed for any individual, a density function for the expected value of each parameter is assumed. The probability of choosing a house then involves integrating over these densities. Again, simulated maximum likelihood can be used. The reason this technique may be more promising in the near term is that McFadden and Train (2000) have shown that any random utility model can be approximated arbitrarily closely with a random parameters logit model. With appropriate assumptions, sampling of alternatives not selected may be possible while maintaining the differences in the consumers. Random parameters logit has been used to allow individual differences even when nothing is known about the demographics of the purchasers. If information is available on the individuals, one can combine fixed effects (discussed above) with these random effects. Such techniques may be promising for property value studies.

Two important empirical applications of the random utility model to housing are Quigley (1985) and Chattopadhyay (2000), although only the latter focuses on an environmental issue. Both use nested logit with the nests being referred to as town or city, neighborhood, and dwelling, although each used data from a single metropolitan area. Quigley used about 600 households, while Chattopadhyay had a little over 3,000. Quigley used household income, while Chattopadhyay used income, race, and number of children. Socio-economic characteristics have to be interacted with housing characteristics since they do not vary by choice otherwise. Quigley found that a linear utility function performed better than logarithmic or income interaction terms. The coefficients on the inclusive value terms were between 0 and 1 and had the correct relative magnitudes (σ higher for the town nest than the neighborhood nest), and were both significantly different than zero thereby rejecting IIA between nests. Chattopadhyay's results were similar using a Cobb-Douglas utility function with a type of demographic translating, although he could not reject IIA between neighborhoods. He also calculated welfare measures and compared them to hedonic results. The RUM welfare measures were generally less than those for the hedonic model and they were sensitive to alternative nesting structures.

There have also been some interesting simulation studies comparing RUM results to hedonic results, although conflicting evidence leaves the issue open. Quigley (1986) and Mason and Quigley (1990) used a generalized CES utility function and a distribution of income to predict a vector of equilibrium prices. They then compared the hedonic model with the discrete choice model when the errors introduced were of varying sizes. When the errors were small, the hedonic tended to underestimate willingness to pay, but the average underestimate was small.

The discrete choice model had little bias, but the dispersion of the estimates was much larger. For larger errors, the dispersion with discrete choice was very large. While these studies seem to favor hedonics, the forms of the utility functions for the two techniques may have favored hedonics.

The alternative simulation was done by Cropper, Deck, Kishor, and McConnell (1993). Using the data generation techniques in their functional form simulation discussed earlier, they compared hedonic and discrete choice models. Their specification of the characteristics of the houses and individuals was more complete than in the Quigley studies. They also experimented with more functional forms. For marginal willingness to pay, the errors with the two models are roughly comparable. However, for non-marginal changes the discrete choice models do much better. The study is well done, but since the results are contrary to those in Chattopadhyay (2000) and the Quigley simulations, further research can probably be justified.

Random Bidding Models

An alternative discrete model was proposed and implemented in Ellickson (1981). He suggested that the researcher could form a hypothesis about the form of the bid function rather than the utility function. This bid function would differ from the true bid, which would be known to the individual. As consumers interact in the housing market, each individual would have the winning bid for a house. Ellickson's interest was in predicting the type of household that would occupy different types of houses.[56] This requires that there be a relatively limited number of types of households. Some demographic attributes are naturally discrete (e.g., renting vs. owning and, to a lesser extent, race), but many are continuous (e.g., income). Categories must be established for these latter types of variables (e.g., income ranges), and this is done at a sacrifice of information. Assume that t = 1,...T indexes the household types that have been created. In the theory section of this chapter, the true bid function for individual *j* was $\theta^j = \theta^j(\mathbf{z},u^j,m^j,\boldsymbol{\alpha}^j)$ with the notation defined earlier. Suppose individual *j* is of type *t*, and let *j* be a member of the set of all type *t* individuals, N_t. The bid function is $\psi_t(z) + \varepsilon_{tj}$, where ψ_t is the common bid function for type *t*, and ε_{tj} is a individual-specific error term that subsumes u and m. Houses go to the winning bidder, so the probability that a family of demographic type *t* will occupy a house with characteristics **z** is

$$\text{prob}(t|z) = \text{prob}\{\psi_t(z) + \varepsilon_t > \psi_{t'}(z) + \varepsilon_{t'} \text{ for all } t' \neq t\} \tag{58}$$

Because the winning bid on a house is the maximum bid, the Extreme Value Distribution, which is usually used, has a theoretical justification in addition to its ease of estimation. That distribution also allowed Ellickson to estimate the model separately for subsets of the household types, an important consideration at the time of his research.

Ellickson correctly made the point that his model could not be used to estimate willingness to pay for characteristics because the parameter estimates were only indentifiable up to scale factor. Only differences in parameters between types and not absolute estimates were meaningful. If this problem could not be avoided, the random bidding model would have little relevance for environmental economics. Fortunately, Lerman and Kern (1983) modified the model to avoid this shortcoming. The researcher has another piece of information, the price for which the house sold, which can be assumed to equal the winning bid. This allows equation (58) to be modified,

$$\text{prob}(t,P|z) = \text{prob}\{\psi_t(z) + \varepsilon_t = p \text{ and } \psi_{t'}(z) + \varepsilon_{t'} \leq p \text{ for all } t' \neq t\} \tag{59}$$

so that all parameters are identified. This modification has increased the interest in the model.

Lerman and Kern also point out that the means of the errors depend on the group size and should be accounted for in the estimation unless a full set of alternative-specific constants is included. They also comment on the loss of information in the aggregation into groups of household types. They suggest an alternative where each individual is considered a type. This would use all data on the individual, but it requires that there be as many types as there are individuals in the sample. This introduces an unmanageable dimensionality. To deal with this, they suggest using a sample of the individuals. This is losing information in a different way, although it would allow comparison of the results obtained using different parts of the information contained in the data. In either case, this loss of information is the major drawback with random bidding models.

There has been one environmental application of the random bidding model.[57] Chattopadhyay (1998) used this third technique to go with his hedonic and random utility studies discussed earlier. Using the same data, he used a logarithmic specification for the bid function. He divided the households into four types by two categories of income and two categories of family size, and estimated all four sets of bid parameters in a single equation using dummy variables. He then compared the results to those from the hedonic model. They were quite close for both marginal and non-marginal willingness to pay for characteristics. The random bidding model did seem relatively stable for alternative categorizations of the demographics. Chattopadhyay's conclusion is that the main advantage of the random bidding model would be if the interest were in the willingness to pay of different demographic groups.

VI. LOCATIONAL EQUILIBRIUM MODELS

The theoretical hedonic model describes an equilibrium, but there has been little formal work on modeling how that equilibrium would change if there were changes in exogenous factors. Empirical work with first-stage hedonic estimation has been limited to estimating a single equilibrium price function or, occasionally, a series of *ex post* equilibrium price functions to trace the shifts. Second-stage hedonic estimation has concentrated on the parameters of utility or bid functions. The discrete choice models have had the same goal. It is only recently that researchers have attempted to incorporate the estimation of preferences into models of the market equilibrium. At the moment there are two main ways in which this is being done. One is motivated by the jurisdictional equilibrium models developed by Dennis Epple and his various colleagues. These models, for example, Sieg, et al. (2003), explicitly consider the environment. The other line of research extends the discrete choice models to incorporate equilibrium. Bayer, et al. (2002) develop the methodology and apply it to residential segregation, but the technique could be applied to environmental issues.

Epple and Sieg (1999) and Epple, et al. (2001) developed a methodology for empirically implementing models of equilibrium with local jurisdictions and housing markets. The earlier theoretical models (e.g., Epple, et al., 1993) had assumed a stratification of communities by income and a resultant distribution of local public goods. Epple and Sieg introduced an unobservable taste parameter that allows a more general distribution of income within communities while maintaining the earlier theoretical insights. Their model concentrated on local public goods such as school quality and crime. The model is now being used not only for local public goods but also to value environmental goods. The model focuses on the equilibrium that is established by the location choices of the residents and recognizes that both continuous and discrete decisions are involved. Thus, the model builds on elements of the hedonic and discrete choice models but is a new approach. Welfare measurement is possible because the underlying utility function is estimated and a new equilibrium can be simulated. The technique also minimizes the requirement of micro data on the residents. However, these attractive features of the model require that it be tightly parameterized.

Sieg, Smith, Banzhaf, and Walsh (1999 and 2003) are good examples of the work by these authors in this area. They use data from Southern California and focus on air pollution. Following Epple and Sieg (1999), they assume that households are mobile and consider housing market conditions, local public goods, and air quality in their location decisions. The local public goods and air quality are assumed to be the same across a community. The households differ by income and an unobservable taste parameter, and there is a joint density function for the two variables. An individual's utility depends on housing, the location good (which is a linear index of the local public goods and air quality), a composite numeraire, and the taste parameter. The indirect utility function, $V(p,g,y,\alpha)$, depends on the price of housing, the location goods index, income, and the taste parameter, in that order. The specific form of the indirect utility function maintains separability between the public goods index and the market goods, which include housing. The functional form is a variant of the CES.

The slope of an indirect indifference curve between p and g is central to the model. The maintained hypothesis of the single crossing property, introduced by Ellickson (1971), says that the slope of this indifference curve changes monotonically as either income or the taste parameter changes. Thus, a change in income will shift this indifference curve so that it intersects the original indifference curve only once. Preferences that are consistent with the single crossing condition will have three characteristics. The boundary indifference property says that some individuals will be indifferent between two adjacent communities in the hierarchy of communities. Stratification implies that, controlling for tastes, households stratify among communities by income, and controlling for income, they stratify by preferences for the public goods. The ascending bundles property says that ranking communities by housing prices yields the same ranking as ordering communities by local public goods provision.

The estimation of the model involves several steps. In the first step, hedonic techniques are used to generate a housing rental price index for each community.[58] The housing expenditures in each community depend on this index and the distribution of income in the community. The distribution of income within each community is obtained from the Census of Population. Data on schooling, crime, and air quality are used to form the public goods index. Then moment conditions for this index, the quartiles of the income distribution, and the quartiles of the housing expenditure distribution are used in a generalized method of moments estimator. The parameters of the joint distribution of income and tastes, the parameters of the indirect utility function, and the parameters of the locational goods index are obtained.

Since these parameters allow them to predict the locational equilibrium, they can predict the changes in houses price as residents move in response to a significant change in air quality. While conceptually this can be done with the hedonic and random utility models as suggested earlier, it has not been implemented. Thus, Sieg, et al. (2003) have taken an important step in modeling the effects of large environmental changes on property values. For example, they are able to estimate the general equilibrium willingness to pay for the significant improvements in ozone between 1990 and 1995 in Southern California.[59] They find that there are significant differences between the partial equilibrium welfare measures and the general equilibrium measures. For the entire study area, the general equilibrium measure is larger. It is also interesting that when the individual counties within the study area are considered, the general equilibrium measures are sometime larger and sometimes smaller. Since the housing price changes and the pollution changes differ between counties, this divergence is to be expected.

Bayer, et al. (2002) incorporate equilibrium concepts in a different way. They start with McFadden's (1978) random utility model for housing. They modify it by allowing for unobserved quality in each housing unit. They also allow the parameters on the housing characteristics to vary with the characteristics of the household. Finally, they allow for social interactions where a person's utility from a house can depend on the socio-economic characteristics of the neighbors.

The equilibrium concept is incorporated in an interesting way. For the moment, abstract from the varying parameters, social interactions, and unobserved quality. The conditional logit model gives the probability that household i selects house h. This probability is summed over all households. The assumption they make is that in equilibrium the sum of the probabilities will equal 1 for each house. They solve iteratively for the vector of house prices such that the sum of the probabilities for each house is equal to 1. They describe this as eliminating the excess demand or supply for each type of house, although it is not the same as having one individual have the winning bid for each house.

In the actual implementation, the varying parameters, social interactions, and unobserved quality make the estimation more complex. They use a contraction mapping similar the Berry, et al. (1995) and develop instruments for the endogenous characteristics using the characteristics of house that are far enough removed to not be neighbors in the social interactions. Once the model has been estimated, it can be used in simulations of new equilibria.

These two locational equilibrium models and similar models yet to be developed should prove useful in designing property value models for use in environmental economics.

CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

This chapter has highlighted the diversity of models and techniques that can be used to study property values as a way of revealing the values individuals put on environmental improvements. In this context, the use of hedonic regressions is well established and widespread. The estimation of the underlying behavioral equations is less common, but recently significant progress has been made here as well. Similarly, the progress on random utility models in other areas is beginning to have an impact on property value studies. Finally, the new locational equilibrium models are a promising area for further research. Each of these last three areas should grow as more and better data, computing power, and econometric techniques become available.

Throughout the chapter I have highlighted areas where future research would be productive. A few can be mentioned here. The spatial nature of environmental problems is central to the use of property value models. The recent availability of geocoded data and spatial econometric techniques should have a major impact on this research area. Significant progress is possible on measuring environmental effects, reconciling subjective and objective perceptions of the effects, and allowing the effects to enter the models in a flexible, perhaps nonparametric, way. We should learn more about the relative strengths and weaknesses of the discrete choice model, the locational equilibrium model, and the hedonic model. A final area that will be very important is refining the welfare measures derived from property value models. This is the goal of the environmental use of these models, and there is still substantial room for improvement.

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Appendix

Exact welfare measures using an estimated log-linear uncompensated inverse demand are given above. If the functional form of the estimates were semi-log or linear, the welfare measures are given below.

For a semi-log uncompensated inverse demand,

$$\frac{\partial P}{\partial z_1} = \exp(\alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma x), \tag{30}$$

the differential equation is again separable, and the bid function is

$$\theta = \frac{1}{\gamma} \ln\left[\frac{\gamma}{\beta_1} \exp(\alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma m) + \gamma u\right]. \tag{31}$$

The direct utility function is

$$u = \frac{1}{\gamma} e^{\gamma P_0} - \frac{1}{\beta_1} \exp(\alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma m), \tag{32}$$

and the economic integrability condition is

$$\frac{\beta_1}{\gamma} < P_z. \tag{33}$$

The welfare measures are

$$CV_q = \frac{1}{\gamma} \ln\left[\frac{\gamma}{\beta_1}\left[\exp(\alpha + \beta_1 z_{11} + \beta_2 z_2 + \gamma x_0) - \exp(\alpha + \beta_1 z_{10} + \beta_2 z_2 + \gamma x_0)\right] + 1\right] \tag{34}$$

$$EV_q = -\frac{1}{\gamma} \ln\left[\frac{\gamma}{\beta_1}\left[\exp(\alpha + \beta_1 z_{10} + \beta_2 z_2 + \gamma x_0) - \exp(\alpha + \beta_1 z_{11} + \beta_2 z_2 + \gamma x_0)\right] + 1\right]. \tag{35}$$

Finally, if the uncompensated inverse demand function is linear,

$$\frac{\partial P}{\partial z_1} = \alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma x, \tag{36}$$

the linear ordinary differential equation can be solve for

$$\theta = \frac{\alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma m}{\gamma} - \frac{\beta_1}{\gamma^2} + e^{-\gamma z_1} u \tag{37}$$

and

$$u = \left(P_0 - \frac{\alpha + \beta_1 z_1 + \beta_2 z_2 + \gamma m}{\gamma} + \frac{\beta_1}{\gamma^2} \right) e^{\gamma z_1} . \tag{38}$$

The condition for economic integrability is

$$\frac{\beta_1}{\gamma} < p_z \tag{39}$$

and the welfare measures are

$$\begin{aligned} CV_q = & \frac{1}{\gamma}\left(\alpha + \beta_1 z_{11} + \beta_2 z_2 + \gamma x_0 - \frac{\beta_1}{\gamma} \right) \\ & - \frac{1}{\gamma} \exp[\gamma(z_{10} - z_{11})] \left(\alpha + \beta_1 z_{10} + \beta_2 z_2 + \gamma x_0 - \frac{\beta_1}{\gamma} \right) \end{aligned} \tag{40}$$

$$\begin{aligned} EV_q = & \frac{1}{\gamma}\left(\alpha + \beta_1 z_{10} + \beta_2 z_2 + \gamma x_0 - \frac{\beta_1}{\gamma} \right) \\ & - \frac{1}{\gamma} \exp[\gamma(z_{11} - z_{10})] \left(\alpha + \beta_1 z_{11} + \beta_2 z_2 + \gamma x_0 - \frac{\beta_1}{\gamma} \right) \end{aligned} . \tag{41}$$

* I would like to thank Kerry Smith, Wally Thurman, and Dan Phaneuf for useful discussions of many of the topics in this chapter.

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1 While these techniques are widely accepted, it is worthwhile to consider the cautionary note expressed by Mäler (1977). Some of the issues raised there are discussed throughout this chapter.

2 One can say that the units are traded in a single market without implying that every consumer is a potential customer for every unit sold in the market. A given consumer may only be interested in a segment of the market. However, all that is required for the market to be integrated is that segments for different consumers overlap.

3 In the hedonic model, if a characteristic *i* is desirable to consumers, then $\partial P/\partial z_i \geq 0$. The degree of curvature in preferences places a limit on the degree of concavity of the hedonic function (for interior solutions), but the former concept is not known *a priori*.

4 McConnell and Phipps (1987) have called this function a marginal rate of substitution function.

5 In the figures the subscript *i* is omitted to simplify the labels.

6 It would be possible that two or more consumers with different $\boldsymbol{\alpha}^j$'s might have equivalent winning bids for identical houses (although given the spatial nature of housing, since two houses cannot be in exactly the same location, they cannot be exactly identical). It would be possible that an individual could be indifferent between making the winning bid on two or more houses. Neither of these possibilities substantively changes the discussion that follows and will not be pursue here.

7 Details of the various estimation techniques are discussed below.

8 Discrete choice models are discussed further in section V below.

9 In all these cases the property value models are only capturing the benefits associated with the place of residence. Benefits occurring away from home must be measured separately.

10 The distance function (also called the transformation function) is a representation of a consumer's preferences that is dual to the more common direct and indirect utility functions and the expenditure function. Accessible discussions are in Anderson (1980), Weymark (1980), or Cornes (1992).

11 Madden (1991) in an excellent paper on demand rationing discusses a similar system where one good is unrationed (the numeraire x here) and all the other goods are rationed (the characteristics **z** here). His interest is in the substitute-complement classification in such a system. He shows that his R-classification corresponds to the Hicksian q-classification. Previous literature had incorrectly suggested an equivalence between the q-classification and a classification based on the Antonelli matrix or traditional distance function.

12 Given the g vector discussed earlier, Luenberger (1996) would call this an adjusted price function.

13 It would be possible to obtain CV_q in terms of the more typical compensating variation. To simplify the notation, let $\theta_{z1}(z_1,z_2,u)$ be denoted as $\pi(z_1,z_2,u)$, the virtual price of z_1. Integrating by parts,

$$CV_q = \int_{z_{10}}^{z_{11}} \pi(z_1,z_2,u)dz_1 = \pi(z_{11},z_2,u)z_{11} - \pi(z_{10},z_2,u)z_{10} - \int_{\pi_0}^{\pi_1} z_1(\pi,z_2,u)d\pi$$

where the last term is a compensating variation measure. However, the compensation must result in the actual quantity changes, so the virtual prices depend on those quantity changes. Also, the calculations are more complex, so it is easier to use CV_q.

14 While some previous researchers have specified a utility function to derive the estimating equations, they have integrated the uncompensated inverse demands to derive welfare estimates rather than deriving exact welfare measures. Palmquist and Israngkura (1999) return to the utility function to estimate exact welfare measures.

15 This would be analogous to the procedure Vartia (1983) developed for integrating ordinary demand systems to an expenditure function. The algorithms would have to be modified for inverse demands integrating to $W(\mathbf{z},u)$ or $\theta(\mathbf{z},u,m)$, but conceptually it is possible.

16 In this theoretical section the stochastic nature of the coefficient estimates is ignored, but the distribution of the welfare measures is discussed in the econometric section.

17 By using the numeraire, the problems of the nonlinear budget constraint are avoided. The information about both income and inframarginal prices is incorporated. This also distinguishes equation 23 from the marginal rate of substitution function discussed in footnote 4.

18 Using discrete choice models to develop welfare measures is easier for housing than it is for recreation as long as residents do not move in response to the environmental change. With recreation models, one has to consider the changes in the sites visited in response to the environmental change. With housing and no moving, this is unnecessary. However, if residents move in response to the change, it is more complex than with recreation. With recreation, the prices (travel costs) do not change. A major environmental change affecting housing will result in a new vector of prices that must be forecast and incorporated into the welfare measures.

19 Retrospective studies designed to analyze the benefits provided by environmental policies that have already been implemented are rare but not unheard of (for example, the retrospective analysis of the Clean Air Act). In a hedonic context, if the policy has already been implemented, the hedonic price schedule could be estimated both before and after the policy. Then the *ex post* hedonic welfare measures discussed in Palmquist (1988) could be used.

20 Sieg, et al. (2000b) have recently used a related technique in a locational equilibrium model. This new technique will be discussed later in this chapter.

21 A good overview of agricultural land markets is Miranowski and Cochran (1993). Urban industrial and commercial lands have received considerably less attention in an environmental context. However, Ihlenfeldt and Taylor (2002) have recently studied the effects of brownfields in such a setting.

22 Smith and Huang (1993, 1995) have conducted an exhaustive meta-analysis of hedonic studies of air pollution over a 25-year period. While they find that a negative relationship between air pollution and property values is probably well established, the magnitude of the effect is influenced by data and modeling decisions. The meta-analysis provides both guidance on those decisions and an overview of one strand of the hedonic literature.

23 See Palmquist (1991) for examples of this range.

24 McCloskey (1985) and McCloskey and Ziliak (1996) strongly advocate shifting from statistical significance to economic significance in exactly such cases.

25 A recent study by Black (1999) on school quality takes this strategy one step further. School quality differs across school boundaries, but the nature of the neighborhood may not. Black focuses on houses within close proximity to boundaries. This natural experiment can control for omitted variables in the specification. This interesting strategy may not be useful for most environmental problems where there is continuous rather than discrete variation, but under the right circumstances it can prove useful. An

excellent example of this type of natural experiment in a hedonic environmental context is Poulos and Smith (2002).

26 A similar strategy has been implemented in Cheshire and Sheppard (1995), although not in an environmental context. A separate Box-Cox parameter is estimated for land area.

27 For a more complete discussion of nonparametric estimation, see Ullah (1988), Hardle (1994), and Yatchew (1998). Pace (1993 or 1995) has a fairly complete discussion in a housing context.

28 However, Stock (1991) did find it made a difference.

29 For example, if the normal, $N(0,h^2)$, is used as a kernel, h is the bandwidth.

30 There are alternatives to kernel estimation, such as k nearest neighbor estimation or spline estimation (see Yatchew, 1998), but they have not been used in property value studies very much.

31 The results in Poor, et al. (2001) could be explained by this. They found that objective measures of water quality performed better in a hedonic regression than the subjective measures given by the resident of each house.

32 Pace and Berry (1997a) use the term "spatially autoregressive error process," but including the word "autoregressive" might be confusing.

33 These functions are the negative exponential, Gaussian, and spherical. See, for example, Dubin, et al. (1999).

34 Actually the weights matrix is usually row-standardized so that the sum of the elements in any row is equal to 1. This standardization is useful for the estimation and eases interpretation (see Anselin and Bera, 1998).

35 Spatial lags and spatial errors can be combined in single equation, but distinctly different weights matrices probably are necessary for successful estimation. This is often referred to as the general spatial model.

36 With direct representation, the sample sizes that can be used are still relatively small. In the early article by Dubin (1988), the number of observations was restricted to 80 to allow the estimation to proceed. In Dubin (1998b) the sample was 1,000.

37 If the characteristics of the house (other than age and environmental quality) have changed between sales, it will be necessary to incorporate those changes in the same manner as the environmental changes.

38 Poulos and Smith (2002) is an example of how one can allow for changes over time in other hedonic characteristics.

39 As in other areas of econometrics, a limited information or two-stage estimator is less efficient than a full information estimator. Simultaneously estimating the hedonic equation and the demand equations has been proposed by Epple (1987). That possibility is also discussed in a recent methodological paper by Tauchen and Witte (2001). A good deal of structure must be assumed in the linear-quadratic structure in Epple model. In important work Ekeland, et al. (2001) point out short-comings with the linear-quadratic model and are working on relaxing that structure. Their work is promising but only deals with a scalar characteristic, so empirical applications are still quite a ways away.

40 Rosen was careful to distinguish between marginal bid functions (which hold utility constant) and uncompensated inverse demands which do not. In the literature that followed, that careful distinction has not always been maintained, perhaps because of Rosen's well-known diagram. However, it is the uncompensated demands or inverse demands that are estimated.

[41] This point is easy to see by thinking about Figure 1 or Figure 2. We observe the chosen point $\{z_0,P(z_0)\}$ or $\{z_0,x_0\}$ and the resulting marginal price at that point. Since we do not observe any other choices the individual would make, we cannot know if the bid function or utility function has the curvature shown. It would be observationally equivalent if these functions had more or less curvature, but that would imply very different demands.

[42] Ekeland, et. al (2001) have recently pointed out that the Brown and Rosen (1982) identification problem has considerably less generality than is usually supposed.

[43] The discussion here focuses on identification of the demand functions and not the hedonic equation because the hedonic equation can always be identified because it does not include the socio-economic variables that are in the demand equations.

[44] One can assume that consumers with the same socio-economic attributes have the same tastes no matter where they live and still expected that the hedonic price schedules will differ between cities. There are substantial frictions in moving firms and individuals between cities. For this reason, the distribution of different types of consumers may differ between cities. Also, because structures are a type of capital that is long lived, the existing stock may differ greatly between cities. These factors will result in different equilibrium price schedules between cities.

[45] Of course, for this to work the hedonic price schedules have to be sufficiently different in the various markets. Markets for a single city over short periods of time might not have sufficient variation. Spatially separated markets might also have this problem, but it seems less likely. Ohsfeldt and Smith (1985) conducted Monte Carlo studies on this issue.

[46] Some studies have used linear hedonic equations in the first stage and then used multiple markets for identification. If the linear functional form is appropriate, this avoids the endogeneity problem. However, it often will not be appropriate, and its use will introduce substantial measurement error in the marginal prices.

[47] It would be possible to avoid the use of instruments if one used full-information maximum likelihood estimation for the entire system including the hedonic equation. Assumptions about the distributions of the error terms (including cross-equation correlations) would be necessary. Such estimation has not yet been implemented.

[48] If the some of the demographics are measured with error, they will be correlated with the error and unacceptable as instruments.

[49] Examples of studies that have used these types of instruments with multiple markets are Bartik (1987), Boyle, et al. (1999), and Palmquist (1984). Chattopadhay (1999) used instruments in a single market. The other empirical studies used linear hedonic regressions. A recent study by Cheshire and Sheppard (1998) has attempted to use a different source for instruments. They use spatial lags in the way temporal lags are used as instruments in time series analysis. Unfortunately, spatial lags are not unidirectional and so cannot serve as valid instruments.

[50] Andrews (1999) has recently developed a method for selecting, from a set of potential moment conditions, the largest vector of moment conditions that is consistent with the data. These moment conditions can then be used in a generalized method of moment (GMM) estimator. The instrumental variable estimator is a GMM estimator. Andrews applies his method to the selection of instruments meeting the orthogonality condition. He shows that asymptotically this method selects all instruments that satisfy the orthogonality condition with probability one, so it may eliminate the potential for data mining.

[51] A related issue is the possibility that the errors in the behavioral equations are correlated for observations in the same group (e.g., block group). In this case, even small correlations can cause spurious reductions in the standard errors when aggregate data are used. Moulton (1990) has raised this problem.

[52] How the choice set is determined will be discussed below.

[53] It is interesting that the frequently cited article where McFadden introduced GEV (and also sampling of alternatives, to be discussed below) was about the choice of residential housing (McFadden, 1978).

[54] Palmquist and Israngkura (1999) experimented unsuccessfully with using the date of purchase to establish a population of houses that sold near that date from which to sample.

[55] This model has also been called mixed logit, random coefficients logit, and error components logit, often by the same authors. While this plethora of names is unfortunate, they do describe the technique. For an overview, see Train (1998) and the references there.

[56] The random utility models, on the other hand, would predict the type of house occupied by a particular individual.

[57] There have been a few non-environmental applications (see Gross, 1988, and Gross, Sirmans, and Benjamin,1990).

[58] For an expanded treatment of this step, see Sieg, et al. (2002a).

[59] The model can also predict the distribution of benefits among individuals. See Smith, et al. (2002).

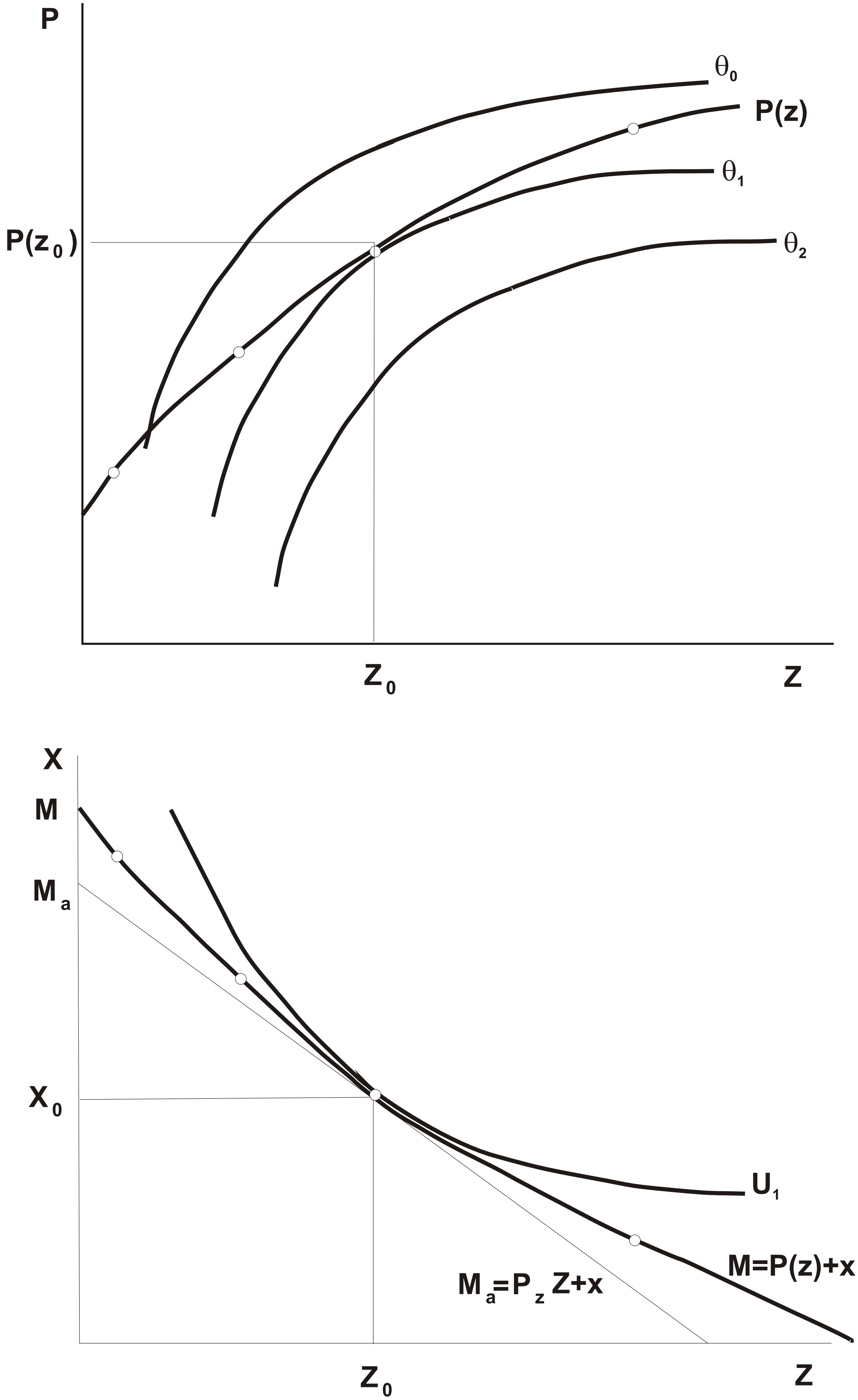

Figure 1

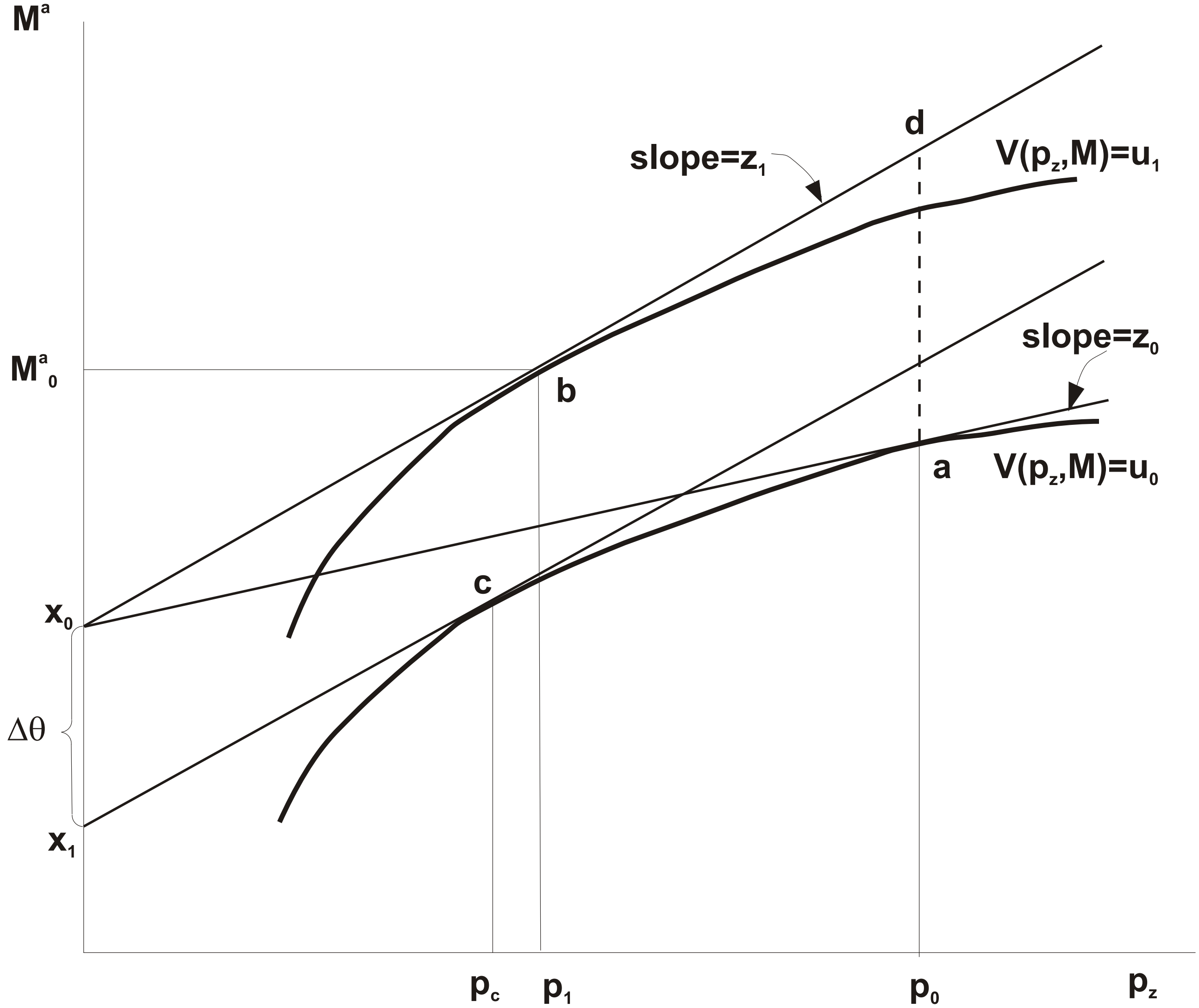

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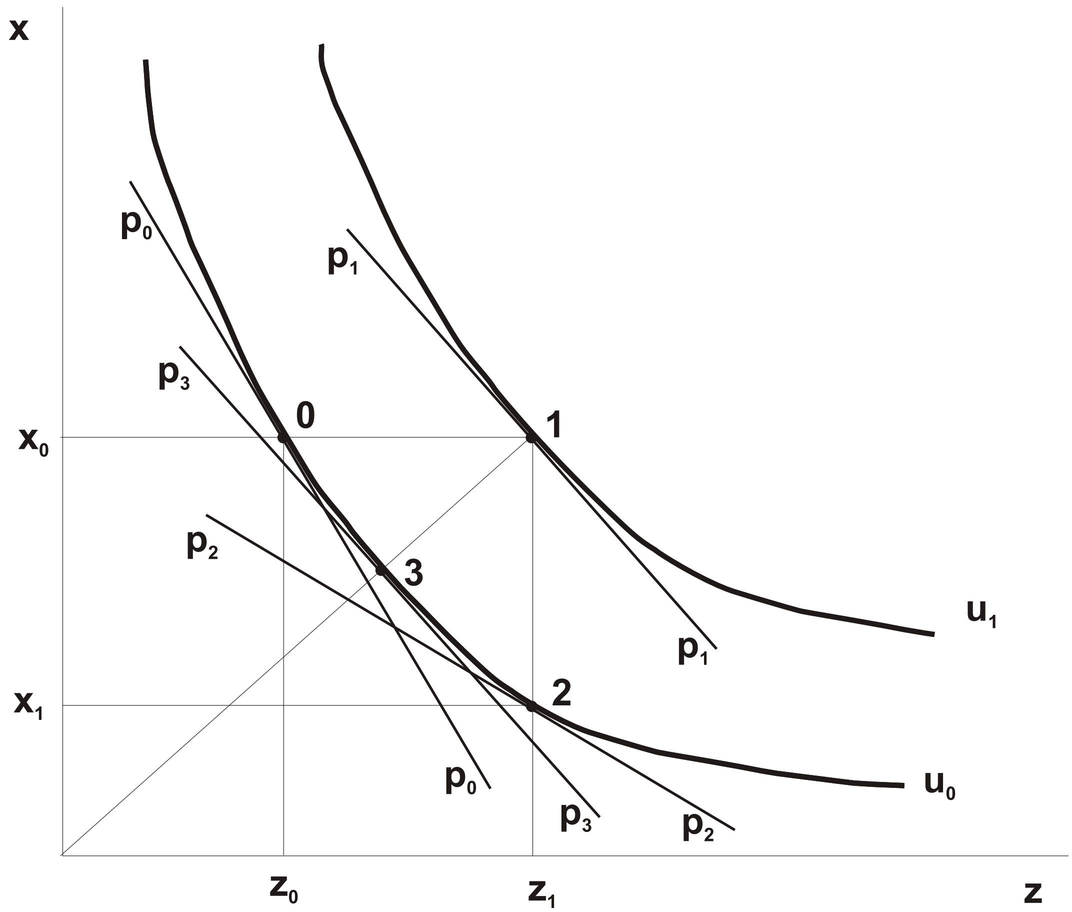
Figure 2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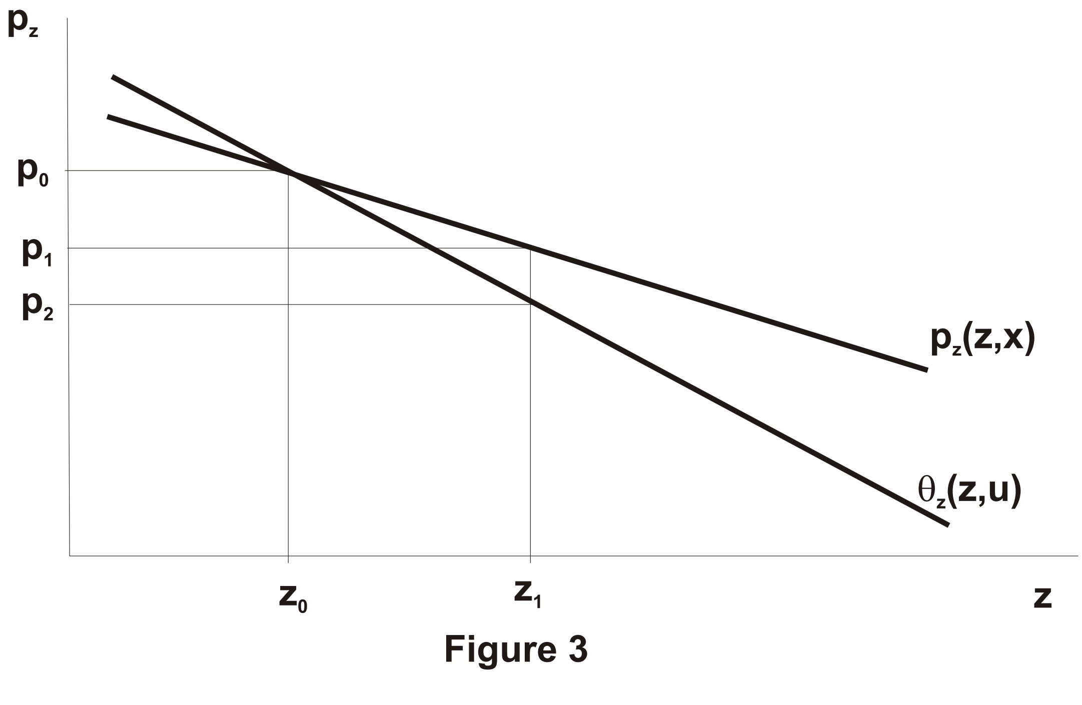
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Figure 3