



Thursday, February 5, 2009
3:00–3:50 p.m.
Harrelson 330

Binomial coefficients and beyond

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If you multiply out the product $(x + y)^n$, the coefficient of $x^k y^{n-k}$ is known as a binomial coefficient. For instance,

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

The coefficient of $x^k y^{n-k}$ counts subsets of $\{1, \dots, n\}$ of size k , so for instance there are 6 subsets of $\{1, 2, 3, 4\}$ of size 2. This coefficient also counts the paths from $(0, 0)$ to $(2, 2)$ in the plane comprised of steps $(1, 0)$ and $(0, 1)$. But what if x and y don't commute? What if $yx = qxy$? It turns out the coefficients now count paths in a more refined way, with the coefficient of $q^A x^k y^{n-k}$ counting paths from $(0, 0)$ to $(k, n-k)$ having area A below the path. I will discuss this result and some other interesting properties of this **q-analogue** of binomial coefficients that can be proven using a mixture of linear algebra and group theory or in an especially slick way using the representation theory of sl_2 .

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