The Implications of Inflation in an Estimated New-Keynesian Model

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Abstract

This paper studies the steady state and dynamic consequences of inflation in an estimated dynamic stochastic general equilibrium model of the U.S. economy. It is found that 10 percentage points of inflation entails a steady state welfare cost as high as 13 % of annual consumption. This large cost is mainly driven by staggered price contracts and price indexation. The transition from high to low inflation inflicts a welfare loss equivalent to 0.57%. The role of nominal/real frictions as well as that of parameter uncertainty is also addressed.

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1 Introduction

Undoubtedly, the period 1990 - 2006 will be remembered as one of relative prosperity in U.S. characterized by high growth and low and stable inflation. Output, for example, averaged an annual growth rate of 3% while the mean annual CPI inflation was 3% for that period. Yet recent developments in international commodity markets have put renewed pressure on inflation. Indeed, we may easily finish 2008 with inflation close to 6%, twice as large as the average inflation over the past few years. International data show that rising prices are not only a problem at home but everywhere with inflation averaging 6% in Belgium, 6.3% in China, 16% in Russia, and 33% in Venezuela. Although it is unlikely that we reach the inflation rates of the 1970s, this fresh spike in prices invites us to revisit some old questions: What are the welfare consequences of inflation? And more importantly, how much would society be willing to sacrifice to bring inflation back to its pre-2006 levels? This paper tries to answer these questions from the perspective of an estimated New Keynesian model.

Evaluating the impact of inflation on society has been a recurrent topic in macroeconomics that traces back to the seminal contributions of Bailey (1956) and Friedman (1969). This research agenda has typically pursued two distinct approaches. The first line follows Bailey (1956) in measuring the welfare cost of inflation as the area under the money demand curve. Under this tradition, money is a special consumption good while inflation is a direct tax on it. Hence, large inflations are welfare reducing as they make holding real balances costly. For example, Bailey established that a 10 percentage point drop in steady state inflation entails a welfare gain equivalent to 1% of annual income. Over the years, authors like Lucas (1981, and 2000), and Fischer (1981) have persistently found welfare estimates smaller than Bailey's. More recently, Ireland (2008) and Khan, King, and Wolman (2003) estimate the welfare cost to be as low as 0.20% and 0.05%, respectively.

The second strand of the literature have explored the cost of inflation in a general equilibrium context. In these models inflation is costly because households must divert productive time into leisure and financial activities aimed to save on real cash balances (Ireland, 1997).

Cooley and Hansen (1989) and Burdick (1997), for instance, find only modest gains (around 0.5%) of low inflation in a highly stylized real business cycle model. Dotsey and Ireland (1996) study inflation in a richer general equilibrium framework and report that an inflation of 4 percentage points entails a welfare cost as high as 1% of annual output. Furthermore, Ireland (1997) shows that the presence of sticky price contracts only exacerbates the negative consequences of inflation. Finally, Schmitt-Grohe and Uribe (2004 and 2005) explore the optimal inflation rate in fully-fledged DSGE models.

The contribution of this paper falls within this second line of research. Specifically, I study the welfare implications of inflation by employing a fairly standard DSGE model entertaining features such as price/wage sluggishness, habit formation, and costly adjustment of investment. The proposed model borrows concepts from Altig, Christiano, Eichenbaum, and Linde (2005, henceforth ACEL) and the important contribution of Christiano, Eichenbaum, and Evans (2005, henceforth CEE). The presence of real and nominal frictions gives the model a more realistic flavor and facilitates comparisons between the predictions of the medium scale new-Keynesian models with those from more parsimonious formulations, e.g. Dotsey and Ireland (1996), and Ireland (1997). As will become clear, the predictions from the two setups can be quite different.

When deciding how many real balances to keep, households confront two tensions in the model. They enjoy utility from directly holding positive money balances as in Sydrausky (1967). However, each dollar kept for utility purposes foregoes a positive return that would be earned if deposited in a financial intermediary. This dual role of money gives rise to a well defined money demand function as in Khan et al. (2003). This money demand equation has two appealing properties: 1) it is well suited for estimation, and 2) it allows us to evaluate the taxational aspect of inflation as in Bailey (1956). Of course, Sydrausky's method is only one of many ways to justify the presence of money in the economy. For example, Cooley and Hansen (1989) propose a cash in advance formulation to analyze the implications of inflation. More recently, Arouba and Schorfheide (2008) study welfare and prices using a search-based model of money balances.

Studying the cost of inflation in an estimated DSGE model poses some interesting challenges. To begin with, ACEL and CEE estimate a small interest rate semi-elasticity of the demand for money, which seems necessary to properly account for the high frequency properties of the data. However, Lucas (1981 and 2000) argues that the long-run semi-elasticity is the right choice for welfare analysis.[1] Hence I propose a flexible money demand formulation, which can simultaneously capture the short- and long-run properties of money demand. The key ingredient in this formulation is that re-balancing the composition of money balances for utility purposes or for bank deposits is costly. To capture this cost, the model assumes households use time-dependent rules to re-optimize their money holdings. The presence of those costs in turn implies that households look forward when re-balancing their portfolios between cash and deposits.

Even though the model gives rise to a rich dynamic money demand equation, minimum-distance estimators tend to recover the short-run properties of the data resulting in the small elasticities reported in the literature (ACEL, and CEE). Therefore, the model is estimated using Bayesian methods similar to those applied in Schorfheide (2000) and Smets and Wouters (2007). This approach has the advantage that priors can be used to simultaneously recover the short- and long-run elasticities of the demand for money. Additionally, the Bayesian methodology allows us to assess the impact of parameter uncertainty in the welfare calculations. It will become clear that this type of uncertainty significantly affects the steady state welfare estimates.

In the benchmark formulation, which include several nominal/real frictions, an annual inflation of 10 percentage points entails a steady state welfare cost equivalent to 13% of annual consumption or 6.6% when measured in annual output. This relatively large cost of inflation arises mainly from staggered price contracts and price indexation, which induce significant steady state price dispersion. Habit formation and the interest semi-elasticity

[1] For the rest of the paper, I will use the terms semi-elasticity of money demand and semi-elasticity as a short cut for interest rate semi-elasticity of money demand.

of money demand also contribute to make inflation costly although in a lesser degree. For example, if money demand is relatively inelastic as in CEE, the steady state welfare cost drops to 9% of annual consumption. Finally, the estimated model implies that Bailey's taxational aspect of inflation imposes a welfare loss of about 1% of consumption, a result consistent with previous studies.

Following Ireland (1997), the estimated model is used to evaluate the transitional costs of moving to a lower inflation state. I find that this transition is welfare reducing but it is only a small fraction of the benefits from living in a low inflation environment. In fact, the transition amounts for a welfare loss of 0.57% of annual consumption in the benchmark specification. The absence of habit formation and sticky prices or the presence of adjustment costs in investment make the transition less costly.

The rest of the paper is organized as follows. Section 2 describes the baseline model including the money demand formulation. I describe the estimation technique and report estimated parameters in section 3. The welfare analysis is presented in sections 4 and 5. Finally, section 6 contains some concluding remarks.

2 Model

The model builds on ACEL, CEE, and Schmitt-Grohe and Uribe (2005). Since this type of environment has been extensively discussed in the literature, I provide a brief discussion, omitting lengthy derivations. The main features of the model can be summarized as follows: The economy grows along a stochastic path; prices, wages, and money holdings are assumed to be sticky à la Calvo; preferences display external habit formation; investment adjustment is costly; and finally, there are five sources of uncertainty: neutral and capital embodied technology shocks, preference, government, and monetary shocks.

2.1 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$ each producing a final good out of capital services, k_j, and labor services, $L_{j,t}$. The technology function is given by $k_{j,t}^{\alpha}\left(S_t^L L_{j,t}\right)^{1-\alpha} - S_t^* \psi$; the term ψ makes profits equal to zero in steady state. S_t^* is the stochastic growth path of the economy (see below for its definition).[2] The neutral technology shock, S_t^L, grows at rate g_t^L which is assumed to follow the process

$$\ln g_t^L = (1-\rho_{g^L}) \ln g_{ss}^L + \rho_{g^L} \ln g_{t-1}^L + \sigma^{g_L} \varepsilon_{g^L,t},$$

where $\varepsilon_{g^L,t}$ is distributed $\mathbb{N}(0,1)$.

Firms rent capital and labor in perfectly competitive factor markets. I assume that workers must be paid in advance. As a consequence, firms must borrow the wage bill, $W_t L_{j,t}$, from a financial intermediary. The loan plus the interest rate, R_t, must be repaid at the end of the period.

Firms choose prices to maximize the present value of profits; prices are set in a Calvo fashion; that is, each period, firms optimally revise their prices with an exogenous probability $1-\xi_p$. If, instead, a firm does not re-optimize its price, then the price is updated according to the rule: $P_{j,t} = (\pi_{t-1})^{\chi} P_{j,t-1}$, where π_{t-1} is the economy-wide inflation in the previous period and $\chi \in [0,1]$. By allowing partial indexation in the price rule, I follow the common practice in the literature (Schmitt-Grohe and Uribe, 2004, and Fernandez-Villaverde and Rubio-Ramirez, 2005). An optimizing firm at time t sets prices according to the program

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{n=0}^{\infty} \left(\xi_p \beta\right)^n \lambda_{t+n} \left[\frac{P_{j,t} \Pi_{\tau=0}^{n-1} \left(\pi_{t+\tau}\right)^{\chi}}{P_{t+n}} y_{t+n}(j) - mc_{t+n} y_{t+n}(j)\right],$$

Here, P_t is the price index, $y_t(j)$ is the aggregate demand for good type j, mc_t is firm j's marginal cost, β is the discount factor, and λ_t is the marginal utility of consumption at time t.

[2] The growth term is needed to have a well-defined steady state around which we can solve the model.

2.2 Households

The economy is populated by a continuum of households indexed by i. Every period households must decide how much to consume, work, and invest. In addition, they must choose the amount of money to be sent to a financial intermediary. I assume agents in the economy have access to complete markets; such assumption is needed to eliminate wealth differentials arising from wage heterogeneity (CEE, and Erceg, Henderson, and Levin, 2000). Households maximize the expected present discounted value of utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[S_t^{Uc} \log(C_{i,t} - bC_{t-1}) - \Phi \frac{L_{i,t}^{1+1/\gamma}}{1+1/\gamma} + \psi_m \left(\frac{M_{i,t}}{S_t^* P_t} \right)^{1-\zeta_m} \right] \tag{1}$$

subject to

$$\begin{aligned} P_t C_{i,t} + \frac{P_t}{S_t^K} \left(I_{i,t} + a(x_t) K_{i,t} \right) + \mathcal{M}_{i,t} &= R_t(\mathcal{M}_{i,t-1} - M_{i,t} + T_t) + R_t^K x_t K_{i,t} + W_{i,t} L_{i,t} + M_{i,t} + A_{i,t}, \\ K_{i,t+1} &= (1-\delta) K_{i,t} + I_{i,t} \left(1 - \Gamma(\frac{I_{i,t}}{I_{i,t-1}}) \right). \end{aligned}$$

Here, S^{Uc} is a preference shock that follows the process $\log S_t^{Uc} = \rho_{Uc} \log S_{t-1}^{Uc} + \sigma^{Uc} \varepsilon_{U_c,t}$ with $\varepsilon_{U_c,t}$ distributed $\mathbb{N}(0,1)$; preferences display external habit formation, measured by $b \in (0,1)$; and Γ is a function reflecting the costs associated with adjusting investment. This function is assumed to be increasing and convex satisfying $\Gamma = \Gamma' = 0$ and $\kappa \equiv \Gamma'' > 0$ in steady state. $\mathcal{M}_{i,t-1}$ is household i's beginning of period t stock of money, whereas T_t is a lump-sum transfer by the government. Households send the amount $\mathcal{M}_{i,t-1} - M_{i,t} + T_t$ to a financial intermediary where it earns the interest rate, R_t. The stochastic trend, $S_t^* = S_t^L \left(S_t^K \right)^{\alpha/(1-\alpha)}$, in the money term is required to have a well-defined steady state. The term S_t^K is an investment specific shock whose growth rate obeys

$$\log g_t^K = (1-\rho_{g^K}) \log g_{ss}^K + \rho_{g^K} \log g_{t-1}^K + \sigma^{g^K} \varepsilon_{g^K,t},$$

where $\varepsilon_{g^K,t}$ is distributed $\mathbb{N}(0,1)$.

As in ACEL, CEE, and Schmitt-Grohe and Uribe (2004), I assume that physical capital can be used at different intensities. Furthermore, using the capital with intensity x_t entails a cost $a(x_t)$, which satisfies $a(1) = 0; a"(1) > 0; a'(1) > 0$. For future reference, define $\varkappa_a = a"(1)$. The term $A_{i,t}$ captures net payments from complete markets and government bonds, and profits from producers. The individual consumption good is assumed to be a composite made of differentiated goods indexed by j according to the aggregator

$$C_{i,t} = \left(\int_0^1 c_t(i,j)^{\frac{\zeta-1}{\zeta}} dj \right)^{\frac{\zeta}{\zeta-1}}, \quad 1 \leq \zeta < \infty,$$

where $c(i,j)$ is the demand of household i for good type j. With this type of composite good, the demand for goods of type j is given by $c(i,j) = \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} C_{i,t}$. Here, The nominal price index is $P_t = \left(\int_0^1 P_{j,t}^{1-\zeta} dj\right)^{\frac{1}{1-\zeta}}$. Similarly, I assume that individual investment obeys $I_{i,t} = \left(\int_0^1 I_t(i,j)^{\frac{\zeta-1}{\zeta}} dj\right)^{\frac{\zeta}{\zeta-1}}$. As with consumption, $I(i,j)$ denotes household i's demand for investment good of type j.

2.3 Wage Setting

Following Erceg, Henderson, and Levin (2000), I assume that each household is a monopolistic supplier of a differentiated labor service, $L_{i,t}$. Households sell these labor services to a competitive firm that aggregates labor and sell it to final firms. The technology used by the aggregator is

$$\widetilde{L}_t = \left[\int_0^1 L_{i,t}^{\frac{\zeta_w-1}{\zeta_w}} dj \right]^{\frac{\zeta_w}{\zeta_w-1}}, \quad 1 \leq \zeta_w < \infty.$$

It is straightforward to show that the relation between the labor aggregate and the wage aggregate, W_t, is given by $L_{i,t} = \left[\frac{W_t}{W_{i,t}}\right]^{\zeta_w} \widetilde{L}_t$.To induce wage sluggishness, I assume that households set their wages in Calvo fashion. In particular, with exogenous probability ξ_w a household does not re-optimize wages each period. If this is the case, wages are set according to the rule of thumb $W_{i,t} = (\pi_{t-1})^{\chi_w} W_{i,t-1}$. Following Schmitt-Grohe (2004) and Fernandez-Villaverde and Rubio-Ramirez (2008), the wage rule allows for partial indexation with parameter $\chi_w \in [0,1]$. Similar to the firms, households set wages according to the program

$$\max_{W_{i,t}} \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_w \beta)^n \left[-\Phi \frac{L_{i,t+n}^{1+1/\gamma}}{1+1/\gamma} + \lambda_{t+n} \frac{W_{i,t} \Pi_{\tau=0}^{n-1} (\pi_{t+\tau})^{\chi_w}}{W_{t+n}} \frac{W_{t+n}}{P_{t+n}} L_{i,t+n} \right].$$

The marginal utility of consumption, λ, is not indexed by i reflecting our assumption of complete markets.

2.4 Demand for Money

As previously discussed, modeling money demand needs to account for two important regularities found in the literature. On one hand, authors such as ACEL and CEE argue that in the context of DSGE models the short-run demand for money is what matters. In particular, they estimate an interest rate semi-elasticity of money demand around 1 (this finding is robust across different econometric techniques). On the other hand, studies about the welfare implications of inflation stress the importance of the long-run properties of money demand. For example, Lucas (2000) estimates that the long-run semi-elasticity of money demand lies between 5 and 7. Based on these numbers, he finds the welfare costs of inflation to be in the order of 1 percent of annual income. Furthermore, an extrapolation of his results implies that for a semi-elasticity of 1 the welfare cost is roughly 0.2 percent. This evidence raises the following dilemma: Too much elasticity delivers sizable welfare costs but worsens the short-run dynamics of money. Alternatively, low elasticities provide the right high frequency description of money at the expense of predicting too low welfare costs.

A simple yet formal way to solve the money demand dilemma is to assume time-dependent portfolio adjustment, i.e. agents re-optimize their money balances, M, infrequently, similar in spirit to the price- and wage-setting models of Woodford (2003) and Christiano, Eichenbaum, and Evans (2005). Specifically, a fraction, $1 - \xi_m$, of randomly chosen households is allowed to re-optimize their balances every period. As far as inactive households, the literature on portfolio choice provides little guidance regarding their behavior (Campbell and Viceira, 2002). Hence, if a household is not allowed to re-optimize today, her money holdings are adjusted according to the rule $M_{i,t} = \pi_{t-1} g^*_{t-1} M_{i.t-1}$, where π_{t-1} represents the last period inflation, and g^* is the growth rate of the aggregate shock S^*.[3] This rule does not allow for partial indexation as initial estimation attempts clearly showed that the indexation parameter was not identified.

As argued in Guerron-Quintana (2008), the sticky money assumption is likely to capture two important aspects of the economy. First, it proxies the degree of access to financial and banking services enjoyed by households. Prior to the widespread use of ATMs, electronic banking, and the branching liberalization of the 1980s, households spent an important amount of resources managing their accounts. Consequently, households had limited access to such services, which is parsimoniously captured in the model by infrequent portfolio re-balancing.

Second, the time-dependent assumption captures the costs faced by households when assessing the uncertainty surrounding the economy and the financial system. The presence of large costs makes it harder for households determine the state of the economy and in particular the risk exposure of banks. As a consequence, households may opt to limit their participation in financial markets. We can also think of the portfolio friction as indirectly capturing the infrequent participation of trading agents in the equity market reported by Vissing-Jorgensen (2003). As before, I interpret this infrequent re-optimization as the result of costs faced by households. The basic idea is that in the presence of these costs, households fully optimize their portfolio only periodically, and follow simple rules for changing their portfolio at other times.

The staggered money setting and the functional forms for the utility function imply that

[3] The presence of g in the indexation rule implies that there are no distortions from portfolio dispersion along the steady state growth path.

an optimizing (active) household at time t chooses money balances according to the program

$$\max_{M_{i,t}} \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_m \beta)^n \left[\psi_m \frac{\left(\frac{M_{i,t} \Pi_{\tau=0}^{n-1} (g^*_{t+\tau} \pi_{t+\tau})}{S^*_{t+n} P_{t+n}} \right)^{1-\zeta_m}}{1-\zeta_m} - \frac{\lambda_{t+n}}{P_{t+n}} (R_{t+n} - 1) M_{i,t} \Pi_{\tau=0}^{n-1} (g^*_{t+\tau} \pi_{t+\tau}) \right].$$

As shown in the appendix, the solution to the previous program gives rise to a money demand for active households, which requires that the expected marginal benefit of an extra dollar (enjoy additional utility), equals its expected marginal cost (foregone interest rate), i.e.

$$\underbrace{\psi_m \left(\frac{M^*_t}{S^*_t P_t} \right)^{-\zeta_m} x^1_{m,t}}_{Marginal\ Benefit} = \underbrace{x^2_{m,t}}_{Marginal\ Cost}, \tag{2}$$

where the terms x^1_m and x^2_m are given by

$$x^1_{m,t} = 1 + \beta \xi_m \mathbb{E}_t \left(\frac{g^*_t \pi_t}{g^*_{t+1} \pi_{t+1}} \right)^{1-\zeta_m} x^1_{m,t+1}, \text{ and } x^2_{m,t} = \lambda_t S^*_t (R_t - 1) + \beta \xi_m \mathbb{E}_t \frac{g^*_t \pi_t}{g^*_{t+1} \pi_{t+1}} x^2_{m,t+1}.$$

Here, M^*_t is the money holdings optimally chosen by active households today. Equation (2) implies that the annualized short- and long-run semi-elasticities of money demand are given by

$$E_{SR} = -\frac{(1-\xi_m)}{4(R-1)\zeta_m}, \text{ and } E_{LR} = -\frac{1}{4(R-1)\zeta_m}, \tag{3}$$

respectively; here, R is the steady state quarterly interest rate (see the appendix for details). As long as $\xi_m > 0$, we see that the short-run elasticity is smaller than its long-term counterpart, $|E_{SR}| < |E_{LR}|$. Consequently, the curvature parameter, ζ_m, can be used to describe the money demand in the long-run as required by welfare analysis. Furthermore, we can control the short-run dynamics of money via the sluggishness coefficient, ξ_m. Precisely this ability to capture the short- and long-run properties of money through separate parameters is exploited in the estimation section.

2.5 Government

The monetary authority sets the quarterly interest rate according to a Taylor rule. In particular, the central bank smooths interest rates and responds to deviations of actual inflation from steady state inflation, π, and deviations of output from its trend level, $(Y/S^*)_t$.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t/S_t^*}{(Y/S^*)_t}\right)^{\phi_y}\right]^{1-\rho_r} \exp(\sigma_m \varepsilon_{m,t}). \tag{4}$$

The term $\varepsilon_{m,t}$ is a random shock to the systematic component of monetary policy and is assumed to be standard normal; σ_m is the size of the monetary shock. Other authors have implemented similar Taylor rules, e.g. Del Negro et al. (2004) and Justiniano and Primiceri (2006).

As in the related literature (ACEL, and Levin et al., 2005), it is assumed that the government has access to lump-sum taxes and debt. Furthermore, the government consumes a stochastic fraction of output $G_t = S_{g,t}Y_t$ (Justiniano and Primiceri, 2006). The law of motion for S_g is $\log S_{g,t} = (1-\rho_g)\log S_g + \rho_g \log S_{g,t-1} + \sigma_g \varepsilon_{g,t}$, where $\varepsilon_{g,t}$ has a standard normal distribution.

2.6 Financial Intermediaries

Financial intermediaries receive deposits from households in the amount $\int (\mathcal{M}_{i,t-1} - M_{i,t})di + T_t$, which includes the monetary transfer T_t from the government. All this money is lent to the good firms so they can pay workers at the beginning of each period. Consequently, the clearing condition in the loan market is $\int W_t L_{j,t} dj = T_t + \int (\mathcal{M}_{i,t-1} - M_{i,t})di$.

3 Estimation

The data come from Haver Analytics database and span from 1984:I up to 2004:IV. I opt for this short sample based on two observations. To begin with, Stock and Watson (2007) report that the stochastic process for inflation have changed around 1984. Second, Fernandez-Villaverde and Rubio-Ramirez (2007 and 2008) argue that either stochastic volatility or parameter drifting are essential features of any DSGE model to capture the pre- and post-1984 features of the data, i.e. to properly account for the Great Moderation. Since a central point in this paper are the implications of inflation in recent years, introducing those features will only complicate the solution and estimation of the model without adding much substance to the subject.

The model is estimated using eight U.S. variables: the growth rates of output, consumption, investment, real wages, and real money balances ($\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta \log (W/P)_t$, $\Delta \log (M/P)_t$), the level of labor, nominal interest rates, and inflation ($\log L_t, i_t, \pi_t$). The series are built as follows: Real GDP per capita results from dividing nominal GDP by population and the GDP deflator. Real consumption is the sum of personal consumption of non-durables and services. Real investment consists of personal consumption expenditures of durables and gross private domestic investment. Both real consumption and real investment are divided by population to obtain per capita measures. The log of hours of all persons in the non-farm business sector divided by population corresponds to labor in the paper. Real wages result from dividing nominal wage per hour in the non-farm business sector by the GDP deflator. Interest rates correspond to the effective Federal Funds Rate while inflation is the quarterly log difference of the GDP deflator.

CEE interpret the utility from money as capturing the transaction role of money. Furthermore, Feenstra (1986) argues that money in the utility function is equivalent to a formulation where money provides liquidity services. These interpretations point to seasonally adjusted $M1$ as the relevant measure of money for estimation purposes. Specifically, the ratio $M1/P$ will be used as the counterpart of aggregate real balances in the model, $M/P = \int M_{i,t}/P di$. In results not reported here, I find that using $M2$ *minus* and its own opportunity cost as measures of money and interest rates deliver similar implications.

Bayesian Inference

Following Schorfheide (2000), Del Negro et al. (2004), and Smets and Wouters (2007), the linearized version of the model is estimated using Bayesian methods. In particular, the posterior distribution of the structural parameters is characterized using a Markov Chain Monte Carlo (MCMC) approach (for details of this algorithm see the appendix and the excellent surveys of An and Schorfheide, 2007, and Geweke, 1999). Since there are eight observable variables and only five structural shocks, I avoid stochastic singularity by following Sargent (1989) in including measurement errors to the state space representation used to estimate the model.[4] These errors are assumed to be iid and distributed $\mathbb{N}(0, \sigma)$. The scale of these errors can vary across the measurement equations. The results in the next sections are based on a Markov chain of 150,000 draws after discarding 10,000 replications from a burn in phase.

Priors

A subset of the parameter space was fixed: $\alpha = 0.36$, $\delta = 0.025$, $\zeta = 6$, $\zeta_w = 21$, $S_g = 0.22$. When I tried to estimate the elasticities of substitution, ζ and ζ_w, I found that their posteriors practically sit on top of their priors. Hence, those parameters are the ones used in Christiano et al. (2005). The steady state fraction S_g was set to match the average share of government expenditure in output in the sample. Since steady state labor, L_{ss}, is estimated, the parameter Φ is endogenously determined.

Based on the discussion in Section 2.4, I set the priors for ξ_m and ζ_m to capture the short- and long-run elasticities of money demand (see Table 1). For the average annualized interest rate of 5.4 percent in the sample, the implied mean elasticities are 6.25 and 12.5, respectively. The long-run value is consistent with the results in Mankiw and Summers (1986) and Ball (1998). On the other hand, the short-run elasticity is large relative to ACEL and Christiano et al. (1999). I choose to do so to keep symmetry among the sticky contract assumptions in the model. Notice that the price, wage, and money contracts share the same prior. As

[4] Since we observe the exact values of interest rates, measurement errors were not included in the equation corresponding to interest rates in the state space representation.

we will see in the next section, the money demand priors are not very informative in the sense that the inference approach uncovers distinct posteriors. From equation (3), it is clear that the parameters ξ_m and ζ_m completely characterize the dynamics of money demand, i.e. the data are silent about the remaining parameter in the money block, ψ_m. Hence, this parameter is set to the value chosen in CEE: 0.055.

The prior distributions for the remaining parameters are reported in Table 1. These priors are loose and consistent with those typically used in the literature (see Del Negro et al., 2004, Levin et al., 2005, and Justiniano and Primiceri, 2006, Smets and Wouters, 2007). For example, the priors for the dispersion parameters χ and χ_w are beta $B(0.5, 0.2)$. The large standard deviation reflects our relative ignorance about those parameters.

Median Estimates

Table 2 reports the median estimates for the structural parameters in my formulation. Numbers in parenthesis correspond to the 5% and 95% percentiles for each parameter (a 90% probability interval). The absence of the price of investment as an observable variable implies that the two trends in the model, S^L and S^K, are not identified separately. Therefore, the steady state growth rate of the investment-specific shock is set to one, $g^K = 1$.

Broadly speaking, the estimates are in line with the results previously found in the literature (CEE, and Smets and Wouters, 2007). For example, the model displays significant habit formation, around 0.93, and adjustment costs of investment in the order of 3.83. The habit formation estimate may seem high relative to that in ACEL; however, the estimate is perfectly in line with the findings in Fernandez-Villaverde and Rubio-Ramirez (2008). The empirical results imply that prices and wages are re-optimized on average every 3 and 1.5 quarters, respectively. It is tempting to contrast the length of the price/wage contracts with the results in Nakamura and Steinson (2008) and Bils and Klenow (2004). Yet the presence of partial indexation makes such comparison unfeasible. The estimated Taylor rule implies that the central bank actively responds to inflation and smooths interest rates with coefficients similar to those found in Justiniano and Primiceri (2006). In terms of the structural errors, I find they display significant serial correlation. The inference approach estimates a Frisch elasticity, γ, of 1.68, a value consistent with that reported in Fernandez-Villaverde and Rubio-Ramirez (2007), and Justiniano and Primiceri (2006). When we turn to inflation, we observe that the inference approach places its steady state value around 2%, which is close to the mean inflation in the sample (2.3%).

The median estimates for the money demand coefficients, ζ_m and ξ_m, are 1.88 and 0.85, respectively. The large value for the Calvo lottery in money reflects the estimation's attempt to capture the high frequency properties of money. In fact, its implied short-run elasticity is 1.73; interestingly, CEE report a close value. On the other hand, the empirical results suggest a long-run elasticity of 11.5, which is well within the boundaries found in the literature (see Goldfeld and Sichel, 1990).[5] The estimates of ζ_m and ξ_m also indicate that the assumptions outlined in Section 2.4 are flexible enough to simultaneously capture the high and low frequency properties of money demand.

4 Welfare Cost of Deflations

Inflation is potentially welfare reducing in the model due to several factors. To begin with, the presence of money demand (equation 2) makes inflation costly because of its tax implications as in Bailey (1956). Indeed, low inflation implies reduced nominal interest rates (Fisher, 1930), which benefits households because consuming real balances becomes inexpensive. A second source of distortion in the economy is staggered price contracts. To see this point, note that, ignoring growth and capital utilization, aggregate output in the model is

$$\begin{aligned} y_t &= \left[k_t^{\alpha}\left(L_t\right)^{1-\alpha} - \psi\right]/s_t, \\ s_t &\equiv \int\left(\frac{P_{i,t}}{P_t}\right)^{-\zeta} di. \end{aligned} \tag{5}$$

[5] The long-run elasticity is somehow larger than that reported in Ireland (2008). Although we use $M1$ and similar time spans, our approaches differ in two dimensions: 1) while Ireland proposes a static money demand, I propose a dynamic formulation; and 2) Ireland estimates his model using dynamic OLS.

Schmitt-Grohe and Uribe (2005) establish that s is bounded below by 1 and captures the degree of price dispersion in the economy. Staggered prices force optimizing firms to heavily review their prices to keep up with inflation, which induces dispersion, i.e. $s >> 1$. Large price stickiness (big ξ) or small price indexation (low χ) exacerbates this dispersion, decreases aggregate output, and ultimately reduces welfare (see Section 5.1). Finally, costly investment adjustment and habit formation make consumption and investment decisions relatively inflexible in the short term. Such inflexibility may also amplify the effects of inflation, especially during the transition from high to low inflation.

A simple way to capture the cost of inflation is to measure households' dislike for high-inflation environments. Following Cooley and Hansen (1989), Ireland (1997), and Lucas (2000), let us define the welfare cost of a high-inflation regime, Λ, as the fraction of consumption in the low-inflation steady state that households are willing to give up to be indifferent between the low and high inflation regimes.[6] To simplify the calculations below, the growth rate of the economy, $g_t^* = S_t^*/S_{t-1}^*$, is set to 1. This assumption is inconsequential for the rest of the analysis as I am solely interested in measuring the welfare costs under perfect foresight. Define the social utility function by

$$\begin{aligned} V &\equiv \int \sum_{t=0}^{\infty} \beta^t \left[S_t^{Uc} \log(C_{i,t} - bC_{t-1}) - \Phi \frac{L_{i,t}^{1+1/\gamma}}{1+1/\gamma} + \psi_m m_{i,t}^{1-\zeta_m} \right] di \qquad (6) \\ &= \sum_{t=0}^{\infty} \beta^t \left[S_t^{Uc} \log(C_t - bC_{t-1}) - \Phi \frac{L_t^{1+1/\gamma}}{1+1/\gamma} + \psi_m \left((1-\xi_m)(m_t^*)^{1-\zeta_m} + \xi_m m_{t-1}^{1-\zeta_m} \right) \right], \end{aligned}$$

where, C, and L correspond to both aggregate and individual consumption, and labor. This is a direct consequence of the complete market assumption. For the money demand block, m^* and m are real balances chosen by active and inactive households, respectively (see Appendix A). The dynamic nature of the model allows us to distinguish two types of welfare costs: in steady state and during the transition. In the absence of uncertainty and using the

[6] Schmitt-Grohe and Uribe (2005) use a related measure to analyze the implications of alternative monetary rules.

functional forms given in Section 3, the social utility function in steady state collapses to

$$V^i\left(C^i, L^i, m^i\right) \equiv (1-\beta)^{-1}\left[\log(1-b)C^i - \Phi\frac{(L^i)^{1+1/\gamma}}{1+1/\gamma} + \psi_m\left(m^i\right)^{1-\zeta_m}\right].$$

Here, the index i indicates whether we refer to the high-inflation steady state ($i = H$), or the low-inflation steady state ($i = L$). In addition, C^i, L^i, and m^i correspond to the steady state consumption, labor, and real balances on regime i. The rule of thumb for money choices implies that households choose the same steady state money balances, i.e. $m = m^*$. With these definitions in place, the steady state welfare gain, Λ_{ss}, is given by

$$\begin{aligned} V^H &= V^L\left((1-\Lambda_{ss})C^L, L^L, m^L\right), \\ V^H &= \log(1-\Lambda_{ss}) + V^L. \end{aligned} \tag{7}$$

The second line is a consequence of the functional forms used in this paper. A positive Λ_{ss} indicates that households prefer the low inflation regime, i.e. they willingly give up consumption to avoid the high inflation equilibrium. In other words, inflation entails a steady state welfare loss if $\Lambda_{ss} > 0$.

As previously argued, nominal and real frictions can make painful the transition from high to low inflation. To quantify their effect on welfare, suppose as in Taylor (1983) and Ireland (1997) that the monetary authority fully commits to a new low inflation policy at time $t = 0$. In the model, such an exercise requires moving the target inflation in the Taylor rule (4) from a high rate, π^H, to a new low inflation π^L. Households and firms observe this change and conclude that the interest rate in the old inflationary regime is large relative to the new steady state.[7] This high interest rate in turn discourages economic activity as it makes the wage bill ($R_t W_t L_t$) more expensive and consumption less attractive (it is more rewarding to send money to the bank). Therefore, from the point of view of the new low-inflation regime,

[7] In steady state, the Taylor rule imposes $R = \pi^{\phi_\pi}\left(Y/S^*\right)^{\phi_y}$. Other things equal, interest rates in the low regime, R_L, are smaller than those in the high regime, R_H, if and only if $\pi^L < \pi^H$.

the old inflationary steady state resembles the initial response of a contractionary monetary shock. It is precisely this contractionary aspect that makes the deflationary path costly, i.e. household must be compensated to undertake the transition. How painful this transition is depends, among other things, on the length of the sticky contracts, habit formation, and the cost of capital adjustment.

Let $\{C_t^+\}_{t=0}^{\infty}$, $\{L_t^+\}_{t=0}^{\infty}$, and $\{m_t^+, m_t^{*+}\}_{t=0}^{\infty}$ denote the sequence of consumption, labor, and real balances associated with the transitional path from the high to the low inflation steady states. These sequences in turn define the transitional social utility function immediately following the adoption of the new policy

$$V^+ \equiv \sum_{t=0}^{\infty} \beta^t \left[\log(C_t^+ - bC_{t-1}^+) - \Phi \frac{(L_t^+)^{1+1/\gamma}}{1+1/\gamma} + \psi_m \left((1-\xi_m) \left(m_t^{*+}\right)^{1-\zeta_m} + \xi_m \left(m_{t-1}^+\right)^{1-\zeta_m} \right) \right],$$

As with the steady-state welfare case, define the transitional cost of the lower inflation policy as the fraction, Λ_+, of the low-inflation regime's consumption that consumers surrender to avoid the transition. That is,

$$V^+ = V^L \left((1-\Lambda_+) C^L, L^L, m^L \right). \tag{8}$$

A negative value of Λ_+ indicates that households must be compensated to face the deflationary path. Using equations (7) and (8), we conclude that the total gains of the policy change are $\Lambda = \Lambda_{ss} + \Lambda^+$. Under the convention previously discussed, positive values of Λ indicate that high inflation is indeed costly.[8] The actual sign of Λ will depend on whether the gains from lower steady state inflation, Λ_{ss}, overcome the welfare loss endured by households during the deflationary transition, Λ^+.

Before fleshing out the results, we must decide the values for the high and low steady

[8] To get a description of the transitional dynamics, I use a first-order perturbation algorithm to evaluate V^+ (see Schmitt-Grohe and Uribe, 2004, and Judd, 1998). The approximation is done about the low-inflation steady state. I compute V^+ using the difference between the high- and low-inflation states as the initial condition for the transitional path. The results from a second-order approximation were similar up to the second decimal to those reported in Table 3.

state inflation rates. Two factors are decisive in selecting the low inflation rate. First of all, note that steady state inflation is an estimated parameter in the model. Furthermore, welfare will be computed using the low inflation regime as the reference point. Therefore, I set π^L to 2%, the value reported in Table 2, in an attempt to keep consistency between the estimation and welfare parts of the model. The high inflation rate is 12%, a number that will make the results comparable to those in the related literature (Ireland, 1997, Lucas, 2000, and Cooley and Hansen, 1989).

5 Results

To estimate the effects of inflation in the model, suppose that the economy is initially in a steady state with an annual inflation of 12 percentage points. At time $t = 0$ the central bank fully commits to bring inflation down to 2%. Table 3 reports the steady state, transitional, and total annualized costs from the deflationary exercise. The first row presents the results when welfare is computed using the median estimates reported in Table 2. The results indicate that 10 percentage points of inflation entail a steady state cost, Λ_{ss}, equivalent to 13% of annual consumption. Using the ratio of consumption to output in steady state, we find that the cost of inflation represents 6.6% of annual income.[9] This result is substantially larger than that reported in Lucas (2000). As we shall see in the next section, frictions like habit formation and price indexation partially explain the difference between the results here and Lucas'.

When we turn to the transitional path, we note that the change from the high to the low inflation environment imposes a significant burden on households, Λ_{+}, which roughly amounts to -0.57% of annual consumption. The negative sign indicates that households have to be compensated to face the deflationary path. To understand such result, recall that reducing inflation requires lowering real economic activity, which is achieved throughout an initial surge in interest rates. Due to the presence of real and nominal frictions, this spike in

[9] The ratio of consumption to output in steady state equals 0.51 in the model.

turn induces a persistent recession in the economy. The transitional welfare loss results from the large and persistent drop in consumption associated with the contractional monetary policy. Unlike Ireland (1997), the transitional cost is only a modest fraction (0.05) of the steady state welfare gain. The next section shows that adjustment costs in investment, a friction absent in Ireland's formulation, seems to ameliorate the welfare cost during the transtion.

The impulse responses (solid lines) in Figure 1 confirm the contractional effects of pushing the economy to the low inflation state.[10] Note that the new policy successfully brings inflation down to 2%; at the same time, the interest rate initially rises to 15%, but, as inflation retreats, interest rates converge to its new steady state of 5%. The initial spike in interest rates makes working capital ($W_t L_t R_t$) expensive, which discourages production. Note, however, that as inflation declines, so does price dispersion. Eventually, this second force takes over and contributes to the recovery of output (equation 5), which ends up in a higher steady state. Consumption reaches its lowest level, -2.5%, about 2 quarters after the adoption of the new policy. The contraction in output reduces the demand for labor and hence increase leisure along the deflationary path. Its surge helps to make the transition less costly because leisure is part of the welfare criterion (equation 6). Finally, the model predicts that it takes less than 20 quarters for most variables to converge to the new steady state (notable exceptions are consumption and real wages). This convergence seems consistent with the evidence from Volcker's deflationary era (It took roughly from 1981 to 1985 for the U.S. inflation to fall from 11% to a value below 4%).

The steady state and transitional results indicate that an inflation of 12%, relative to an equilibrium with 2%, entails a welfare cost of 12.4% of annual consumption or 6.2% of annual income. Although this estimate looks out of touch with the results in Cooley and Hansen (1989) and Lucas (2000), it is consistent with the findings in a recent paper by Arouba and Schorfheide (2008). Indeed, these later authors find that 10 percentage points of inflation

[10]The impulse responses for inflation and interest rates are expressed as percentage points. For all other variables, the impulse responses are percentage deviations from the steady state with 2% inflation.

can represent as much as 16% of annual consumption. Yet our welfare estimates are still large relative to those in the sticky-price formulation pursued in Ireland (1997). Hence it seems necessary to understand whether the additional nominal/real frictions in the model drive the different welfare estimates. In the experiments to follow, one parameter will be changed at a time while the remaining ones are kept at their median estimates.[11]

5.1 Role of Frictions

Schmitt-Grohe and Uribe (2005) argue that partial price indexation induces significant price dispersion in DSGE models. With partial indexation, inactive firms cannot fully incorporate changes in past inflation as prices are adjusted according to $P_{i,t} = (\pi_{t-1})^{\chi} P_{i,t-1}$. Hence, once a firm happens to re-optimize it does very aggressively to keep up with inflation. High inflation and low price indexation (low χ) induce stronger price revisions by active firms leading to substantial price dispersion. But equation (5) shows that as price dispersion raises output declines, which is potentially welfare reducing. Figure (3) precisely illustrates this interaction between inflation and indexation and their effects on steady state consumption and output.

To fully characterize the effects of indexation, the second row in Table 3 reports the welfare results when χ and χ_w are set to 1. Full price/wage indexation drives down the steady state welfare cost by a factor of 3. Indeed, the welfare cost is equivalent to 1.75% of annual output, which is surprisingly close to the welfare cost reported in the sticky-price model of Ireland (1997). This finding stresses the importance of a better understanding of the mechanisms behind price setting at the micro level. In terms of the deflationary path, we observe that full indexation amplifies the dynamic welfare effect although by a small margin. As shown by Figure (1), this increase results from the strong decline in real balances following the deflationary shock.

The third row in Table 3 shows that eliminating wage dispersion has essentially no impli-

[11] This way of analyzing the role of different parameters in an estimated model is suggested by CEE and ACEL.

cations on the welfare estimates. To understand this unexpected result, recall that complete markets allow households to buy insurance against events such as extended periods of no wage revisions. Consequently, complete markets make the welfare measure (equation 6) insensitive to developments in the labor market. Similarly, flexibility in wage setting has no impact on the welfare cost of inflation due to the households' ability to smooth consumption via insurance markets (row 8 inTable 3).

Lucas (2000) emphasizes the crucial role of the elasticity of money demand for welfare analysis. Indeed, he finds that a small elasticity is typically associated with negligible welfare costs of inflation. To assess the implications of Lucas' observation in the context of DSGE models, I set the parameter ζ_m such that the implied long-run semi-elasticity matches that of CEE, 1, a value 10 times smaller than in the baseline case. By shrinking ζ_m we are effectively reducing the area underneath the demand for money curve. Bailey's (1956) theory in turn suggests that inflation should become less costly with the reduced elasticity. Accordingly, the results in Table 3 show that the welfare cost in steady state is two-thirds of that under the benchmark formulation. This finding concurs with those of Lucas but it also highlights the fact that other frictions, such as price indexation or habit formation, play an even more important role in the welfare calculations.

The fifth row of Table 3 presents the welfare results in the absence of sticky money. Flexible real balances ($\xi_m = 0$) imply that the short- and long-run elasticities both equal 11.5 (equation 3). This result combined with the absence of steady state dispersion in money holdings explains why the steady state welfare is unaffected. That money has little impact on the transition is an unexpected outcome. To understand this finding, recall that complete markets and the Taylor rule pin down the dynamics of the monetary part of model (this is Woodford's, 1998, cashless economy result). As a consequence, consumption and labor become insulated from fluctuations in the money market implying the small change in the transitional welfare cost. We conclude that sticky money demand is important for welfare analysis in DSGE models because it allows the estimation procedure to recover a high long-run elasticity of money demand while preserving the short-run dynamics of real balances consistent with the data.

The results in Table 3 indicate that if habit formation vanishes, the steady state cost is smaller than in the baseline scenario. To understand this finding, note that steady-state real balances, m, and marginal utility of consumption, λ, are given by

$$\begin{aligned} m &= \left(\frac{\psi_m}{(\pi/\beta - 1)} \frac{1}{\lambda} \right)^{1/\zeta_m}. \\ \lambda &= \frac{1}{C(1-b)} \end{aligned} \tag{9}$$

Clearly as habit formation declines, so does the marginal utility of consumption. To compensate for the lost utility, households substitute consumption with real money balances. Since inflation acts as tax on real balances, households have a stronger desire for real balances in the low inflation environment. Therefore, the substitution effect is larger in the low inflation scenario, i.e. $\partial m^H/\partial b < \partial m^L/\partial b$. These arguments in turn indicate that the difference between utility from money in the low and high states, $\frac{(m^L)^{1-\zeta_m} - (m^H)^{1-\zeta_m}}{1-\zeta_m}$, is increasing in habit formation. But this difference is precisely what matters for steady state welfare comparisons (equations 6 and 7). Hence the lower habit formation is, the lower the steady state welfare costs of inflation.

Notice that the transitional welfare cost almost disappears in the absence of habit formation. This finding is the product of two forces. First, smaller habit formation makes consumption more flexible allowing it to quickly adjust to the new steady state. Second, households heavily substitute consumption with leisure when habit formation is low (an explanation along the lines of the previous paragraph applies). This intuition is readily confirmed by the impulse responses reported in Figure 1. The strong substitution towards leisure is apparent from the large decline in labor. Furthermore, consumption returns to its pre-shock steady state in less than three years. In the baseline scenario, consumption has not reached its steady state even five years after the shock.

Table 3 shows that price flexibility makes the welfare cost of inflation decline in steady state as well during the transition. As previously argued, the absence of sticky price contracts completely eliminates price distortions; firms can freely adjust prices every period, which increases output and hence decreases the welfare cost of inflation. In fact, the welfare cost without sticky prices is almost identical to that computed with full price indexation. Figure 2 depicts the transitional responses in the absence of sticky prices (dashed line). Relative to the baseline formulation, we note two main features: 1) consumption, output, and investment are less responsive and, 2) all variables converge more quickly to the new steady state. For example, inflation falls below 4 percent 5 quarters after the monetary shock, which is 2 quarters faster than in the benchmark case.

The row labeled "No Invest Adj Cost" displays the welfare results in the absence of adjustment costs in investment. The cost function Γ is designed to have non-zero effects only during dynamic paths, which explains why steady state welfare remains unchanged. Figure (2) shows that investment is highly responsive to the deflationary shock (dotted lines). As a consequence, capital displays a strong and persistent decline. Other things equal, this contraction induces a significant drop in labor productivity causing a persistent decline in real wages as well as in consumption. It is precisely this lasting contraction in consumption that worsens the transitional welfare cost. From this experiment, we conclude that the deflationary transition is more costly in Ireland (1997) because his formulation does not display adjustment costs in investment.

When we eliminate all real/nominal frictions in the model, the steady-state inflation entails a modest welfare loss of 1% of annual consumption (0.5% of annual income). Recall that in a frictionless economy, inflation is solely costly due to its tax implications as in Bailey (1956). Therefore, it is not surprising that the welfare estimates concurs with the findings of Lucas (2000), who argues that inflation is welfare reducing via its effects on money demand. Figure 2 (starred lines) show that inflation adjusts to its new steady state without any real effect on the economy resulting in the nil welfare cost during the transition reported in Table

3. Without any frictions, the model displays Modigliani's (1963) real dichotomy, which explains the costless deflationary path.

To conclude this section, it is worth briefly mentioning the role of parameter uncertainty. The 90% probability intervals reported in Table 3 indicate this type of uncertainty primarily affects the steady state welfare estimates. In fact, the upper bound of this welfare cost can be as high as 34 percentage points of annual consumption in the baseline model. The results in Table 2 and 3 suggest that uncertainty around the sticky price parameter, ξ, and price indexation, χ, drive the large probability intervals associated with welfare. Note that when those frictions vanish the 90% probability intervals for welfare shrink by a significant amount. This situation does not happen if we suppress, for example, habit formation or sticky wages.

6 Conclusion

What is the welfare cost of inflation? How much should society give up to live in a low inflation environment? Answering these questions has been a major endeavor in economics for more than half a century. This paper has revisited those questions but from the perspective of an estimated New-Keynesian model. According to the benchmark model, which features real and nominal frictions, 10 percentage points of inflation entails a total welfare cost of 12.4% of annual consumption (6.2% in annual output). From the point of view of a policy maker, this result suggests that environments with low inflation are desirable. The result is even more appealing for it comes from the type of models now being used for policy analysis around the world.

A second important contribution of this paper is the analysis of the different frictions on both the static and dynamic welfare costs of inflation. The results indicate that price inflexibility increase the transitional welfare cost by roughly 0.40%. Furthermore, habit formation and price indexation are crucial components for welfare analysis in steady state as well as during the transition.

As in Levin et al. (2005), I find that parameter uncertainty does indeed influence our inference of the welfare costs, in particular those of inflation. As a consequence, the results in this paper suggest that parameter uncertainty must be incorporated into policy analysis to have a better understanding of the welfare costs of deflations.

In the experiments outlined in this paper, price indexation has emerged as an essential determinant of the welfare implications of inflation. This finding, therefore, calls for a better understanding of the microfundations behind price setting. At stake is whether inflation imposes a large burden on society or not.

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7 Appendix A: Money Demand Equation

Active households in the money market set their real balances according to the program

$$\max_{M_{i,t}} \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_m \beta)^n \left[\psi_m \frac{\left(\frac{M_{i,t} \Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau})}{S_{t+n}^* P_{t+n}} \right)^{1-\zeta_m}}{1-\zeta_m} - \frac{\lambda_{t+n}}{P_{t+n}} (R_{t+n}-1) M_{i,t} \Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau}) \right].$$

The presence of the terms $g_{t+\tau}^* \pi_{t+\tau}$ is a consequence of the rule of thumb followed by inactive households (non-optimizing households). This is so because a household given the option to re-optimize money balances must take into account that with positive probability she may not re-optimize ever again. Taking derivatives with respect to $M_{i,t}$ we arrive to the following first order condition

$$\mathbb{E}_t \sum_{n=0}^{\infty} (\xi_m \beta)^n \left[\psi_m \left(\frac{M_{i,t} \Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau})}{S_{t+n}^* P_{t+n}} \right)^{-\zeta_m} \frac{\Pi_{\tau=0}^{n-1} \pi_{t+\tau}}{S_{t+n}^* P_{t+n}} - \frac{\lambda_{t+n}}{P_{t+n}} (R_{t+n}-1) \Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau}) \right] = 0.$$

Next, re-write the first order condition as a restriction on money holdings, M_t^*, which requires that the expected marginal benefit of an extra dollar, equals its expected marginal cost, i.e.

$$\underbrace{\psi_m \left(\frac{M_t^*}{S_t^* P_t} \right)^{-\zeta_m} x_{m,t}^1}_{\textit{Marginal Benefit}} = \underbrace{x_{m,t}^2}_{\textit{Marginal Cost}} . \tag{10}$$

where the terms x_m^1 and x_m^2 are given by

$$\begin{aligned} x_{m,t}^1 &\equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_m \beta)^n \left(\frac{\Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau})}{\Pi_{\tau=1}^{n} (g_{t+\tau}^* \pi_{t+\tau})} \right)^{1-\zeta_m}, \\ x_{m,t}^2 &\equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_m \beta)^n \lambda_{t+n} S_{t+n}^* (R_{t+n}-1) \frac{\Pi_{\tau=0}^{n-1} (g_{t+\tau}^* \pi_{t+\tau})}{\Pi_{\tau=1}^{n} (g_{t+\tau}^* \pi_{t+\tau})}. \end{aligned}$$

After some tedious algebraic manipulations these last equations collapse to

$$\begin{aligned} x_{m,t}^1 &= 1 + \beta \xi_m \mathbb{E}_t \left(\frac{g_t^* \pi_t}{g_{t+1}^* \pi_{t+1}} \right)^{1-\zeta_m} x_{m,t+1}^1, \\ \text{and } x_{m,t}^2 &= \lambda_t S_t^* (R_t - 1) + \beta \xi_m \mathbb{E}_t \frac{g_t^* \pi_t}{g_{t+1}^* \pi_{t+1}} x_{m,t+1}^2. \end{aligned}$$

Given the complete market assumption, the fraction $(1-\xi_m)$ of households re-optimizing money holdings today chooses the same level, M_t^*, governed by equation (10) . Moreover, the random nature of the Calvo lottery implies that inactive households this period have on average the same money levels as yesterday, M_{t-1}, adjusted by technology growth and inflation (Woodford, 2003). Hence, aggregate money balances, M_t, result from the combination of both active and inactive money:

$$M_t = \int M_{i,t} di = (1-\xi_m) M_t^* + \xi_m \pi_{t-1} g_{t-1}^* M_{t-1}. \tag{11}$$

Let $\widetilde{\lambda}_t = \lambda_t S_t^*$ and $m_t = \frac{M_t}{S_t^* P_t}$, then the aggregate equation (11) becomes $m_t = (1-\xi_m) m_t^* + \xi_m \frac{g_{t-1}^* \pi_{t-1}}{g_t^* \pi_t} m_{t-1}$. In steady state this condition implies $m = m^*$. Furthermore, the optimal condition for money in steady state collapses to $\psi_m (m)^{-\zeta_m} = \widetilde{\lambda} (R-1)$. From this equation is clear that the annualized long-run interest semi-elasticity of money demand is

$$E_{LR} = -\frac{\partial \log m}{\partial R} = \frac{1}{4(R-1)\zeta_m}.$$

In the short term, contemporaneous changes in interest rates, R_t, affect aggregate money holdings, M_t, only through its influence on active households in the financial market, i.e. those who re-optimize money balances today, M_t^*. This observation in turn implies that the short-term elasticity of money demand is $E_{SR} = -\frac{\partial \log m_t}{\partial R_t} = -(1-\xi_m) \frac{\partial \log m_t^*}{\partial R_t}$. Finally, optimizing households understand that 1) the change in interest rates is permanent, and 2) they may not re-optimize real balances ever again. Hence, households who re-optimize at time t adjust their real balances to the new steady state, i.e. $-\frac{\partial \log m_t^*}{\partial R_t} = \frac{1}{4(R-1)\zeta_m}$. The combined effect is that the short-run semi-elasticity is given by

$$E_{SR} = (1-\xi_m) \frac{1}{4(R-1)\zeta_m}.$$

8 Appendix B: MCMC Algorithm

Let $p(\varphi)$ and $p(Y_T|\varphi)$ be the prior distribution of the parameter vector φ and the likelihood of the data conditional on the parameter vector, respectively. I use the data, a state-space representation of the model, and the Kalman Filter to evaluate the posterior distribution $p(\varphi|Y_T)$. A random walk Metropolis-Hasting algorithm is applied to generate 150000 draws $\varphi_{(n)}$ from the posterior distribution. At each iteration n, a candidate parameter vector $\widetilde{\varphi}$ is drawn from the distribution $\mathbb{N}(\varphi_{(n-1)}, c_o^2 \sum)$ and the acceptance ratio, r, is computed $r = \frac{p(Y_T|\widetilde{\varphi})p(\widetilde{\varphi})}{p(Y_T|\widetilde{\varphi}_{(n-1)})p(\widetilde{\varphi}_{(n-1)})}$.The new draw $\widetilde{\varphi}$ is kept with probability $\min(r, 1)$ and reject it otherwise. To characterize the variance of the jumping distribution, $c_o^2 \sum$, I proceed as follows. First, I apply Christopher Sims' *csminwel* code to compute the mode of the posterior distribution. To that end, 1000 draws from the prior distribution were used to calculate the posterior. The 10 draws achieving the highest posteriors are the initial points for Sims' minimization algorithm. Second, after checking that the algorithm delivers similar modes, at least for three different initial conditions, I compute the inverse Hessian at the mode and use it as variance of the jumping distribution. Third, the constant c_o is set to achieve an acceptance rate close to 0.35, a value typically suggested in the literature (Casella and Roberts, 2004).

To check convergence of the resulting algorithm, I run three separate chains each starting from a different random draw of the jumping distribution centered at the mode. The medians of the resulting chains lied within 2% of each other. To further confirm convergence, I compute the potential scale reduction factors, which were less than 1.005.

Table 1: Priors Densities for Structural Parameters

σ_m	σ^{g_L}	σ^{g_K}	σ^{U_C}	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
IG [2,2]	IG [2,2]	IG [2,2]	IG [2,2]	B [0.5,0.1]	B [0.5,0.1]	B [0.5,0.1]	N [1,0.15]	B [0.75,0.1]	N [1.70,0.3]
ϕ_y	κ	ξ_m	ζ_m	$100(g_L-1)$	$100(g_K-1)$	$100(\pi-1)$	L	$\varkappa_a$	β
G [0.12,0.1]	N [3,1]	B [0.5,0.1]	N [1.5,0.15]	N [0.5,0.1]	N [0.5,0.1]	N [0.5,0.1]	N [52.89,5]	N [0.17,0.1]	B [0.99,0.002]
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}	g_K				χ	χ_w	σ_{error}
B [0.5,0.15]	B [0.5,0.15]	B [0.5,0.15]	N [1.01,0.003]				B [0.5,0.2]	B [0.5,0.2]	IG [0.05,0.03]

Notes: IG~Inverse Gamma, B~Beta, N~Normal, G~Gamma

Mean and Standard Deviation in square brackets

All measurement errors have priors as described by σ_{error}

Table 2: Estimated Parameters Baseline Case

σ_m	σ_{g_L}	σ_{g_K}	σ_{U_C}	σ_{gov}	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
0.18 [0.15,0.21]	0.36 [0.29,0.45]	0.32 [0.21,0.42]	3.06 [2.05,4.62]	0.52 [0.36,0.73]	0.92 [0.87,0.96]	0.34 [0.25,0.43]	0.67 [0.59,0.77]	1.68 [0.87,2.66]	0.82 [0.73,0.86]	1.80 [1.66,1.95]
ϕ_y	κ	ξ_m	ζ_m	$100(g_L-1)$	$100(\pi-1)$	L	χ	χ_w	$\varkappa_a$	β
0.034 [0.021,0.050]	3.83 [2.67,5.13]	0.85 [0.81,0.89]	1.88 [1.46,2.19]	0.39 [0.29,0.50]	0.5 [0.37,0.68]	58.22 [51.66,63.16]	0.40 [0.20,0.65]	0.67 [0.44,0.85]	0.25 [0.12,0.45]	0.994 [0.992,0.995]
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}	ρ_g	σ_{out}	σ_{cons}	σ_{invest}	σ_{labor}	σ_{wage}	σ_{inflat}	σ_{money}
0.67 [0.56,0.76]	0.82 [0.71,0.97]	0.98 [0.97,0.99]	0.54 [0.30,0.78]	0.36 [0.31,0.42]	0.32 [0.25,0.39]	1.67 [1.45,1.92]	0.053 [0.03,0.10]	0.15 [0.12,0.18]	0.48 [0.41,0.55]	1.14 [0.94,1.32]

Notes: L: steady state labor; π: steady state inflation; g_L: growth rate of neutral technology

g_k: growth rate of investment specific shock

Table 3: Welfare Cost Estimates[a]

	Steady State (Λ_{ss})	Transition (Λ_+)	Total (Λ)
Baseline	13.05 [5.74,34.44]	-0.57 [-1.40,-0.20]	12.36 [5.38,33.16]
Full Indexation (χ=1,χ_w=1)	3.46 [2.26,7.22]	-0.66 [-1.25,-0.39]	2.80 [1.52,6.56]
Full Wage Indexation (χ_w=1)	13.05 [5.74,34.44]	-0.56 [-1.37,-0.19]	12.38 [5.74,33.19]
Small Elasticity: $E_{LR} = 1$	8.97 [2.81,30.69]	-0.59 [-1.41,-0.22]	8.33 [2.12,30.16]
No Money Stickiness	13.05 [5.74,34.44]	-0.58 [-1.40,-0.21]	12.36 [5.38,33.16]
No Habit Formation	10.08 [3.55,31.42]	-0.17 [-0.63,-0.05]	9.90 [3.49,30.79]
No Price Stickiness	3.46 [2.26,7.22]	-0.24 [-0.45,0.08]	3.18 [1.99,7.03]
No Wage Stickiness	13.05 [5.74,34.44]	-0.46 [-1.45,-0.14]	12.46 [5.38,33.15]
No Invest Adj Cost	13.05 [5.74,34.44]	-1.30 [-2.59,-0.64]	11.44 [4.51,32.48]
No Frictions	1.02 [0.60,2.89]	-1.1$e-5$	1.02 [0.60,2.89]

[a]Estimates expressed as percentage of annual consumption.

90% probability interval in square brackets.

Figure 1: Impulse Responses Deflationary Shock

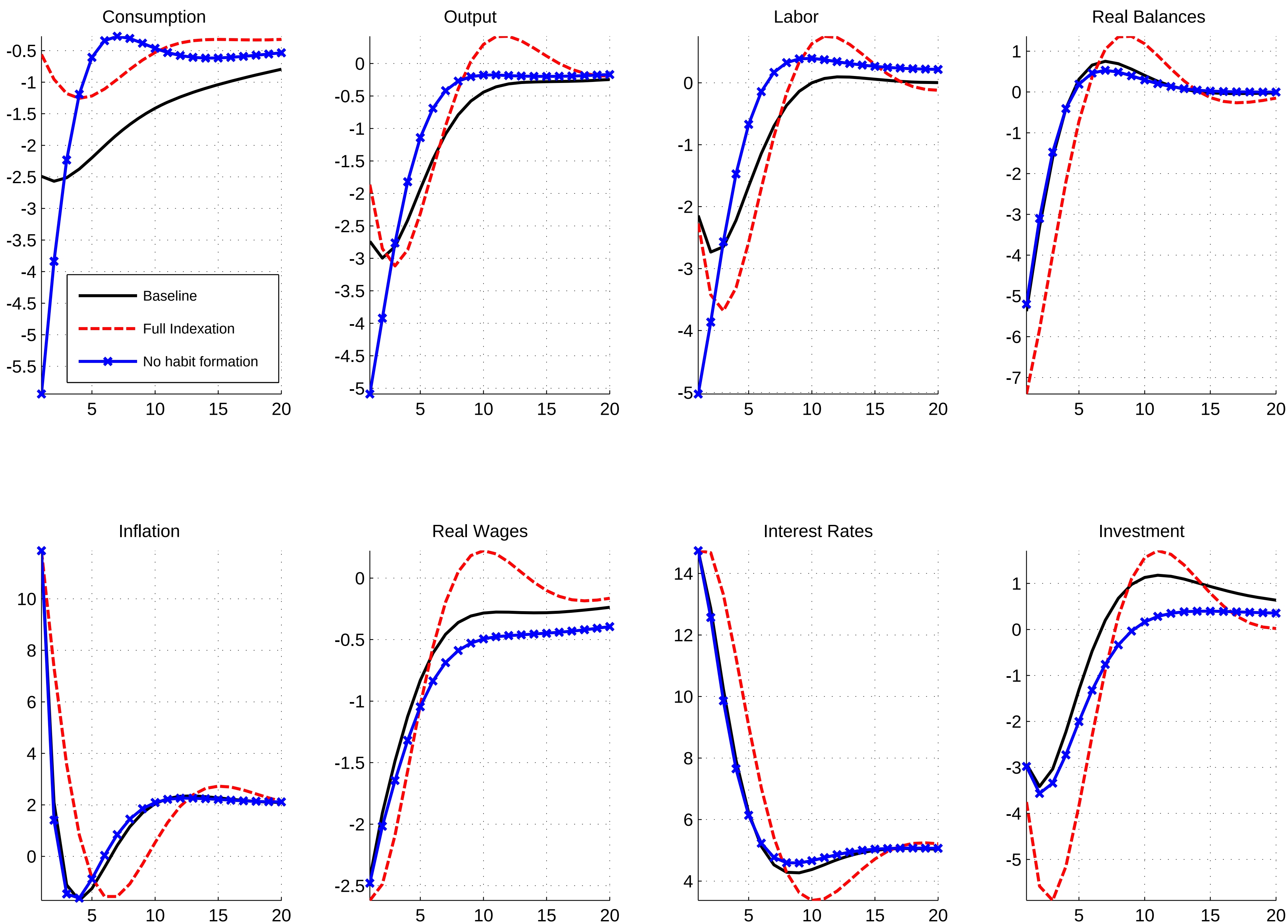

Figure 2: Impulse Responses Deflationary Shock

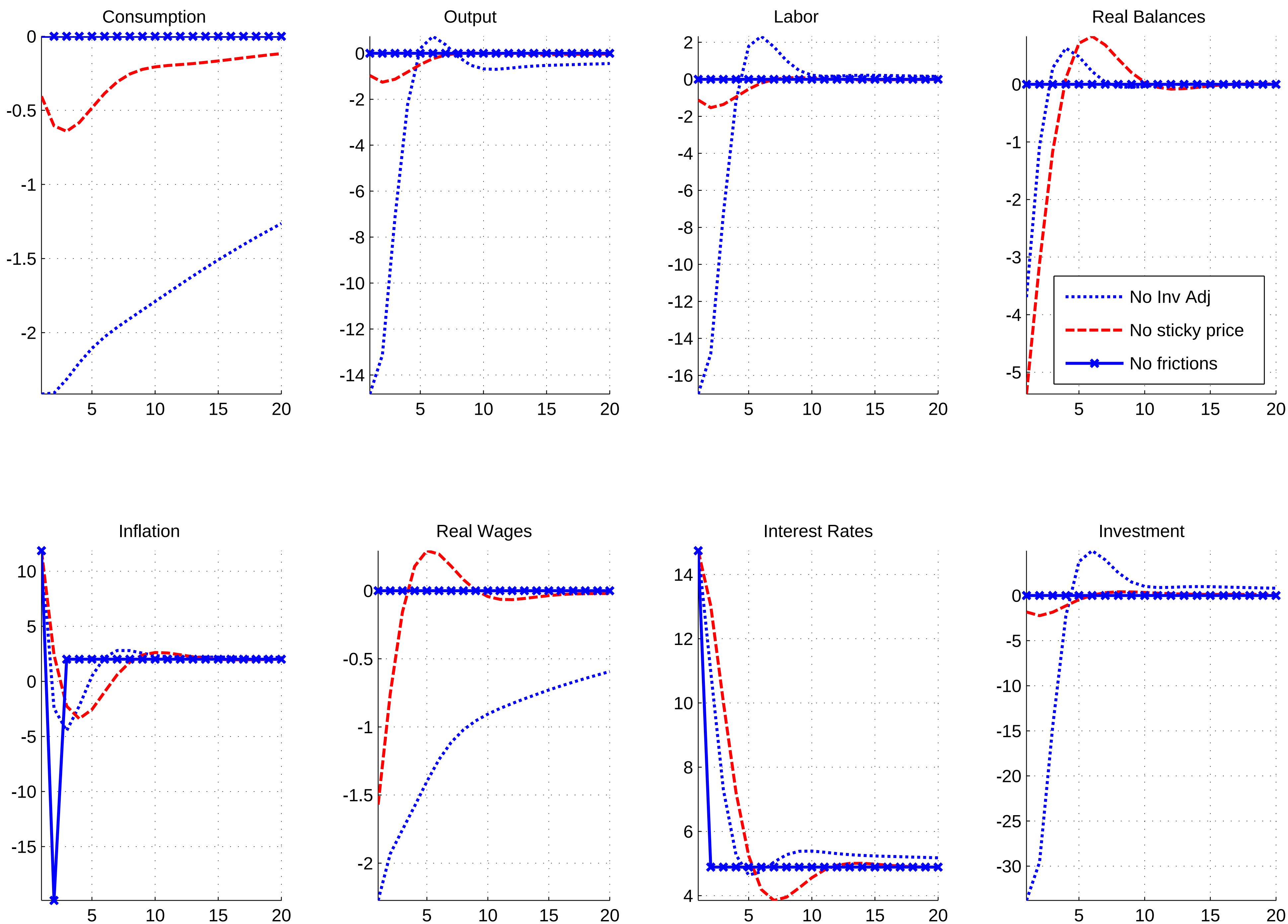

Figure 3: Effects of Price Indexation in Steady State