Heterogeneous Portfolio Adjustment, the Demand for Money, and the Great Moderation*

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Abstract

The demand for money has been a long standing puzzle in the monetary economics literature. Researchers consistently have estimated low short-run interest rate semi-elasticities, usually around 1, and high long-run interest rate semi-elasticities of 10. To understand why the literature report different estimates, I formulate and estimate a model of the demand for money that simultaneously accounts for low short- and high long-run semi-elasticities. In my formulation, re-balancing money holdings between money for purchases and money for financial investment is costly. I model this re-balancing cost by assuming that households re-optimize their money holdings subject to an exogenous probability. I use the equilibrium condition implied by my formulation to estimate the model's parameters. My estimates for the short-run and long-run interest semi-elasticities are 1.04 and 13.16, respectively. Finally, a simulated version of my model suggests that the late-1970s financial innovations contributed to the smoothing of the U.S. economy.

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1. Introduction

The demand for money has been a long standing puzzle in the monetary economics literature. Indeed, since the seminal papers of Friedman (1956) and Meltzer (1963), the literature has yielded different estimates for the coefficients of the money demand equation. For example, whereas Lucas (1988) suggests that the long-run interest rate semi-elasticity of money demand falls between 5 and 10,[1] other researchers (e.g., Goldfeld and Sichel 1990, and Altig, Christiano, Eichenbaum, and Linde 2004) find that the short-run semi-elasticity lies around 1. This paper investigates what frictions might account for the considerable difference between the short- and long-run semi-elasticities and the implications of such frictions on the dynamics for money demand and the Great Moderation.

Understanding the demand for money is critical for several reasons. First, Lucas (2000) stresses that the interest semi-elasticity of money demand is the key parameter in determining the welfare cost of inflation in steady state. Using a semi-elasticity of 7 he finds that driving inflation down from 13 to 3 percent would yield a benefit equivalent to an increase in real income of about 0.8 percent. However, this gain would shrink to 0.18 percent for a semi-elasticity of 1.[2] Second, in Dotsey, King, and Wolman's (1999) state-dependent pricing model, the interest elasticity of money demand heavily influences the response of output and inflation to a monetary shock. Finally, the semi-elasticity is essential in Engel and West's (2005) discussion of exchange rates and fundamentals.

To study the money demand puzzle, I develop a model of the demand for money that features time-dependent portfolio setting as its main friction. In my formulation, households divide their money holdings into two parts: money for consumption purchases and money for financial investment. Re-balancing the composition of their money holdings is costly, however. To capture this cost, my model assumes households use time-dependent rules to re-optimize their money holdings, a supposition that yields nontrivial heterogeneity across households.

My formulation implies that in equilibrium velocity depends on its own lag value and households' expectations of the short-term interest rate. I use this equilibrium condition to estimate the parameters of my model by using the generalized method of moments. My estimates for the short-run and long-run interest semi-elasticities are 1.04 and 13.16, respectively; I find that households re-optimize their money balances once every 4.5 quarters. Notably, the estimated short-run

[1] Meltzer (1963) and Ball (2002) estimate a money demand equation delivering long-run semi-elasticities consistent with Lucas' findings.

[2] Elasticities of 1 and 7 are the competing estimates available in the literature.

semi-elasticity agrees with the findings of Christiano, Eichenbaum and Evans (2005), and the long-run elasticity is in the range reported in Lucas (1988), and Stock and Watson (1993). Moreover, the portfolio sluggishness I uncover is consisten with the microdata-based evidence reported in Vissing-Jorgensen (2002). I find little statistical evidence against my model.

My model of the demand for money also explains why pre-1984 real balances is twice as volatile as post-1984 real balances. Financial innovation of the late 1970s and early 1980s facilitated portfolio adjustment. With these innovations, households could re-optimize their portfolios easily in response to shocks, thereby increasing the volatility of real balances. To support this hypothesis, I re-estimate my model over the pre-1984 and post-1984 samples.[3] Whereas the short- and long-run interest semi-elasticities equal 3.00 and 16.67, respectively, in the first subsample, those elasticities rise to 0.42 and 4.72, respectively, in the second subsample.[4] Furthermore, the frequency of portfolio re-optimization declines from six quarters in the pre-1984 sample to three quarters in the post-1984 sample. This subsample analysis suggests that financial innovation did indeed reduce the distinction between the short- and long-run dynamics of money demand, thereby increasing the volatility of money holdings.

Increasing flexibility in the financial sector allows households adjust more efficiently monetary resources allocated for good purchases. A flexible portfolio in turn might facilitate consumption smoothing. Consequently, my model for the demand for money also suggests that financial innovations of the 1970s and 1980s contributed to the Great Moderation. To explore this argument, I simulate my model of money demand using different degrees of portfolio sluggishness. I find that moving from high portfolio sluggishness, in which households re-balance their portfolios on average every 4 quarters, to complete flexibility the model accounts for almost 25 percent of the observed decline in the volatilities of output, and consumption. Moreover, the model explains 8 percent of the decline in inflation and most of the increase in the volatility of real balances.

From the methodological standpoint, this paper is related to the heterogeneous price setting literature (see, for example, Altig, Christiano, Eichenbaum, and Linde, 2005; Christiano, 2004; Woodford, 2005). Specifically, I show that the approach outlined in Woodford (2005) can be extended to manage heterogeneity in households. This method involves guessing and verifying that individual consumption relative to economy-wide consumption is a function of individual money

[3] The results in Sims and Zha (2004) suggest 1984 as the most likely year when the Great Moderation started.

[4] In the rest of this manuscript I will use semi-elasticity and elasticity interchangeably. The reader, however, must remember that the numbers reported corresponds to the semi-elasticity of money demand.

holdings relative to aggregate money holdings.

The rest of this paper is organized as follows. Section two analyzes the elasticity puzzle in more detail. Section three outlines the basic model and its implications for money demand. Section four estimates the basic money demand equation and several variants using a partial information generalized method of moments, and explains how my model of money demand partially captures the change in the volatilities of output and consumption. Finally, Section five provides concluding remarks.

2. The Money Demand Puzzle

A direct approach to solving the money puzzle would entail formulating and estimating a dynamic money demand equation that allows for lags of real balances; this approach would create a wedge between real balances in short and long run. Goldfeld (1976) and Goldfeld and Sichel (1990) estimate this type of equation to improve the forecasting power of Meltzer's (1963) canonical money demand. Specifically, they suggest replacing Meltzer's equation with the formulation:

$$\ln m_t = b_0 + b_1 \ln y_t + b_2 \ln r_t + b_3 \ln m_{t-1} + b_4 \pi_t \tag{1}$$

where m refers to real balances, y to real output, and r to short term interest rate. Although this alternative specification worked reasonably well in terms of forecasting ability for the pre-1974 period, it widely overpredicted money balances for the post-1974 period. In fact, Meltzer's model outperformed Goldfeld's. Given this puzzling behavior, Goldfeld (1976) concluded that M1 was no longer the appropriate variable, this finding is what he called the missing money period. He argued that a new monetary aggregate was needed.

The problem with (1) is that the estimated coefficient on lagged money, b_3, does not differ significantly from 1 in the post-1974 sample. According to Goldfeld's (1976) partial adjustment mechanism, however, b_3 should be smaller than 1. Table 1 presents the estimation of (1) as reported in Goldfeld and Sichel (1990). Note that the coefficient b_3 is significantly less than 1 in the pre-1974 sample, but it equals approximately 1 when the sample is extended.[5]

These findings have implications for the elasticities of money demand. Let us define the short-run semi-elasticity as the immediate effect of an interest-rate increase on money demand. Ad-

[5] Given the presence of serial correlation, claiming that the estimated coefficients are significant is misleading. However, I follow the Goldfeld's original interpretation.

ditionally, let us define the long-run semi-elasticity as the cumulative effect of this increase on money demand.[6] The pre-1974 short-run elasticity falls between 1.6 and 3.0, a range consistent with Christiano, Eichenbaum, and Evans' (2005) results. The results are contradictory for long-run elasticities, however. For the initial period, the elasticity is 7.5 or 14 depending on whether a short- or a long-run interest rate is used. For the extended sample 1952 - 1986, on the other hand, the implied elasticity falls between $-3,333$ and 600.

To further investigate Goldfeld's (1976) findings we can divide his analysis into two parts. First, he uses M1 as the monetary aggregate. During the 1970s, however, financial instruments, like NOW accounts, and electronic banking were introduced, making M1 a poor measure of money used in transactions. The second part of the puzzle hinges on Goldfeld's (1976) partial adjustment formulation. The results in Meltzer (1963), Stock and Watson (2003), and especially Lucas (1988) indicate that the income elasticity of money demand equals one. Arguing that the ratio of trends of real balances to real income, which is roughly 1 for the period 1960-2001, drives this elasticity, Lucas suggests that properly recovering the interest semi-elasticity requires imposing an income elasticity of 1.[7] His suggestion in turn implies that the coefficient of y is one or that the dependent variable in (1) is velocity, not real balances.[8]

This paper's approach accounts for the two components of Goldfeld's (1976) analysis. I incorporate a unit elasticity of income by using velocity as the measure of transaction costs incurred when purchasing goods. Moreover, I use M2 minus (M2MSL) as the monetary aggregate. Finally, I use a general equilibrium formulation to derive a forward-looking money demand equation whose coefficients depend on structural parameters.

3. Basic Setup

3.1. Household's Problem

I assume there is a continuum of households indexed by i. At the beginning of the period, and before any uncertainty is disclosed, households have access to insurance provided by perfectly competitive insurance companies. A Calvo mechanism then decides which households can re-optimize their

[6] Operationally, these definitions imply that the short- and long-run elasticities are given by $\zeta_{short} = -100b_2$ and $\zeta_{short} = -100\frac{b_2}{1-b_3}$,respectively.

[7] Lucas also points out that Goldfeld's model widely overpredicted money balances because of the high interest elasticity imposed in his model. .

[8] The idea of using velocity in the analysis of the demand for money goes back to Friedman and Schwartz (1982).

money balances, and the rest of the uncertainty is disclosed. Third, households optimally choose consumption expenditure and other relevant variables (such as labor and investment). Households allowed to re-optimize decide how much money to send to the bank, and how much to keep for transaction purposes. By sending money to the bank, households are entitled to collect a return at the end of the period. Finally, households receive payments from insurance, capital and labor markets, and payments from the bank, including money transfers made by the government.

I introduce money in the model by assuming that purchasing a good involves a proportional transaction cost that depends positively on individual velocity. In this way, I incorporate Lucas' (1988) observation that the income elasticity of money is essentially 1. A typical household in this economy faces the following problem

$$\max_{c,M,Q} E_t \sum_{j=0} \beta^j U(c_{i,t+j}), \tag{2}$$

subject to

$$(1+\eta(V_{i,t+j}))p_{i,t+j}c_{i,t+j} + M_{i,t+j+1} \leq R_{t+j}(M_{i,t+j} - Q_{i,t+j} + (x_{t+j}-1)M^a_{t+j}) + Q_{i,t+j} + A_{i,t+j}$$

where $A_{i,t}$ is household i's net cash inflow from the insurance, labor, and capital markets,[9] $Q_{i,t}$ is the stock of money kept for transaction purposes, $M_{i,t+1}$ is the end-of-period, individual stock of money, and R_t is the quarterly gross interest rate paid by the bank. The variable x_t represents the gross growth rate of the economy-wide per capita stock of money, M^a_t. The quantity $(x_t - 1)M^a_t$ is a lump-sum transfer made by the monetary authority. The term $\eta(V_{i,t})$ represents transaction costs per dollar spent in consumption. Individual velocity is defined as $V_{i,t} \equiv \frac{p_t c_{i,t}}{Q_{i,t}}$. Denote $\eta' > 0$ and $\eta'' > 0$ as the first and second derivatives of the transaction function.[10]

3.2. Derivation of the Demand for Money

The main friction in the basic formulation (2) comes from time-dependent portfolio adjustment; agents re-optimize their money balances, Q, infrequently, similar to the models of Woodford (2003) and Uribe and Schmitt-Grohe (2005). With this friction, I try to incorporate in a parsimonious way the theoretical developments in Alvarez, Atkeson, and Edmond (2003) and the empirical evidence

[9] In general, this term is given by $A_{i,t} = -p_{t+j}I_{i,t+j} + w_{t+j}h_{i,t+j} + R^k_{t+j}k_{i,t+j} + D_{i,t+j}$, where I is investment, h is labor supply, w is nominal wage, R^k is the rental rate of capital, and D is dividends.

[10] This way of modeling money originated in Sims (1994) and has been applied widely. See Schmitt-Grohe and Uribe (2005) and Altig et al. (2004) for examples.

reported in Vissing-Jorgensen (2002). Atkeson et al. discuss an inventory theoretic model for money demand in which staggered portfolio adjustment is a key source of inflation inertia. On the empirical side, Vissing-Jorgensen finds that roughly half of active investors (i.e., trading agents) in the equity market annually modify their portfolios.[11] I interpret this infrequent re-optimization as the result of costs faced by households. The basic idea is that in the presence of these costs, households fully optimize their portfolio only periodically, and follow simple rules for changing their portfolio at other times. The type of costs I have in mind are those associated with optimization (e.g., costs associated with information gathering, decision making, negotiation and communication).

Interestingly, infrequent participation in the financial markets is widely accepted among practitioners. For example, Swensen (2000) argues that a large fraction of investors prefers to remain inactive because of the high cost and uncertain returns associated to active portfolio management.[12] He argues[13]

Pursuing active management involves competing in an extremely tough arena, since markets tend to price assets accurately. Enormous sums of money deployed by intelligent, motivated asset managers seek to exploit perceived mispricings at a moment's notice. Serious investors consider carefully the certain results of low-cost passive strategies before persuing uncertain returns from high-cost active management activities. Significant costs raise the hurdle for active strategies.

Introducing sluggish behavior in asset markets is cumbersome, however, as it creates severe heterogeneity across agents. I propose simplifying the analysis by assuming that money balances display a sluggish pattern. This modification in turn allows us to use an insurance-market assumption to alleviate the heterogeneity stemming from wealth differentials.

Assumption 1: Only a fraction, $1-\xi_{po}$, of randomly chosen households are allowed to re-optimize their money balances, Q, every period. The rest of the population follows a rule outlined in Assumption 2.

Assumption 1 captures the idea that households do not adjust their money balances immediately because of costs associated to the decision making process. Financial sophistication might help reduce these costs and increase the frequency of portfolio re-optimization. Although the lit-

[11] A report by the Investment Company Institute (1999) supports these results. Furthermore, The Economist, in its January 28th 2006 issue, reports that in the period 2000-2004 roughly 75% of the trading volume on the stockmarket can be explained by an investing strategy that retains unchanged its portfolio structure.

[12] David F. Swensen is Yale University's Chief Investment Officer and manages the university's $10 billion dollar endowment.

[13] Swensen, 249.

erature on portfolio choice provides little guidance regarding how to model inactive households,[14] one possibility is that they follow a rule of thumb. I use an approach similar to that proposed in Christiano, Eichenbaum, and Evans (2005), who suggest households increase their monetary holdings at the rate of inflation.

Assumption 2: If a household is not allowed to re-optimize today, then her money holdings are adjusted according to the rule $Q_{i,t} = \pi g_c Q_{i.t-1}$, where π represents the steady state economy-wide inflation, and g_c is the growth rate of consumption in steady state.

The presence of g_c in the indexation rule implies that there are no distortions from portfolio dispersion along the steady state growth path (see the appendix for more details).

Assumption 3: There exists insurance provided by competitive companies. Moreover, agents can contract upon any uncertainty, including that created by Calvo, with the insurance companies.

This last assumption (used in papers like Erceg, Henderson, and Levin 2000 and Woodford 2003) aims to reduce the degree of heterogeneity across households resulting from time-dependent portfolio adjustment. By employing this assumption, I focus on the direct effects of the Calvo friction on consumption and money balances. Using standard arguments (see the technical appendix to Christiano, Eichenbaum, and Evans 2005), we can show that the presence of competitive insurance companies imply that different households value an extra dollar today equally: $\lambda_{i,t} = \lambda_{i',t}$ for households i and i', where λ is the budget multiplier.

To derive the optimal money holdings for household i that re-optimize at time t, we take the FONC for Q_t:

$$\left[-R_t + 1 + \eta'(V_{i,t}) \left(\frac{p_t c_{i,t}}{Q_{i,t}}\right)^2\right] + E_t^i \sum_{j=1}^{\infty} (\xi_{po}\beta)^j \frac{\lambda_{t+j+1}}{\lambda_t} \left[\begin{array}{c} (-R_{t+j}+1)(\pi g_c)^j + \\ \frac{\left(\eta_{i,t+j}^t\right)'}{(\pi g_c)^j} \left(\frac{p_{t+j} c_{i,t+j}}{Q_{i,t}}\right)^2 \end{array}\right] = 0, \quad (3)$$

where $\eta_{i,t+j}^t \equiv \eta(V_{i,t+j}^t)$, $V_{i,t+j}^t \equiv \frac{p_{t+j} c_{i,t+j}}{(\pi g_c)^j Q_{i,t}}$ and E_t^i is the expectation operator upon the event that household i does not re-optimize her money balances after period t. The Lagrangian multiplier, λ, is not indexed reflecting my assumption of insurance markets.

In the absence of time-dependent portfolio adjustment, the optimal condition for money balances requires the cost of an extra unit of balances, R_t, to equal the benefits of the decrease in the transaction cost: $R_t = 1 + \eta'(V_t) \left(\frac{p_t c_t}{Q_t}\right)^2$. No extra costs or gains arise under these circumstances because households can choose a new portfolio freely in the next period. In contrast, with time-dependent portfolio adjustment, the costs and benefits of an extra unit of money balances extends

[14] For an authoritative survey see Campbell and Viceira (2002).

well beyond the current period because with positive probability households must retain their current nominal balances, indexed by inflation, next period. In the second period, losses come from the foregone interest income $\pi g_c R_{t+1}$.[15] On the other hand, gains come from the savings on transaction costs $\pi g_c + \frac{\left(\eta_{t+1}^t\right)'}{\pi g_c}\left(\frac{p_{t+1}c_{t+1}}{Q_t}\right)^2$. Because these costs and gains arrive during the next period with probability ξ_{po} they must be discounted using the household's stochastic discount factor $\xi_{po}\beta\frac{\lambda_{t+1}}{\lambda_t}$. The combination of these terms corresponds to the second element of (3). The argument can be extended readily to subsequent periods.

Deriving a tractable money demand equation from (3) presents a couple of challenges. First, as in the Calvo pricing literature, we must evaluate the conditional expectation $E_t^i\left(\eta_{i,t+j}^t\right)'\left(\frac{p_{t+j}c_{i,t+j}}{Q_{i,t}}\right)^2$ only across those histories in which household i has not re-optimized its money balances. However, household consumption, $c_{i,t+j}$, depends on all continuation histories including those in which the household can re-optimize its balances. Second, in spite of the insurance market assumption, the presence of the transaction function η implies that consumption is still heterogeneous across agents. As contingent markets equate the marginal utility of wealth across households we have that:

$$\frac{U_{c,t}(i)}{[1+\eta(V_{i,t})+\eta'(V_{i,t})V_{i,t}]} = \frac{U_{c,t}(j)}{[1+\eta(V_{j,t})+\eta'(V_{j,t})V_{j,t}]}, \tag{4}$$

for households i and j. Because money holdings differ across households, so too does consumption.

To manage these complications, I extend the procedure outlined in Woodford (2005). First, to remove the nonlinear terms, I take a log-linear approximation of (3) about steady state. Proposition 1 in the appendix shows that this equation indeed admits a first-order Taylor approximation under mild assumptions. Second, because only active households absorb additional funds in the market after a monetary shock, other things equal, velocity of active households is smaller than that of inactive households. The only source of this differential lies in the sluggish adjustment setup. Thus, individual velocity, V_i, must be a function of economy-wide velocity, V^*, plus a term that depends on individual money holdings, Q_i, relative to economy-wide money balances, Q^*. Therefore, I guess and verify that, as a log-linear approximation, the velocity for household i is:

$$\widehat{V}_{i,t} = \widehat{V}_t^* + \Psi\widehat{\left(\frac{Q_{i,t}}{Q_t^*}\right)}, \tag{5}$$

where Ψ is a coefficient to be determined. Note this solution is valid under the first-order Taylor

[15] The indexation rule stated in assumption 2 implies that money balances, Q, growth at rate πv in steady state. Then the extra term in front of the interest rate.

approximation used to solve equation (3) and is accurate up to an error term of order $o\left(||\varepsilon||^2\right)$.[16] In the appendix, I show that by combining the insurance market assumption, (3), (4) and (5), the money demand equation and the coefficient Ψ are given by:

$$\begin{aligned} \widetilde{R}_t &= c_1\widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + E_t[c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})], \qquad (6) \\ \Psi &= -\frac{\rho}{\rho+\tau}. \end{aligned}$$

Here the reduced-form coefficients, c_i and τ, depend on the structural parameters; a hat corresponds to log deviations from steady state; starred variables correspond to aggregate variables; ρ is the coefficient of risk aversion; and $\widehat{g}_{c,t}^*$ is the growth rate of consumption. In (6), the gross interest rate, $\widetilde{R}_t$, represents deviations of the interest rate from its steady state value and is computed using quarterly figures. The negative coefficient in front of relative money balances confirms the suspicion that households with larger money holdings have smaller individual velocities.

Equation (6) resembles Goldfeld's (1976) partial adjustment model discussed in Section 2. This new formulation differs from his model in several ways, however. First, unlike the partial adjustment model, (1), money balances do not appear explicitly in the last equation; instead, they enter indirectly through their influence on velocity. Second, my formulation involves the growth rate of consumption, whereas the original model focused on the level of output. Third, the new dynamic money equation includes a forward looking term, V_{t+1}, which is absent from Goldfeld's formulation.

3.3. Interest Semi-elasticity of Money Demand

I define the interest semi-elasticity of money demand as the percentage change in real balances stemming from an increase of a hundred basis points in the annualized interest rate. Operationally, this definition implies that the semi-elasticity, ζ, is given by:

$$\zeta = -\frac{1}{4}\frac{\partial(\log(Q/P)_t)}{\partial R_t}. \qquad (7)$$

Equation (7) accounts for the quarterly time period of the model. For this definition to be meaningful, however, we must find a representation in which real balances are a function of interest rates. To that end, I proceed in two steps. First, I solve (6) for velocity as a function of interest rates and the other variables. Using standard techniques (see Hamilton, 1994 or Sargent, 1987), I

[16] This factor represents all terms that are of second order or higher in the deviations of the variables from their steady state

show that velocity is governed by the stationary solution to the difference equation (6):

$$\widehat{V}_t^* = \lambda_1 \widehat{V}_{t-1}^* + \frac{1}{c_2\lambda_2} E_t \sum_{j=0}^{\infty} (\lambda_2)^{-j} \left[-\widetilde{R}_{t+j} + c_3(\widehat{\pi}_{t+j} + \widehat{g}_{c,t+j}^*) - c_2(\widehat{g}_{c,t+j+1}^* + \widehat{\pi}_{t+j+1}) \right], \quad (8)$$

where λ_1 and λ_2 are the inverse of the roots in the polynomial $\frac{c_3}{c_2}z^2 + \frac{c_1}{c_2}z + 1 = 0$.[17] The estimated coefficients satisfy $c_1 > 0, c_2, c_3 < 0$ and are such that $0 < \lambda_1 < 1 < \lambda_2$ as required for (8) to be a valid solution. In the appendix, I show that the structural parameters satisfy that relation between the characteristic roots. Second, because velocity and real balances are linked through the equation $V = \frac{PC}{Q}$, I combine this condition and the stationary solution for velocity and arrive at a condition involving real balances as a function of interest rates as desired.

The idea behind (8) is straightforward. First, the term in front of c_3 is a consequence of the relation $\widehat{V}_t^* = \widehat{V}_{t-1}^* + \widehat{\pi}_t + \widehat{g}_{c,t}^* - \widehat{q}_t^*$ used in the derivation of the equation for velocity (see the appendix for details.) According to this condition, today's velocity differs from yesterday's because innovations in inflation, consumption growth, and money growth. The higher the first two factors are, the higher velocity is today. Second, the terms $\widehat{g}_{c,t+j+1}^* + \widehat{\pi}_{t+j+1}$ reflect the forward looking behavior of velocity stemming from sluggish money balances and transaction costs. If households expect higher consumption rates in the future, they will forecast an increase in velocity that will drive up transaction costs. To counterbalance the rise in costs and afford the additional consumption, households must increase their money balances in the future. As the Calvo friction implies, however, this will be impossible for some households. So these households raise their money balances today, when they can re-optimize. Moreover, higher inflation in the future implies that consumption will be more expensive requiring higher money balances to afford the same level of consumption. In either case, current velocity declines thanks to higher current money balances.

Equation (8) also indicates the positive relation between velocity and present and future interest rates. If households expect high interest rates in the future, holding money balances, Q, is more expensive than sending a dollar to the bank. Therefore, households decrease current money balances, sparking an increase in velocity. Furthermore, the increase in velocity, and the decline in real balances, depend on the persistence of interest rates. Consider, for example, the effect of a temporary increase in interest rates at time t. The immediate effect on velocity, keeping $\widehat{\pi}$ and $\widehat{g}_c^*$ constant, is measured by the term $-\frac{1}{\lambda_2 c_2}$. This result indicates that, other things equal, a 1-percent increase in interest rates raises velocity by $-\frac{1}{\lambda_2 c_2}$ percent. Considering this outcome, we can define

[17] I have imposed a boundedness condition to eliminate terms of the form $a_1\lambda_1^{-t}$ and $a_2\lambda_2^t$.

an elasticity measuring the effect of a temporary increase in interest rates:

$$\zeta^T = -\frac{1}{4c_2\lambda_2}.$$

Here, the index T stands for "temporary" to reflect this definition's correspondence to a temporary increase in interest rates. Unlike temporary increases, permanent rises in interest rates have short- and long-term consequences on real balances. After the increase in interest rates, velocity declines because households forecast a permanent change in interest rates. Moreover, when the economy reaches its new steady state, velocity settles at a value different from its initial value. This latter effect happens because of the backward looking term in (8). Using the same logic as before, we can define the following semi-elasticites:

$$\begin{aligned}\zeta^P_{short} &= -\frac{1}{4c_2\lambda_2(1-\lambda_2^{-1})},\\ \zeta^P_{long} &= -\frac{1}{4c_2\lambda_2(1-\lambda_1)(1-\lambda_2^{-1})}.\end{aligned}$$

The index P stands for "permanent." A direct comparison between the temporary and permanent definitions reveals that the elasticities from a permanent increase in interest rates are larger than those from a temporary increase, $\zeta^P > \zeta^T$. The reason is that a more persistent change in interest rates generates a larger change in real balances because transaction costs, η, and the Calvo adjustment in money balances induce agents to be more forward looking than in the absence of time-dependent adjustment.

Using the results in this section, we can argue that the estimates in Christiano et al. (2005) corresponds to what my formulation identifies as a short-run semi-elasticity while the elasticities reported in Lucas and others reflect a long-run elasticity. For the last statement to be true, we need to show that $0 < \zeta_{short} < \zeta_{long}$. This condition follows trivially from the observation that $0 < \lambda_1 < 1 < \lambda_2$ and $c_2 < 0$ (see the appendix for details.)

Note that (6) is the results of optimal behavior from rational agents. Hence, the next section uses GMM to estimate that moment condition.

4. GMM Estimation

Following the recent literature on Calvo pricing,[18] I estimate equation 6 using GMM. One advantage of this approach is that specifying the rest of the economy is unnecessary to obtain inference on the coefficients c_i. I study several cases that consider the varying information sets available to households, as well as some extra assumptions discussed below.

4.1. Case I

A direct and testable implication of equation 6 and rational expectations is

$$E[(-\widetilde{R}_t + c_1\widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1}))X_t] = 0, \tag{9}$$

where X_t is any variable on the household's information set at time t, I_t. I use the GMM estimation as outlined in Hansen (1982) to estimate the reduced parameters c_i. Note that the last condition implies that the term in square brackets has an MA(0) representation. Therefore, I choose the weighting matrix for GMM to yield a consistent estimate of

$$\Omega = \sum_{k=-\tau}^{\tau} E[\psi_{t+1+k}X_{t+k-\tau}][\psi_{t+1+k}X_{t+k-\tau}]', \tag{10}$$

where $\psi_{t+1} \equiv -\widetilde{R}_t + c_1\widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})$ and $\tau = 0$. The weighting matrix is computed using the Newey-West estimator. Burnside and Eichenbaum (1996) show that imposing restrictions implied by the underlying model greatly improves the estimate of the weighting matrix.

Estimates for Case I are reported in Table 2 (this table also displays the results for a dynamic indexation rule that is discussed extensively in the appendix). In the robustness section, I explore the effects of different specifications of the information set.

4.2. Case II

For Case II, I consider the situation in which households choose their optimal portfolio one period in advance; in other words, if a household is allowed to re-optimize today, it does so based on information available in the last period. These circumstances might arise given a lag between

[18] See for example Eichenbaum and Fisher (2004) and the references therein.

the time when households choose a new portfolio and when they actually implement it. Therefore, the new moment condition is:

$$E[(-\widetilde{R}_t + c_1\widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})) \cdot X_{t-1}] = 0. \tag{11}$$

This moment condition implies that the money demand equation has an MA(1) representation. This requirement, in turn, signifies that the weighting matrix be estimated with one lag, $\tau = 1$ in (10). The next section discusses details of the estimation, and Table 2 presents the results from the estimation of (11).

4.3. Data

Data are U.S. quarterly and the sample period covers 1960:I-2005:IV. All data come from the St. Louis Fed. Nominal consumption is measured by personal consumption expenditure on nondurable goods (GCN), plus personal consumption expenditure on services (GCS). The price index is measured by the ratio of nominal to real output. The gross interest rate, R_t, is computed using the 3-month treasury bill rate (TB3MS). Output corresponds to real, chain-weighted output (GDPQ).

The results in Goldfeld (1976) and Goldfeld and Sichel (1990) indicate that M1 is an inappropriate measure of money for transaction purposes. Although M2 might seem an obvious replacement for M1, this monetary aggregate has the disadvantage of including small time deposits issued by financial institutions, which do not measure money spent in transactions. I propose measuring transaction balances, Q_t in the model, with M2-minus (M2MSL). This monetary indicator, available from the St. Louis Fed, comprises M2 minus small-denomination time deposits. Furthermore, M2-minus based income velocity is stationary, which is necessary for GMM estimation. In Section 8.2, I perform a series of tests checking the stability of velocity.

4.4. Results

4.4.1. Estimating the Semi-elasticity of Money Demand

Choosing the right instruments for estimating (9) is crucial. Although the obvious choice corresponds to the variables in the equation to be estimated, we still must decide how many lags to include. From the estimation of the new Keynesian Phillips Curve, we know that using a large set of instruments can mislead us regarding the plausibility of the overidentifying restrictions implied by the model (see Gali and Gertler 1999). Furthermore, Staiger and Stock (1997) advocate using smaller instrument sets to minimize the estimation bias that can surface in small samples. Given this considerations, I propose the following set of instruments: $X_t = \{1, g_{y,t-j}, V_{t-j}, R_{t-j}, j = 0, 1\}$ for Case I and $X_{t-1} = \{1, g_{y,t-j}, V_{t-j}, R_{t-j}, j = 1, 2\}$ for Case II; g_y is the growth rate of output. This parsimonious instrument set applies across different cases without extra lags or variables. In Sections 4.4.2 and 8.4, however, I analyze the effects of including additional lags and alternative instruments on the coefficients of the money demand equation.

Table 2 shows the results from the estimation of Cases I and II. J_T corresponds to the p-value of Hansen's statistic for overidentifying restrictions. Values in parentheses are standard deviations. Optimal weighting matrices are used in the computation of the GMM estimates. These matrices are computed taking into account the MA representation implied by the different cases. Table 2 also reports the results of the dynamic indexation case summarized in the appendix.

In Case I, the coefficients c_1, c_2, c_3, and c_4 correspond to the coefficients of the reduced-form equations used in the GMM estimation , and ζ^T and ζ^P correspond to temporary and permanent elasticities as defined in the previous section. Table 2 clearly reveals the data's lack of support for the assumption of immediate implementation. Although, the coefficients are statistically significant, Hansen's J statistic rejects the validity of the model.[19]

The results from Case II are more interesting. According to the static indexation case, all coefficients are statistically significant. Moreover, Hansen's J statistic does not reject the model.[20] The estimated elasticities reflect my theory's predictions. First, temporary elasticities are smaller than permanent ones. Second, there is a significant difference between short- and long-run elasticities; a permanent increase in interest rates of a hundred basis points implies an immediate decline of 1.04 percent and a long-run decline of 13.16 percent in real balances. The first figure is close to the low estimates reported in literature (e.g., Altig et al., 2004; and Goldfeld and Sichel, 1990). Taking into account sampling error, the long-run estimate also falls in line with values found in the literature. Mankiw and Summers (1986), for example, find an interest elasticity of 11 percent. Furthermore, Stock and Watson's (1993) study reports long-run semi-elasticities around 12. In the remainder of this paper, I take Case II with static indexation as the benchmark model.

[19] The estimated model under dynamic indexation indicates a negative coefficient for current velocity, a finding that contradicts the model's theoretical results. However, the coefficients do imply a difference between the point estimates of short- and long-run elasticities as predicted by my theory.

[20] Most of the Durbin-Watson statistics associated with the different cases in this paper are in the range $1.70 - 2.50$ suggesting that there is no serial correlation. For this reason, I do not report that statistic.

Given that the approach I pursue in this paper departs completely from the techniques in the money-demand literature, the results of this first experiment are a promising signal of the model's potential. As shown in the next section, these results are quite robust to different specifications and estimation techniques. Overall, the static indexation with delayed implementation, Case II, offers better estimates than Case I.

A well-known issue with Hansen's J statistic is its low power against alternative formulations (see Hall 2005 for a theoretical discussion; and Gali and Gertler 1999 for empirical analyses). Therefore, the results from Case II might be misleading. To investigate this possibility, I assume that active households' money demand roughly describes economy-wide money balances. I show in the appendix that this alternative formulation can be derived from the original formulation by assuming that the growth rate of money and inflation are equal. The results from the reduced model are reported in Table 9. A quick look at Hansen's J statistic and the coefficient significance levels reveals that the reduced model performs poorly.

The results described in this section and in Table 2 illustrate this paper's first main point: different elasticities reported in the literature are not puzzling after taking into account the short- and long-run behavior of money. Furthermore, the statistical and economical significance of the static indexation model indicates that the new partial adjustment formulation successfully describes the data covering $1960 - 2005$. This new equation is characterized by short-run and long-run elasticities of 1.04 and 13.16, respectively.

4.4.2. Robustness Checks

There are many ways to check the robustness of this paper's results, I concentrate on two checks that are important both from the perspective of both the GMM estimation and the data: an alternative measure of the interest rate, and sub-sample analysis. The appendix provides a more comprehensive set of checks including different instruments, and additional lags in the instrument set. The checks in this section and in the appendix indicate that the findings from the previous section are robust.

M2-minus's Opportunity Cost

The results from Section 4.4 were derived under the assumption that the relevant opportunity cost for M2MSL is the 3-month treasury bill rate. This assumption makes sense for the pre-1980 sample because the M2MSL own rate was close to zero and relatively flat. However, financial deregulation in the late seventies removed caps on interest rates paid on savings and other accounts. Therefore, the 3-month T-bill interest rate might not be valid for measuring the true opportunity cost of M2MSL after 1980. To account for this possibility, I repeat the estimation of equation 6, but this time I use the difference between the 3-month T-bill interest rate and M2MSL own rate.[21]

The second panel of Table 3 shows the results from the estimation of equation 6 under Case II when M2MSL own rate is used. The short- and long-run elasticities are 1.70 and 16.67, respectively; thus, a significant difference between the two elasticities remains. Moreover, the new point estimates lie within one standard deviation of the estimates reported in Table 2, which presents Case II static indexation.

A similar conclusion arises from the results when the fed fund rate is used, as shown in the first panel of Table 3. In this case, the short- and long-run elasticities are 0.78 and 12.50, respectively. As before, the new estimates significantly differ from zero and are close to the benchmark estimates.

The results from this section show that the distinction between the short- and long-run elasticities is robust in the face of alternative opportunity costs. In the appendix, I use the Aaa Corporate Yield Bond as an additional measure of the opportunity cost for holding money. The results are consistent with those found here. Based on this conclusion, I use the 3-month treasury bill rate as the relevant interest rate throughout the remainder of this paper.

Subsample Analysis

The late 1970s and early 1980s featured continuous financial innovations, including electronic banking, more flexible deposit accounts, and new banking regulations.[22] Additionally, several authors have argued that a significant change occurred in the economy in the early 1980s resulting in what Stock and Watson (2002) call the Great Moderation. Therefore, a subsample analysis is necessary to test whether those innovations modify the conclusions drawn from the complete sample.

Table 4 reveals that these elasticities are closer to each other after 1984. To observe this pattern, compare $\zeta^P_{short} = 0.42$ and $\zeta^P_{long} = 4.72$ in the final sample with $\zeta^P_{short} = 3.00$ and $\zeta^P_{long} = 16.67$ in the initial period. The difference between the short- and long-run elasticities declines by a factor of

[21] My choice of this opportunity cost follows Teles and Zhou (2005), Goldfeld and Sichel (1990) and the refences therein.

[22] Dylan et al. (2005), Teles and Zhou (2005) and Guerron (2006) provide a comprehensive description of the financial innovations experienced in the U.S. economy at this time.

three in the post-1984 sample. As Table 4 indicates, this decline is still present when we consider the more recent sample: $1996-2005$.

The elasticities' closeness in the second sample suggests that financial innovations in the late 1970s and early 1980s might entail reduced transaction costs, making it easier for households to adjust their money balances. Easier portfolio adjustment implies the reduction of differences between the short-run and long-run dynamics of money in my model. This reduction is equivalent to a decline in the Calvo probability. As the next section discusses this decline is exactly what happens with the implied Calvo probabilities.

4.5. Implied Frequency of Re-optimization

This section discusses the values of the structural parameters implied by the reduced-form model. The model's expected time of reoptimization for a household is given by $\frac{1}{1-\xi_{po}}$. Therefore, we can use the results from the estimation section to infer the implied frequency of reoptimization. I only consider the case in which there is identification (i.e., the number of structural parameters equals the number of coefficients in the reduced version). I compute the structural parameters by solving the following non-linear mapping:

$$\widetilde{c} - c(x) = 0, \tag{12}$$

where $x = [\eta\ \eta''\ \xi_{po}]'$, $\widetilde{c}$ is the vector of estimated coefficients from our GMM procedure, and $c(x)$ is the vector mapping the structural parameters space into the reduced form coefficients space. To achieve identification I set the following parameters: $\rho = 1$, $\beta = 0.99$, and R, π, and V are taken from sample analogs. The slope of the transaction cost function is computed from the steady state relation $R = 1 + \eta' V^2$.

Table 5 displays the results from the nonlinear equation (12). The probability of portfolio adjustment, ξ_{po}, ranges from 0.67 to 0.84, meaning that households re-optimize their portfolios every six quarters on average in the upper bound, whereas they review their financial decisions every three quarters in the lower bound. The benchmark case (static indexation with delayed implementation) attributes a Calvo probability of 0.77. This large probability (and its implied frequency of reoptimization) is close to values reported in Vissing-Jorgensen (2002), who uses microdata from the PSID to show that traders participating in the stock market modify their portfolios once every 6 quarters. Table 5 also indicates the Calvo probability is larger in the initial sample than in the final sample, implying that households waited longer before re-optimizing their portfolio in the pre-1984 sample versus the post-1984 sample.[23] Alternatively, this result suggests that re-balancing money holdings became easier after 1984, further confirming the hypothesis that money balances are more flexible in the second rather than the first part of the sample. Notably, different specifications do not affect the results.

The estimated transaction function, η, is close to the 0.036 found in Altig et al. (2004). For the curvature of the transaction function, n'', the model yields numbers close to zero, the largest being 0.004. This number differs considerably from that estimated in Altig et al. (2004).[24] In my model, the smaller the curvature of η is, the larger the response of money balances to interest rates. To see this effect, take a log-linear approximation to the money demand equation under the assumption that there is no sluggish adjustment. Under these circumstances, we get $\widehat{V}_t = \frac{R}{2\eta' V^2 + \eta'' V^3} \widehat{R}_t$ from which my claim follows.

The discrepancy around η'' may help to explain why the elasticity found in Altig et al. differs considerably from that estimated in this paper and in Lucas (1988). From simulations not reported here, I find that the former paper uses an econometric technique placing all emphasis in the high frequency properties of the data. This restriction in turn requires that η'' be a number around 1 which is the only value consistent with the moderate initial decline of interest rates and velocity following a monetary shock (see Altig et al. 2004). On the other hand, the approach outlined here, GMM, makes no distinction between the short- and long-run properties of the data; η'' close to zero seems to be the best choice to match the data properties captured by the moment condition (9).

4.6. Financial Innovations and the Great Moderation

Advances in computer science and changes in financial regulation in the early 1980s are likely responsible for the portfolio flexibility enjoyed by households after 1984. Interestingly, these financial innovations precede the beginning of the Great Moderation. We are therefore left with the question of whether such innovations contributed to the decline in the volatilities of other variables such as output and consumption (the last two columns in Table 6 report the smoothing of the U.S.

[23] In fact, we cannot rule out the possibility of complete portfolio flexibility in the post-1980 sample as the Calvo probability is not significantly different from zero.

[24] Altig et al. do not report η'' directly. However, if we recover this parameter from their calculations, the result is close to 1.

economy; to facilitate comparison, the pre-1984 volatilities are normalized to 1).

As explained above, a decrease in the Calvo probability is equivalent to a decline in the costs associated to portfolio re-balancing which in turn might result from financial innovations. To measure the effect of the declining Calvo probabilities on output and consumption, I propose the following experiment: First, I embed my model for the demand of money in Altig, Christiano, Eichenbaum, and Linde's (2004) New-Keynesian DSGE model. For the sake of brevity, I omit the model's outline and refer the interested reader to the companion paper "Financial Innovations: An Alternative Explanation of the Great Moderation" for details.[25]

Second, I choose 5 Calvo probabilities corresponding to different degrees of financial innovation with the lowest probability reflecting the highest degree of innovation. Third, I simulate the model using monetary, neutral technology, and investment-specific shocks recovered following the VAR approach described in Altig et al. (2004). Finally, I compute the standard deviations for interest rates, and the growth rates of consumption, investment, and output. I repeat the procedure for different degrees of portfolio sluggishness.

Table 6 reports the model's results for different degrees of portfolio sluggishness. To facilitate comparison with the data the volatilities for the highest degree of inflexibility, $\xi_{po} = 0.75$, are normalized to 1.[26] We see that moving from an inflexible portfolio towards a completely flexible one, the volatility of output declines by 25 percent and that of consumption by 20 percent. Almost half of the total decline happens when households move from adjusting their portfolio on average every 4 quarters, $\xi_{po} = 0.75$, to 3 quarters, $\xi_{po} = 0.67$.

Next, note that the volatility of real balances increases by 40 percent. Unlike the real variables, the volatility of money increases evenly with the increase in portfolio flexibility but the largest decline still happens during the first drop in the probability. Finally, interest rates become much smoother as the Calvo probability declines. We expect this result because interest rate is the only variable which absorbs monetary and technology shocks in the presence of portfolio inflexibility. Therefore, a substantial jump in interest rates is required to restore equilibrium after the monetary authority injects additional fund into the economy.

The intuition behind these results is quite simple. With infrequent portfolio re-balancing households cannot fully smooth out consumption as money for transaction purposes is determined in

[25] The paper is available at http://www4.ncsu.edu/~paguerro/research.htm

[26] A probability of $\xi_{po} = 0.75$ implies that households review their decisions on average every 4 quarters. This figure is consistent with the results in the previous section.

the past. Consequently, consumption and then output are very volatile. However, as portfolio re-balancing becomes more frequent, households transfer volatility out of consumption to real balances. Alternatively, optimal behavior by households relates the growth rate of consumption to interest rates.[27] Portfolio flexibility decreases the volatility of interest rates which, in turn through the first order conditions, implies smoother consumption. The smoothing of output is a direct consequence of less volatile consumption.

5. Conclusion

This paper has expanded the field's understanding of money demand in three ways. First, I put forth a micro-founded dynamic money demand equation , which is a reformulation of Goldfeld's partial adjustment money (1). To derive this new equation, I show that the tools from price-setting models can help manage the heterogeneity on the economy's household side. This paper's second and main contribution, however, was to show that my model of money demand simultaneously accounts for low short-run interest elasticities and high long-run elasticities of money demand. Estimates of these elasticities are in line with the findings in the literature. Finally, this paper offered a simple explanation for the change in real balances, output, and consumption volatility after 1984. Given estimates from the pre- and post-1984 samples, real balances seems to have become more volatile and consumption smoother since financial innovations of the late 1970s facilitated monetary transactions.

To follow up on this paper's results, researchers should investigate the estimation of (6) using micro data. This endeavor poses a great challenge, however: obtaining data on households' opportunity cost of capital. Mulligan and Sala-i-Marti (2000) stress this difficulty in their estimation of a static money demand equation; they use the marginal tax rate as a proxy of the interest rate but their computations indicate that the tax rate does not provide enough dispersion to derive significant estimates.

Finally, this paper describes a simple way to manage heterogeneity on the household side of the economy. Future research therefore could apply this method to issues like that nonseparability of the utility function in the presence of idiosyncratic shocks to labor supply.

[27] Suppose for a moment that there is no cost involved in buying goods, $\eta = 0$. Then, combining the optimal conditions for consumption and that for money holdings we get $c_{t+1}/c_t = \beta R_{t+1}/\pi_{t+1}$.

6. Appendix

This section heavily draws on Lang (1993), Woodford (1998, and 2003). The interested reader is referred to those papers for additional details on the proofs and definitions.

Proposition 1 *Suppose* η *is twice continuously differentiable and* $\xi_{po}\beta\pi < 1$*. Then (3) has a locally unique deterministic solution given by* $R = 1 + \eta' V^2$ *and admits a first-order Taylor approximation around this steady state.*

Proof. The proof basically consist of showing that the conditions for the inverse and implicit mapping theorems hold. For simplicity, I consider a perfect forseight equilibirum, i.e. the entire sequences $\{V_{i,t}\}, \{R_t\}, \{\lambda_t\}$, and $\{\pi_t\}$ for $t \geq 0$ are known at time $t = 0$. Re-write (3) as follows

$$\left[-R_t + 1 + \eta'(V_{i,t})V_{i,t}^2\right] + \sum_{j=1}^{\infty}(\xi_{po}\beta)^j \frac{\lambda_{t+j+1}}{\lambda_t}(\pi g_c)^j \left[(-R_{t+j} + 1) + \left(\eta_{i,t+j}^t\right)' \left(V_{i,t+j}^t\right)^2\right] = 0, \quad (13)$$

where $\eta_{i,t+j}^t$ and $V_{i,t+j}^t$ are defined in the main section. Next, consider a deterministic steady state in which $R_t = R$, $\lambda_t = \lambda$, $g_{c,t} = g_c$ and $\pi_t = \pi$, then it is straightforward to show that (13) has a solution, in which $V_{i,t} = V_{i,t+1}^t = V$ and $R = 1 + \eta' V^2$. Since $\eta \in \mathbb{C}^2$ and $\eta' \neq 0$ the implicit function theorem implies there exist a continuous function $\aleph : A \to B \subset \mathbb{R}$, with A an open ball around R, such that $V = \aleph(R)$ and $R = 1 + \eta'(\aleph(R))\aleph(R)^2$. Moreover, this solution is locally unique since the inverse theorem holds. Note that the sequence of equilibrium conditions (13) can be written in the form

$$\Gamma(V_i, X) = 0,$$

where V_i, X refer to the sequences of values for the variables $V_{i,t}, X_t = \{R_t, \lambda_t, \pi_t\}$ in periods $t = 0, 1, 2...$, and Γ maps a four-tuple of infinite sequences into an infinite sequence. The t element of Γ, Γ_t, is given by the left-hand side of (13). Define the derivative mapping $D\Gamma(V_i, X)$ as the linear operator that maps the sequences V_i, X into the sequence

$$\widehat{\Gamma}_t = \sum_{k=0}^{\infty}[\Gamma_{R,k}\widehat{R}_{t+k} + \Gamma_{V,k}\widehat{V}_{i,t+k}], \quad (14)$$

where $\Gamma_{R,k} = -R\left(\xi_{po}\beta\right)^k$, and $\Gamma_{V,k} = \left(\xi_{po}\beta\right)^k(\eta'' V + 2\eta')V^2$.[28]

Since $\xi_{po}\beta < 1$, then $\sum_{k=0}^{\infty}[|\Gamma_{R,k}| + |\Gamma_{V,k}|] < \infty$ and

$$\widehat{R}_t = \left(1 - \xi_{po}\beta L^{-1}\right)\left[\widehat{\Gamma}_t - \sum_{k=0}^{\infty}\Gamma_{V,k}\widehat{V}_{i,t+k}\right], \quad (15)$$

[28] The term $\Gamma_\lambda \lambda_{t+k}$ is omitted because $\Gamma_\lambda = 0$ in the deterministic steady state.

where L is the lag operator. Assuming the sup norm topology[29] on the linear space of sequences, the last two conditions imply D maps bounded sequences into bounded sequences and has continuous inverse. Therefore, the inverse and implicit mapping theorems hold and (15) is an first-order Taylor approximation to the solution to (13). This approximate solution is accurate up to an error term of order $O\left(||V||^2\right)$ .Moreover, the solution given by (15) is locally unique. ■

The next section uses the results from proposition 1 to derive the aggregate implications of the time-dependent portfolio adjustment assumption.

6.1. Derivation of the Basic Money Demand Equation

I derived the equilibrium dynamics of velocity, V_t, in the case of small enough disturbances around the steady state defined in proposition 1. Log-linearizing the FONC for money balances (3) we obtain:

$$\begin{aligned} \widetilde{R}_t &= (2\eta' V^2 + \eta'' V^3)\widehat{V}_t + \\ &+ E_t^i \sum_{j=1}^{\infty} (\xi_{po}\beta)^j \left(\frac{1}{\pi g_c}\right)^j \left\{-(g_c\pi)^j R\widehat{R}_{t+j} + (g_c\pi)^j (2\eta' V^2 + \eta'' V^3)\widehat{V}_{t+j}^t\right\}, \end{aligned} \tag{16}$$

where v is the growth rate of consumption and $\widetilde{R}_t = R_t - R$. In (16), I have suppressed the household subindex to make the derivation clearer. However, we must remember velocity refers to household i. The problem with the last expression is that V_{t+j}^t depends on consumption at time $t + j$, conditional on not having re-optimized for the last j periods. Moreover, the expectation operator, E_t^i, must be taken only over those histories in which household i does not re-optimize. As discussed in Christiano (2004) and Woodford (2005), a direct evaluation of the last expression is troublesome. Following, Woodford's insight, I guess and verify that individual velocity obeys the following relation

$$\widehat{V}_{i,t} = \widehat{V}_t^* + \Psi \widehat{\frac{Q_{i,t}}{Q_t^*}}, \tag{17}$$

[29] Definitions of the sup norm and proofs of the Implicit and Inverse Mapping theorems are provided in Lang (1993). Woodford (1998 and 2003) applies those theorems to the existance and uniqueness of a dynamic model characterized by equations of the type $f(p_{t-1}, p_t, p_{t+1}; u_t) = 0$ for some vector p and disturbance u.

where V_t^* is economy-wide velocity and Ψ is a coefficient to be determined. Individual velocity can be expressed in terms of relative variables and aggregate velocity:

$$\widehat{V}_{i,t} = \widehat{V}_t^* + \widehat{\widehat{c}}_{i,t} - \widehat{\frac{Q_{i,t}}{Q_t^*}},$$

then use this last expression and (17) to show that $\widehat{\widehat{c}}_{i,t} \equiv \widehat{\left(\frac{c_{i,t}}{c_t}\right)} = (\Psi + 1)\widehat{\frac{Q_{i,t}}{Q_t^*}}$. To recover Ψ use the fact that the multiplier is the same for everybody. That is, take a log-linear approximation to (4) to show:

$$-\rho(\widehat{\widehat{c}}_t(i) - \widehat{\widehat{c}}_t(l)) = \tau(\widehat{V}_{i,t} - \widehat{V}_{l,t}).$$

Combining the last two equations we obtain:

$$-\rho((\Psi+1)\widehat{\frac{Q_{i,t}}{Q_t^*}} - (\Psi+1)\widehat{\frac{Q_{l,t}}{Q_t^*}}) = \tau(\widehat{V}_t^* + \Psi\widehat{\frac{Q_{i,t}}{Q_t^*}} - \widehat{V}_t^* - \Psi\widehat{\frac{Q_{l,t}}{Q_t^*}}).$$

The last condition must be satisfied for households i and l regardless of when they re-optimized their money balances. There are two possible cases. The first and trivial solution indicates that the last equation is satisfied for any pair of households if their hold the same money balances, i.e., $Q_i = Q_l$. But this condition is not true in the presence of sluggish portfolio adjustment. The second solution requires the equality of the individual terms. That is,

$$-\rho(\Psi+1)\widehat{\frac{Q_{i,t}}{Q_t^*}} = \tau\Psi\widehat{\frac{Q_{i,t}}{Q_t^*}} \text{ and } -\rho(\Psi+1)\widehat{\frac{Q_{l,t}}{Q_t^*}} = \tau\Psi\widehat{\frac{Q_{l,t}}{Q_t^*}}.$$

But these conditions are satisfied if the unknown coefficient is given by:

$$\Psi = -\frac{\rho}{\rho+\tau},$$

where ρ is the coefficient of risk aversion and $\tau = \frac{(2\eta' V + \eta'' V^2)}{(1+\eta+\eta' V)}$. Note that Ψ is independent of individual variables as desired. The evaluation of (16) requires the knowledge of $\widehat{V}_{t+j}^t$. This variable can be readily expressed in terms of aggregate terms and money holdings at time t:

$$\widehat{V}_{t+j}^t = \widehat{V}_{t+j}^* + \Psi\left(\widehat{\frac{Q_{i,t}}{Q_t^*}} - \sum_{k=1}^{j}\widehat{\mu}_{q,t+j}\right)$$

where μ_q is the growth rate of money balances. With an indexation rule of the type $Q_{i,t} = \pi Q_{i,t-1}$, there would be a term $\left(\frac{\pi}{\mu_q}\right)^j$ in front of $\sum_{k=1}^{j} \widehat{\mu}_{q,t+j}$. As a consequence, the model would lack of a stationary steady state in which all households choose the same portfolio. A related phenomenon is present in Altig et at. (2004). Now combine the last results with (16) to show

$$-R\widehat{R}_t + \tau_1(\widehat{V}_t^* + \Psi\widehat{\frac{Q_{i,t}}{Q_t^*}}) + E_t^i \sum_{j=1}^{\infty} (\xi_{po}\beta)^j \left\{ -R\widehat{R}_{t+j} + \tau_1 \left[\widehat{V}_{t+j}^* + \Psi \left(\widehat{\frac{Q_{i,t}}{Q_t^*}} - \sum_{k=1}^{j} \widehat{\mu}_{q,t+j} \right) \right] \right\} = 0, \tag{18}$$

where $\tau_1 = (2\eta' V + \eta'' V^2)$. In equation (18), the expectation operator is taken over aggregate variables.

Let L^{-1} denote the forward operator. Then (18) implies

$$0 = E_t \left[(-\widetilde{R}_t + \tau_1 \widehat{V}_t^* + \omega \widehat{\frac{Q_{i,t}}{Q_t^*}})(1 - \xi_{po}\beta L^{-1}) + \xi_{po}\beta \left(-\widetilde{R}_{t+1} + \tau_1 \widehat{V}_{t+1}^* - \frac{\Psi\tau_1}{1-\xi_{po}\beta} \widehat{\mu}_{q,t+1} \right) \right], \tag{19}$$

where $\omega = \tau_1 \Psi \frac{1}{1-\xi_{po}\beta} < 0$. In (19) the expectation operator E_t^i has been replaced by the regular expectational term since with aggregate variables those operators are the same. I am interested in a symmetric equilibrium in which active households choose the same levels of consumption and nominal balances. Moreover, individual and aggregate money holdings are related by $Q_t^* = \int_A Q_{i,t} da + \int_I Q_{i,t} di$ where A and I are the sets of active and inactive households, respectively. Then we can show that relative money balances for active households obey $\widehat{\frac{Q_{i,t}}{Q_t^*}} = \frac{\xi_{po}}{1-\xi_{po}} \widehat{\mu}_{q,t}$. Combining this equation and (19) and collecting terms we can obtain an expression similar in structure to the new Phillips curve equation,

$$-\widetilde{R}_t + \tau_1 \widehat{V}_t^* - \omega \frac{\xi_{po}}{1-\xi_{po}} \left(\widehat{\mu}_{q,t} - \xi_{po}\beta E_t \widehat{q}_{t+1} \right) - \frac{\Psi\tau_1}{1-\xi_{po}\beta} \xi_{po}\beta E_t \widehat{\mu}_{q,t+1} = 0.$$

Finally, the growth rate of money holding can be rewritten in terms of aggregate velocity and inflation using

$$\widehat{\mu}_{q,t} = \widehat{V}_{t-1}^* - \widehat{V}_t^* + \widehat{\pi}_t + \widehat{g}_{c,t}.$$

Combine the last two equations to arrive to $\widetilde{R}_t = c_1\widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + E_t[c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})]$,with the reduced parameters given by

$$
\begin{aligned}
c_1 &= \tau_1 - \frac{\xi_{po}}{1-\xi_{po}}\omega - \frac{\xi_{po}\omega}{1-\xi_{po}}\xi_{po}\beta - \frac{\Psi\tau_1}{1-\xi_{po}\beta}\xi_{po}\beta > 0, \\
c_2 &= \frac{\xi_{po}\omega}{1-\xi_{po}}\xi_{po}\beta + \frac{\Psi\tau_1}{1-\xi_{po}\beta}\xi_{po}\beta < 0, \\
c_3 &= \frac{\xi_{po}}{1-\xi_{po}}\omega < 0.
\end{aligned}
$$

It follows that $c_1 + c_2 + c_3 = \tau_1 > 0$ and $c_1 > \tau_1$ because $\omega < 0$ and $\Psi < 0$.

Next, I show that the roots in the polynomial $\frac{c_3}{c_2}z^2 + \frac{c_1}{c_2}z + 1 = 0$ are such that $\lambda_i = z_i^{-1}$ satisfy $-1 < \lambda_1 < 1 < \lambda_2$. The solution to the polynomial is given by:

$$
z_1 = \frac{-\frac{c_1}{c_2} + \sqrt{\left(\frac{c_1}{c_2}\right)^2 - 4\frac{c3}{c_2}}}{2\frac{c_3}{c_2}}, z_2 = \frac{-\frac{c_1}{c_2} - \sqrt{\left(\frac{c_1}{c_2}\right)^2 - 4\frac{c3}{c_2}}}{2\frac{c_3}{c_2}}.
$$

$z_1 > 1$ is true if and only if $\sqrt{\left(\frac{c_1}{c_2}\right)^2 - 4\frac{c3}{c_2}} > 2\frac{c_3}{c_2} + \frac{c_1}{c_2} > 0$. Squaring both sides of the inequality and canceling out common terms the last inequality holds if and only if $c_1 + c_2 + c_3 > 0$; but this condition holds for the relevant parameters in this study. Therefore, the root with positive sign in front of the square root is larger than one or $0 < z_1^{-1} < 1$. For the negative root, we have $-\sqrt{\left(\frac{c_1}{c_2}\right)^2 - 4\frac{c3}{c_2}} < 0 < 2\frac{c_3}{c_2} + \frac{c_1}{c_2}$ which implies that $0 < z_2 < 1$ since $\frac{c_1}{c_2} > \sqrt{\left(\frac{c_1}{c_2}\right)^2 - 4\frac{c3}{c_2}}$. Once again, this implies $1 < z_2^{-1}$. In all these derivations, I have assumed that the term inside the square root is positive. I could not find enough conditions on the structural parameters to guarantee this assumption. However, the estimated coefficients reported in tables 2 through 10 satisfy that assumption.

6.2. Dynamic Indexation

As stressed in section 3, there is no evidence about the behavior of inactive households. However, the way these households choose money balances is an important part of the money demand equation. Therefore, I propose an alternative indexation rule for money balances. This alternative rule will allow us to explore the effects that inactive households have on the demand for money and consequently on the elasticity puzzle.

Assumption 2' (Dynamic Indexation): If Calvo does not visit today, then $Q_t = g_c \pi_{t-1} Q_{t-1}$.

The difference between assumptions 2 and 2' lies in the inflation term. According to assumption 2', inactive households continuously update their portfolio using the most recent inflation, π_{t-1}. During periods of relative calm, there is no much difference between the two updating rules. However, I consider it important to analyze the effects of dynamic indexation rules because the 1970s and early 1980s were characterized by significant innovations in inflation. Following the same steps as with the static model, we can log-linearize the new FONC condition about steady state. The dynamic money demand equation is given by:

$$\widetilde{R}_t = c_1 \widehat{V}_t^* + c_4 \widehat{\pi}_t + c_3[\widehat{V}_{t-1}^* + \widehat{g}_{c,t}^* - \widehat{\pi}_{t-1}] + E_t[c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})]. \tag{20}$$

The main difference between this equation and the static indexation one comes from the presence of the lagged inflation term $\widehat{\pi}_{t-1}$ at time t and the independent inflation term $\widehat{\pi}_t$. The results in table 2 reveal similar findings as those from the main section although the point estimates are not significant at standard confidence values. However, we still see that there is a difference between the short- and long-run response of money balances to the interest rate.[30]

6.3. Unit Root Test

Section 4 argues that velocity seems to be stationary although highly persistence (see figure 1). Traditional Dickey-Fuller tests, however, fail to reject the null that velocity has a unit root even at 10%. On the other hand, it is widely accepted that the DF test lacks power so I supplement my analysis using the Covariate Adjusted Dickey Fuller test (CADF) proposed in Hansen (1995). Monte Carlo studies show that the inclusion of covariates in the regression used in the unit root test increases the power of the test.

The CADF test I implement includes lagged inflation, lagged consumption growth, and lagged money growth as covariates. The t statistic from the CADF test is -3.38 while the critical value at 1% significance level is -2.57.[31] We can see that compared to the traditional DF test, the CADF test easily rejects the presence of a unit root in velocity.

[30] I also tried the alternative indexation rule $Q_t = \pi_{t-1} v_{t-1} Q_{t-1}$. Under this assumption, the money demand equation is given by $\widetilde{R}_t = c_1 \widehat{V}_t^* + c_4 \widehat{\pi}_t + c_5 \widehat{v}_t^* + c_3[\widehat{V}_{t-1}^* - \widehat{\pi}_{t-1}] + E_t[c_2(\widehat{V}_{t+1}^* - \widehat{v}_{t+1}^* - \widehat{\pi}_{t+1})]$. The results were statiscally insignificant, however; for this reason I choose to present the case in which only past inflation matters.

[31] The critical values are from Hansen (1995) table 1, standard case, page 1155.

When I include interest rates as a covariate, I reject the null of unit roots in velocity only at the 10% level. The t statistic for the CADF test is -1.60 while the 10% critical value is -1.51. This value is taken from Hansen (1995), Table 1. This second results is not surprising since interest rates are highly persistent. However, I consider that the combined evidence is enough to reject the presence of a unit root in velocity.

6.4. Additional Robustness Checks

6.4.1. Case III

As before, let μ_q denote the growth rate of the economy-wide nominal balances, i.e. $Q_t^* = \mu_q Q_{t-1}^*$. This equation combined with $Q^* = \int Q_i di + \int Q_a da$, where a stands for active households while i for inactive, implies

$$Q_t = \frac{1}{1-\xi_{po}} \frac{\mu_q - \pi\xi_{po}}{\mu_q} Q_t^*.$$

Here, Q_t represents balances chosen by active households at time t. We can see that if the growth rate of nominal balances equals the inflation rate, then balances chosen by active households correspond exactly to the economy-wide balances. It turns out that in the sample period under consideration those two rates are reasonably close to each other, $\mu_q = 1.017$ and $\pi = 1.01$. Then we can replace individual variables for economy-wide figures in (3) such that V_t becomes V_t^*. With this additional assumption, the new static condition to be estimated are:

$$E[(-\widetilde{R}_t + b_1\widehat{V}_t^* + b_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1}))X_t] = 0.$$

The resulting equations can be understood as rough approximations of the true models from cases I and II, but with the advantage that fewer estimated parameters are estimated. Moreover, from the empirical literature on Phillips Curves, it is well known that Hansen's J statistic has low power against alternative formulations, see Eichenbaum and Fisher (2004). Therefore, we can use Case III as a robustness check for the results derived from Cases I and II. This experiment will help to discover whether the results are robust to alternative specifications. The results from this experiment are reported in Table 7. A quick look at Hansen's J statistic and the coefficient significance reveals that the reduced model does a poor job. The explanation is quite simple: the assumption that only active households matter implies that interest rates obey the following equation:

$$\widetilde{R}_t = b_1 \widehat{V}_t^* + b_2 E_t(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1}),$$

which in turn requires interest rates to be a purely forward looking function of real balances, inflation, and consumption growth. From the Taylor rule literature, however, we know that interest rates are better described by backward looking rules.[32] Overall, Case II under the assumption that households use a static indexation rule offers the best description of the data.

6.4.2. Using Inflation as Instrument

In my initial analysis, I do not include inflation as an instrument. The reason is that velocity and consumption growth taken together can be used to derive an indicator of both monetary activity and inflation. The problem is that this indicator might not distinguish between inflation and money. To account for this possibility, I repeat the estimation of the model but this time I include inflation as an additional instrument. Table 8 reports the results from the new estimation. As with the case of additional lags, the point estimates are a little bit larger than those in the benchmark case. The static formulation, for example, indicates that the short- and long-run elasticities, under the permanent definition, are 0.89 and 25, respectively. The Hansen statistic indicates that we reject the model at 10% significance level.

Comparing the results in Tables 2 and 8 we conclude that the point estimates, and therefore their economic interpretation, do not significantly change with the inclusion of inflation as instrument.[33] The larger coefficients obtained in this subsection tend to confirm the popular view that larger instrument sets may deliver biased estimates in small samples (see Staiger and Stock (1997)).

6.4.3. Additional Lags in the Instrument Set

As argued in Section 4, my choice of the set of instruments is driven by two factors. On one hand, I need a parsimonious set which can be maintained throughout the estimation of 2, 3 or 4 coefficients. On the other hand, a small instrument set decreases the small sample bias due to weak instruments reported in Staiger and Stock (1997). In this section, I consider the effects of including

[32] Woodford (2003), chapter 4, uses a generalized Taylor rule of the form $R_t = \overline{R}_t + \rho(R_{t-1} - \overline{R}_{t-1}) + \theta_\pi(\pi_t - \pi) + \frac{\theta_y}{4}(x_t - x)$,where x is a measure of economic activity like deviation from output. This equation clearly show why any attempt to fit interest rates in terms of future variables will systematically fail.

[33] Note that the confidence intervals implied by the parameters in this section include the point estimates in the benchmark case.

additional lags in the set of instruments. Specifically, I repeat the estimation of Case II, but this time using four lags of the instruments. Table 9 presents the results. For the static indexation version, we see that the coefficients are statistically significant and the elasticities are slightly larger than those reported in Table 2. However, the new estimates are well within one-standard deviation of those reported previously. For instance, the long-run elasticity is 19.23 which falls in the two-standard deviation interval associated with the long-run elasticity for the benchmark instrument set. In terms of Hansen's J statistic, we see that the model is rejected at standard confidence levels.

6.4.4. Normalization

Hamilton et al. (2003) provide examples in which the way an economic model is normalized matters for estimation. Their examples involve nonlinear, GMM, and likelihood-based methods. Moreover, Yogo (2003) shows that the estimation of the elasticity of intertemporal substitution is affected by different normalizations. The conclusion from this evidence is that GMM is not invariant to transformations in small samples. Furthermore, the choice of normalization for the moment condition may affect not only point estimates but also confidence intervals.[34]

To confront this econometric issue Table 10 presents the results of the estimation of Case II static indexation under three alternative transformations. In the first panel, the coefficient in front of current velocity, V_t, has been normalized to one. The coefficients correspond to the equation $\theta\widetilde{R}_t = \widehat{V}_t^* + c_3(\widehat{\pi}_t + \widehat{g}_{c,t}^* + \widehat{V}_{t-1}^*) + E_t[c_2(\widehat{V}_{t+1}^* - \widehat{g}_{c,t+1}^* - \widehat{\pi}_{t+1})]$. The other two panels are interpreted in a similar way. Several features are worth emphasizing. First, we cannot reject either normalization based on Hansen's J statistic. Moreover, all coefficients are significantly different from zero at standard confidence levels. Second, the long-run elasticities are larger than those reported in Table 2. The largest difference happens when the forward or backward looking coefficients, c_2 and c_3, are normalized to one. However, if we take into account sampling error, the elasticities are not statistically different from those previously found. Finally, the coefficient in front of interest rates, θ, is estimated with less precision relative to the remaining coefficients. This in turn affects the precision of the elasticities.

In summary, the results are robust to different normalizations. This conclusion is not unexpected since the instrument set was chosen to avoid the weak instrument problem (see Staiger and Watson

[34] Gali and Gertler (1999) estimate a New-Keynesian Phillips Curve in which normalization is not an issue. Their results, however, seem to be the exception rather than the rule.

(1997)). As argued in Yogo (2003), strong instruments makes GMM less vulnerable to normalization bias.[35]

6.4.5. Alternative Interest Rate: Aaa Corporate Yield Bond

The results from Section 4.4 were derived under the assumption that the relevant opportunity cost for M2-minus is the T-bill Rate. However, this rate is not the interest rate earned on saving accounts. Moreover, investment accounts may yield higher and more volatile returns than the T-bill rate. To account for this possibility, I repeat the estimation of equation 6, but this time I use the Aaa Corporate Yield Bond as an alternative measure of the opportunity cost for M2-minus. According to the estimation, the short- and long-run elasticities are $\underset{(0.48)}{1.69}$ and $\underset{(1.23)}{9.67}$, respectively. Furthermore, the Hansen's statistic does not reject the model's validity at 5 percent. As before, there is a significant difference between the point estimates and these are close to the estimates found by Christiano, Eichenbaum, and Evans (2005) and Lucas (1988), respectively.

[35] To complement the analysis, I implemented the F statistic suggested in Stock and Yogo (2003) and implemented in Yogo (2003) to test for null hypothesis of weak instruments. This statistic soundly rejected the hull.

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Table1: Goldfeld's Estimation

Variable	1952:III-1974:I	1952:III-1986:IV	1974:II-1986:IV
Intercept	0.381 (1.7)	−0.313 (2.6)	−0.451 (2.6)
y	0.131 (5.0)	0.039 (5.0)	0.044 (1.3)
RCP	−0.016 (5.9)	−0.013 (5.0)	−0.018 (2.6)
$RCBP$	−0.030 (3.3)	−0.02 (0.4)	0.100 (1.0)
m_{t-1}	0.788 (12.4)	1.007 (47.9)	0.997 (32.6)
P_t/P_{t-1}	−0.711 (6.9)	−0.889 (9.5)	−0.823 (2.6)
ζ_{short}	$1.6^a,\ 3.0^b$	$1.3^a,\ 2.0^b$	$1.8^a,\ -1.0^b$
ζ_{long}	$7.5^a,\ 14.2^b$	NA	$600^a, -3.3e3^b$

t-statistics in parentheses

RCB: commercial paper rate, RCBP: commercial bank rate

^{a} using RCB, ^{b} using RCBP

Table 2: GMM Estimates Complete Sample

Case I: Static Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
0.48 (0.19)	−0.11 (0.10)	−0.35 (0.09)	NA	0.68 (0.18)	0.99 (0.19)	27.78 (7.49)	0.00

Case I: Dynamic Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
−0.21 (0.09)	0.20 (0.19)	0.04 (0.03)	0.36 (0.09)	NA	NA	NA	0.00

Case II: Static Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
1.59 (0.72)	−0.70 (0.36)	−0.88 (0.35)	NA	0.26 (0.11)	1.04 (0.40)	13.16 (3.34)	0.14

Case II: Dynamic Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
0.94 (0.94)	−0.17 (0.64)	−0.76 (0.35)	−1.15 (0.92)	0.32 (0.15)	0.42 (0.29)	19.23 (11.93)	0.02

Standard errors in parentheses

Instruments: two lags of velocity, output growth and interest rates

Sample: 1960:I - 2005:IV

Table 3: GMM Estimates Complete Sample

Fed Fund Rate

c_1	c_2	c_3	ζ^T_{short}	ζ^P_{short}	ζ^P_{long}	J_T
1.79	−0.76	−1.01	0.23	0.78	12.50	0.08
(0.82)	(0.42)	(0.42)	(0.09)	(0.29)	(3.68)	

M2-minus Own Rate

c_1	c_2	c_3	ζ^T_{short}	ζ^P_{short}	ζ^P_{long}	J_T
1.20	−0.56	−0.63	0.36	1.70	16.67	0.48
(0.56)	(0.29)	(0.27)	(0.15)	(0.70)	(4.90)	

Standard errors in parenthesis, p-value for Hansen's statistic

Instruments: two lags of velocity,

consumption growth, and interest rate

Sample: 1960:I - 2005:IV

Table 4: Sub-Sample Analysis

Case II: Static Indexation 1960:I - 1983:IV

c_1	c_2	c_3	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
0.91	−0.45	−0.44	0.46	3.00	16.67	0.27
(0.42)	(0.21)	(0.21)	(0.19)	(1.72)	(3.92)	

Case II: Static Indexation 1984:I - 2005:IV

c_1	c_2	c_3	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
1.47	−0.46	−0.95	0.24	0.42	4.72	0.87
(0.53)	(0.29)	(0.27)	(0.06)	(0.11)	(1.23)	

Case II: Static Indexation 1996:I - 2005:IV

c_1	c_2	c_3	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
2.08	−1.01	−1.05	0.20	0.48	5.68	0.35
(0.95)	(0.51)	(0.46)	(0.07)	(0.11)	(2.10)	

Standard errors in parenthesis

Instruments: two lags of velocity, output growth,

and interest rates

Table 5:

Implied Coefficients from GMM

	Case II			
	β	η	η''	ξ_{po}
Static	0.99	0.054 (0.54)	0.00 (2e−4)	0.77 (2e−3)
	Sub-Sample Analysis			
1960:I-1983:IV	0.99	0.051 (0.24)	0.00 (1e−4)	0.84 (0.19)
1984:I-2005:IV	0.99	0.034 (0.07)	0.004 (5e−4)	0.67 (0.01)
1996:I-2005:IV	0.99	0.001 (0.38)	0.002 (0.01)	0.70 (0.01)

Standard errors in parentheses

Standard errors computed using Delta Method

Table 6: Volatilites

	0.75 (4 quarters)	0.67 (3 quarters)	0.5 (2 quarters)	0.35 (1.5 quarters)	0.01 (1 quarters)	*Data* Pre-1984	*Data* Post-1984
σ_m	1	1.21	1.31	1.36	1.38	1	1.14
σ_y	1	0.84	0.78	0.76	0.75	1	0.44
σ_c	1	0.87	0.82	0.79	0.79	1	0.52
σ_r	1	0.50	0.30	0.21	0.18	1	0.50

Table 7: GMM Estimates Complete Sample

Case III: Static/Lagged Information					
b_1	b_2	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
−0.10 (0.07)	0.11 (0.07)	0.00	12.95 (1.95)	12.95 (1.95)	0.00
Case III: Dynamic/Lagged Information					
b_1	b_2	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
−0.01 (0.02)	0.03 (0.02)	0.00	12.50 (1.90)	12.50 (1.90)	0.00

Standard errors in parenthesis, p-values for Hansen statistic

Instruments: four lags of velocity, consumption

growth and interest rate

Sample: 1960:I - 2005:IV

Table 8: GMM Estimates with Inflation as Instrument

Case II: Static Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
1.15 (0.49)	−0.45 (0.24)	−0.70 (0.26)	NA	0.34 (0.12)	0.89 (0.27)	25.00 (9.75)	0.07

Case II: Dynamic Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
1.30 (0.55)	−0.46 (0.28)	0.82 (0.29)	−1.47 (0.58)	0.37 (0.11)	0.74 (0.17)	13.15 (2.90)	0.10

Standard errors in parentheses, p-values for Hansen statistic

Instruments: two lags of velocity, output growth

inflation and interest rate

Sample: 1960:I - 2005:IV

Table 9: GMM Estimates with Four Lags

Case II: Static Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
1.10 (0.24)	−0.51 (0.13)	−0.58 (0.12)	NA	0.39 (0.07)	1.89 (0.76)	19.23 (5.17)	0.00

Case II: Dynamic Indexation

c_1	c_2	c_3	c_4	ζ^T	ζ^P_{short}	ζ^P_{long}	J_T
0.12 (0.75)	0.13 (0.38)	−0.23 (0.39)	−0.22 (0.379)	1.03 (1.04)	0.68 (2.54)	20.83 (4.62)	0.02

Standard errors in parentheses, p-values for Hansen statistic

Instruments: four lags of velocity,

consumption growth, and interest rates

Sample: 1960:I - 2005:IV

Table 10: Different Normalizations

Case II: Static Indexation

$\widehat{V}_t^*$

θ	c_2	c_3	ζ^P_{short}	ζ^P_{long}	J_T
0.22	−0.49	−0.51	1.14	18.00	0.56
(0.16)	(0.04)	(0.04)	(0.88)	(14.7)	

Case II: Static Indexation

$\widehat{V}^*_{t+1} - \widehat{\nu}^*_{t+1} - \widehat{\pi}_{t+1}$

c_1	θ	c_3	ζ^λ_{short}	ζ^λ_{long}	J_T
−1.98	−0.38	0.98	1.67	23.68	0.53
(0.12)	(0.35)	(0.16)	(0.45)	(11.35)	

Case II: Static Indexation

$\widehat{\pi}_t + \widehat{v}^*_t + \widehat{V}^*_{t-1}$

c_1	c_2	θ	ζ^P_{short}	ζ^P_{long}	J_T
−1.99	0.99	−0.63	1.59	19.47	0.65
(0.15)	(0.15)	(0.39)	(1.08)	(13.59)	

Standard errors in parentheses,

Instruments: two lags of velocity,

output growth, and interest rates

Sample: 1960:I - 2005:IV

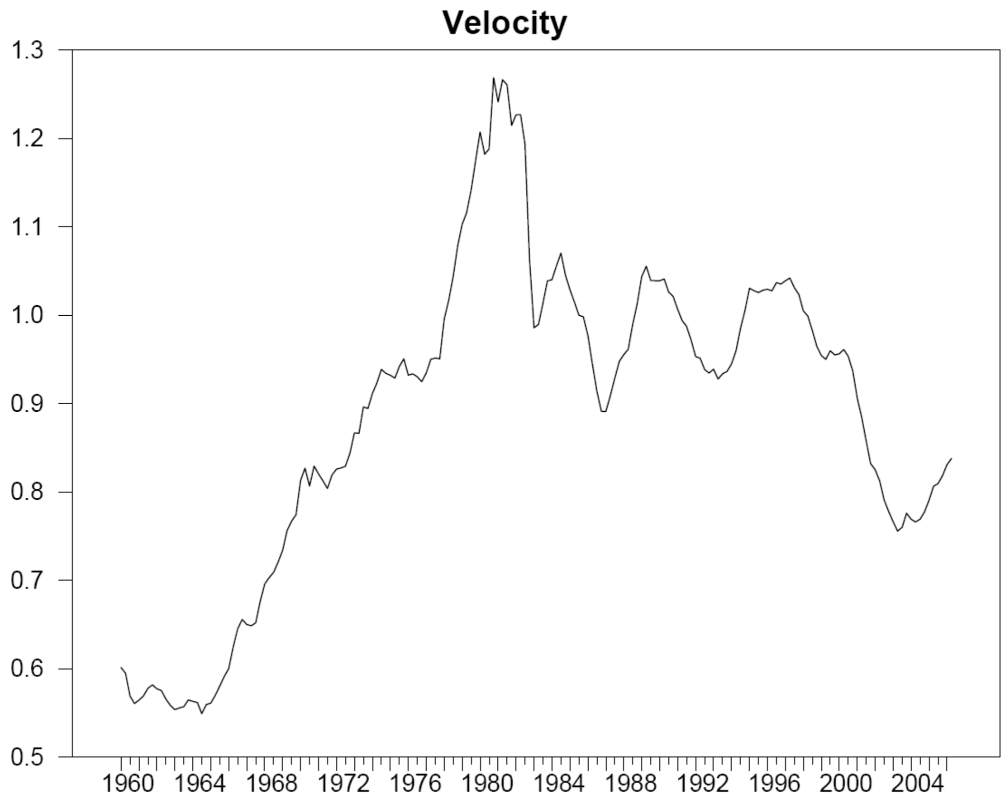
Velocity
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004

Figure 1: