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Integrated deterministic and stochastic simulation of electronic circuits: Application to large signal–noise analysis

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SUMMARY

The ability to model the effect of non-negligible levels of white noise superimposed on a carrier is investigated when this signal–noise combination is fed to the input of an MMIC power amplifier. Transient simulation using stochastic differential equations is introduced here to handle large levels of noise of arbitrary frequency characteristics. The effectiveness of the modelling is ascertained by looking at measured gain-compression plots at the output of the amplifier and comparing these with simulated results. It is found that increasing levels of noise introduce increased compression of the output power characteristic.

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KEY WORDS: large signal–noise; stochastic integration; transient noise simulation; MMIC amplifier

1. INTRODUCTION

The simulation of noise in electronic circuits and systems has generally treated noise models as being negligibly small sources of random terms generated in a particular distribution. This has enabled separation of the random terms from the deterministic terms during analysis. While this approach works well for most circuits, there are certain applications where the noise is comparable with the signal level and impacts the performance of circuits. In these cases, increased levels of noise can affect the overall performance of the system under consideration. As will be shown, it is possible to analyze a circuit with high levels of noise using numerical simulation by bringing together several advanced capabilities. Specifically, high simulator

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dynamic range is required (fREEDA™‡, used here, has a dynamic range in transient simulation exceeding 160 dB [1]); precise nonlinear models for elements of the system [2]; and, as presented here, a stochastic solver integrated into the general-purpose transient simulation framework. Even though total noise power may be large, perturbations from time point to time point can be very small and high simulation precision is required to track signals and separate stochastic from deterministic results after each time step and each nonlinear iteration.

This paper explores the effect of high levels of noise superimposed on a carrier on the gain-compression characteristics of a linear X-band MMIC driver amplifier. A series of measurements are performed each with varying levels of noise and the corresponding gain-compression curves are generated. These curves are compared with simulated results obtained from the high dynamic range simulator fREEDA™. Section 2 is an introduction to the theory of stochastic differential equations (SDEs) and highlights the different interpretations that are possible when dealing with equations of this nature. Section 3 is an introduction to the circuit simulator fREEDA™ and highlights the basic technique behind the transient noise analysis routine and the formulation of the corresponding error function. Section 4 explains the measurement technique used to explore the effects of large levels of noise on amplification and compares the results with simulation. Section 5 makes an attempt to fill some gaps that arise while comparing measurement and simulation in the previous section and opens up new areas of exploration in nonlinear device modelling. Finally, Section 6 presents conclusions as a result of this work.

2. STOCHASTIC DIFFERENTIAL EQUATIONS

2.1. Introduction

SDEs extend ordinary differential equations (ODEs) with the presence of random terms. Langevin was among the first to transform the ODE

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a(t, x) \tag{1}$$

in to a noisy differential equation containing an additive perturbative term resulting in the SDE of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}X_t = a(t, X_t) + b(t, X_t)\xi_t \tag{2}$$

The term $a(t, X_t)$ is the deterministic drift coefficient and the term $b(t, X_t)\xi_t$, an intensity factor, represents the perturbative effect. ξ_ts are normal random processes whose covariance has a constant spectral density and hence is referred to as a white-noise process. As a white-noise process assigns an equal weight to each of its Fourier frequency components, the covariance is deduced to be a constant multiple of the Dirac delta function δ_t. Inserting $a = 0$ and $b = 1$ in the above equation gives

$$\frac{\mathrm{d}}{\mathrm{d}t}B_t = \xi_t \tag{3}$$

where B_t is the well-known Brownian motion process. The fact that white noise is the derivative of Brownian motion process has been utilized. Although the sample paths of the Brownian

‡ http://www.freeda.org

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motion process are nowhere differentiable, Japanese mathematician Itô [3] was able to provide a way out of this problem by introducing a new stochastic process, the Itô stochastic integral, which enables (2) to be expressed in differential form as

$$\mathrm{d}X_t = a(t, X_t)\,\mathrm{d}t + b(t, X_t)\,\mathrm{d}B_t \tag{4}$$

or in integral form as

$$X_t(\omega) = X_0(\omega) + \int_0^t a(s, X_s(\omega))\,\mathrm{d}s + \int_0^t b(s, X_s(\omega))\,\mathrm{d}B_s \tag{5}$$

where the second integral is not a conventional Riemann integral but an Itô stochastic integral and is evaluated with respect to the Brownian motion. When trying to approximate a stochastic sum to the above integral it can be shown [4] that convergence can only be achieved in the mean-square sense. For a suitable class of random functions $h:[0,T]\rightarrow\Re$ and partitions $0 = t_0 < t_1 < t_2 \cdots < t_N = T$ with maximum spacing Δ, the Itô integral is defined as the mean-square limit of Itô sums in which the integrand is evaluated at the lower end point of the partition t_j of each sub-interval $[t_j, t_{j+1}]$ and is expressed as

$$\int_0^T h(t,\omega)\,\mathrm{d}B_t = \mathrm{ms} - \lim_{\Delta\to 0} \sum_{j=0}^{N-1} h(t_j,\omega)(B_{t_{j+1}} - B_{t_j}) \tag{6}$$

Stratonovich proposed an alternative interpretation wherein the function is evaluated at the mid-point of each interval [5]. It is this interpretation that is essential for incorporating large signal–noise analysis in a general-purpose transient circuit simulator. The convergence formula for the Stratonovich form can be expressed as

$$\int_0^T h(t,\omega)\circ\,\mathrm{d}B_t = \mathrm{ms} - \lim_{\Delta\to 0} \sum_{j=0}^{N-1} h((t_j + t_{j+1})/2,\omega)(B_{t_{j+1}} - B_{t_j}) \tag{7}$$

and the Stratonovich SDE is expressed in differential form as

$$\mathrm{d}X_t = a(t, X_t)\,\mathrm{d}t + b(t, X_t)\circ\,\mathrm{d}B_t \tag{8}$$

The symbol $\circ$ is used to indicate that the equation under consideration is being interpreted in the Stratonovich sense and the Stratonovich integral equation becomes

$$X_t = X_0 + \int_{t_0}^t a(s, X_s)\,\mathrm{d}s + \int_{t_0}^t b(s, X_s)\circ\,\mathrm{d}B_s \tag{9}$$

This rule makes no assumptions on the nature of the interactions of the stochastic and deterministic terms. More importantly Stratonovich showed that this interpretation allows the system of differential equations to be solved using the conventional rules of calculus.

2.2. Itô versus Stratonovich forms

Both Itô and Stratonovich forms are mathematically accurate but interpreting an SDE in either form will give different end results. This is a paradox, which must be overcome if one is to favorably accept the results of the stochastic differential framework. This is not the first instance in which this conundrum has been addressed. There are several approaches that have been used to provide a justification for choosing an interpretation. Here, we refer to the work of West *et al.* in [6] where they trace back the origin of SDE to Langevin's equation, see (2). This equation in

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its original form is linear in the dependent variable of the system and the stochastic term is purely additive. The probability distribution function of this dependent variable can be determined by the so-called Fokker–Planck equation of evolution [7]. Obtaining this Fokker–Planck equation requires the use of the ordinary rules of calculus. If the Langevin equation is linear and the stochastic term is purely additive, then the final form of the Fokker–Planck equation obtained using Itô's rules of calculus is identical to the form obtained using the ordinary rules of calculus, or the form proposed by Stratonovich. However, when the fluctuations are non-additive, as in the more general case, the two interpretations produce different results although one can convert one form to another. The key to understanding the right approach to use requires an understanding of the underlying assumptions behind each approach. When SDE involves a multiplication between the deterministic term and the stochastic term, the differences between the two interpretations become immediately evident. In order to generalize this discussion, let us write the stochastic integrand of (5) or (9) in the form $g(t, X_t)f(t)$, where $f(t)$ represents the fluctuating noise term that is multiplied with $g(t, X_t)$. Itô's classical form considers zero correlation between the deterministic and stochastic components, or in other words $\langle g(t, X_t)f(t)\rangle$ vanishes, where the angular brackets represent the correlation function. The other form, or the form employed by Stratonovich, assumes that there is a finite amount of correlation between the deterministic and stochastic terms, or in other words $\langle g \times (t, X_t)f(t)\rangle$ is non-zero. It is argued in [6] that there will always be, in general, a finite amount of correlation between the deterministic and stochastic terms because the fluctuations are never purely delta-correlated. This correlation can be neglected only if the correlation time is small compared with the time scales of the rest of the system, which can be assumed in the special cases of the modelling of genetic and population dynamics (see [8]) or when modelling delta-correlated white-noise processes in electronic circuits (see [9]), but not in the general case. Also, it is well known that a $1/f$ noise process is infinitely correlated with its past history [10]. Furthermore, as shown in [6], the probability density function obtained with the Itô assumption is not normalizable and is therefore not a valid probability density function. Starting out with the Stratonovich assumption will, on the other hand, produce a probability density function that is normalizable. The solution of a nonlinear system with multiplicative fluctuations and non-zero correlation between the stochastic and deterministic terms can therefore only be appropriately determined in the Stratonovich sense. This has also been validated experimentally in [11] wherein the authors consider line broadening in harmonic oscillators and systems described by higher-order chemical reactions employing Langevin models with non-additive fluctuations and find the Stratonovich assumption to be accurate. Further experimental validation can be found in [12] wherein the authors compare simulation and measurement curves of an analogue simulator comprising noise generators, integrators and multipliers and find the Stratonovich form to be the more accurate one. To allow for consideration of nonlinear, correlated and multiplicative effects, our approach has therefore made use of the Stratonovich assumption.

3. IMPLEMENTATION IN *f*REEDA™

The circuit simulator *f*REEDA™ is a state-variable-based simulator which is developed on object-oriented principles [13], which allows for a clean separation of data and algorithm. This permits a device model developer to be ignorant of the details of the implementation of the

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algorithm used to analyze the circuit. Similarly, the algorithm developer can be fairly uninvolved in the model-creation process and still produce a robust analysis routine. *f* REEDA™ uses several off-the-shelf libraries to perform several computing/mathematical tasks. As of this writing, *f*REEDA™ uses a combination of the *BLAS*[§] and *SuperLU*[¶] libraries to perform linear algebra and sparse matrix operations, respectively. It uses the *FFTW*[‖] library to perform fast-Fourier transform operations and the *NNES*[**] library to solve nonlinear systems of equations and the package *ADOL-C*[††] to automatically compute derivatives for its device models; for further details, see [2]. There is support for several device models that include conventional electronic models such as resistors, capacitors and inductors, voltage and current sources, semiconductor diodes, BJT, JFET and several levels of MOSFET transistors. Additionally, there are microwave elements such as gyrators, circulators, microwave diodes and MESFETs. Behavioral models such as analog mixers, filters and Foster's canonical *N*-port formulation are also supported.

There is support for several analysis types ranging from an operating point or DC analysis, a basic fixed-time-marching analysis to more complex SPICE-like variable time step routines and steady-state frequency-domain-based harmonic balance analysis. The underlying approach *f* REEDA™ can be thought of as equation-based process modelling as opposed to the more conventional sequential modular approach to process modelling in the sense of [14]. This enables *f*REEDA™ to implement models that are not just restricted to electronic devices but become a universal simulator with a generic model formulation system that can use any appropriate variable as a state variable. This allows for a heterogeneous nature of variables to be present in a single error function. In other words, it allows for the simulation of different types of sub-systems to be connected together in useful configurations. A generic example is shown in Figure 1, which connects spatially distributed elements to linear and nonlinear electronic and thermal elements.

The idea behind transient analysis routines in *f*REEDA™ is to convert the differential equations describing the circuit into an algebraic system of nonlinear equations using time-marching integration methods. The circuit is partitioned into separate groups that contain linear elements and sources in one group and nonlinear elements in the other group. This is represented schematically in Figure 2. The linear elements are handled with traditional modified nodal-admittance techniques. In order to formulate the error function, the nonlinear elements are replaced by nonlinear voltage or current sources. For every nonlinear element, one terminal is assumed to be the reference and the element is replaced by a set of sources connected between the remaining terminals and the reference terminal. The error function is formulated at the interface between the linear elements and the nonlinear elements/sources. In general, this is a reduced number of variables when compared with having to solve for every state variable in the circuit. The error function combines the contributions from the linear and nonlinear sides as

$$\mathbf{f}(\mathbf{x}_\xi) = \mathbf{v}_\mathrm{L}(x_\xi) - \mathbf{v}_\mathrm{NL}(x_\xi) = \mathbf{0} \tag{10}$$

and contains a mixture of deterministic terms and stochastic noise terms that can be both additive and multiplicative. $\mathbf{v}_\mathrm{L}(\mathbf{x}_\xi)$ and $\mathbf{v}_\mathrm{NL}(\mathbf{x}_\xi)$ correspond to the contributions of the linear and

[§] www.netlib.org/blas.
[¶] http://crd.lbl.gov/xiaoye/SuperLU/.
[‖] www.fftw.org.
[**] www.netlib.org/opt.
[††] http://www.math.tu-dresden.de/adol-c/.

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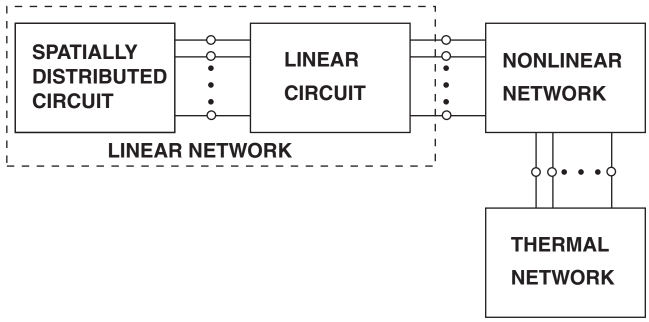

Figure 1. Connections between a heterogeneous collection of elements.

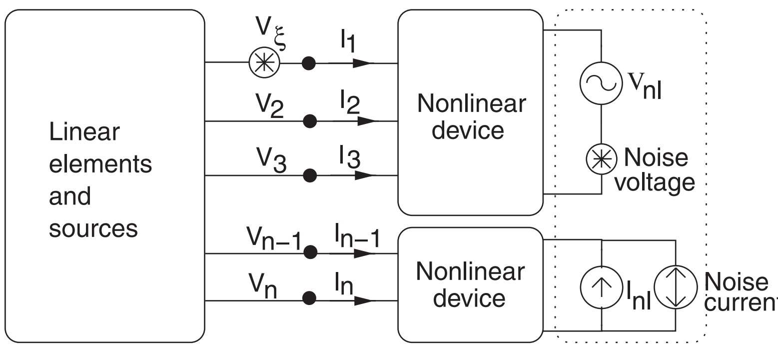

Figure 2. A partitioned network of linear and nonlinear elements and sources along with internal sources of noise.

nonlinear parts of the circuit and depend on stochastic state variables, $\mathbf{x}_\xi$. For the deterministic case [15], the device models would not contain any sources of random noise and the variables of the system would depend on the deterministic state variables, $\mathbf{x}$, although the general form of the error function would remain unchanged, see [16] for further details. The stochastic terms need not necessarily be small signal as they may produce significant instantaneous components upon interaction with large deterministic signals in a circuit. If the resulting system of SDEs is interpreted in the Stratonovich sense, it can be solved by the same numerical techniques used to solve a deterministic system of equations. This allows for easy implementation of a concurrent deterministic and stochastic framework and enables the use of pre-existing robust and established numerical solution techniques.

4. VERIFICATION

The theory outlined here was experimentally verified by considering a carrier plus noise presented to the input of an X-band PHEMT MMIC amplifier (Filtronic LMA411), the layout of which is shown in Figure 3. The MMIC operates between 8.5 and 14 GHz and has an output

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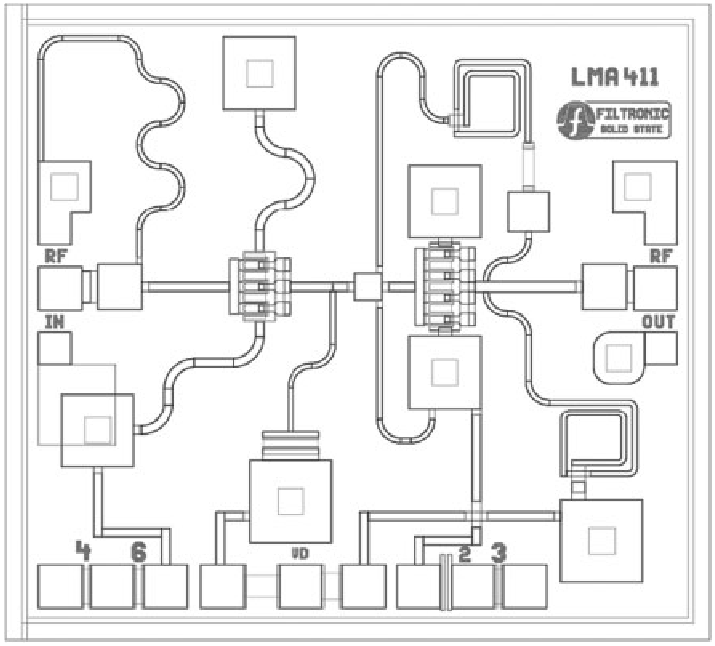

Figure 3. Layout of the two-stage X-band MMIC.

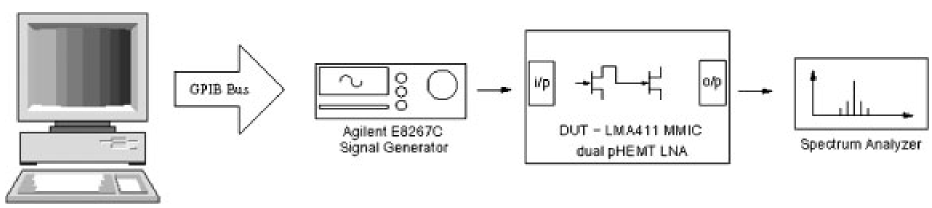

Figure 4. Measurement setup for the X-band MMIC amplifier.

power of +14 dBm at 1 dB gain compression and 17 dBm at 3 dB gain compression. At 10 GHz, the noise figure is 2 dB. The circuit consists of two cascaded pHEMT stages each in a class-A common source configuration. The device was biased at a DC voltage of 6 V and a current of 94 mA. The measurement setup used is shown in Figure 4. A white-noise sequence was generated digitally and transferred via a GPIB bus to the Agilent E8267C signal generator that was set to a carrier frequency of 10 GHz. The signal generator is capable of accepting externally generated signals in a band of 100 MHz about a preset center frequency. This procedure creates a composite signal consisting of a 10 GHz sinusoid and a synthesized 100 MHz wide-band-limited white-noise sequence. The white-noise sequence was generated using the `randn` function in Mathworks's MATLAB, which uses the Ziggurat technique [17]. This approach is known to generate white-noise samples with a very large period, roughly 2^{1492}, thus ensuring that the random sequence of numbers so generated will be truly random for almost all lengths of data sets of interest.

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The amplitude of the composite signal was varied from −20 dBm to 5 dBm in steps of 1 dBm and that of the gain of the amplifier was measured at 10 GHz with two different input conditions: first one with no noise at the input and second with the signal-to-noise ratio maintained at −20 dBc. A plot of measured gain *versus* input power with no input noise is compared with the plot of measured gain with noise level maintained at −20 dBc and is shown in Figure 5. In this plot, the total noise power in the 100 MHz noise bandwidth is specified. High levels of noise suppress the effective power gain of the amplifier with respect to the carrier. The suppressed power at the center frequency is dissipated into frequency bands around 10 GHz. This is represented schematically in Figure 6. It should be noted that a noise level at −20 dBc means that the total input noise is maintained at a constant level of 20 dB below the carrier.

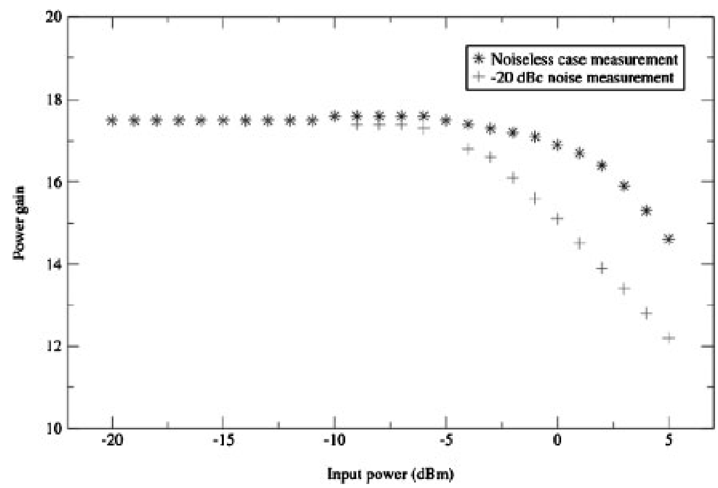

Figure 5. Comparison between measured curves of gain with no input noise and noise maintained at −20 dBc.

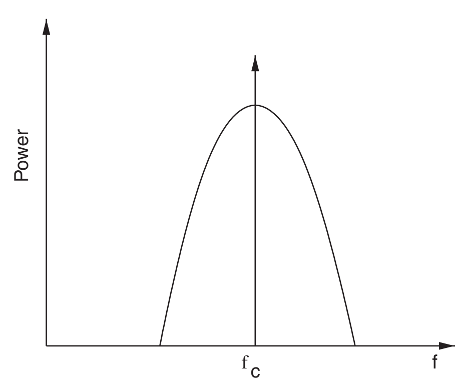

Figure 6. Power transferred from the center frequency into the sidebands.

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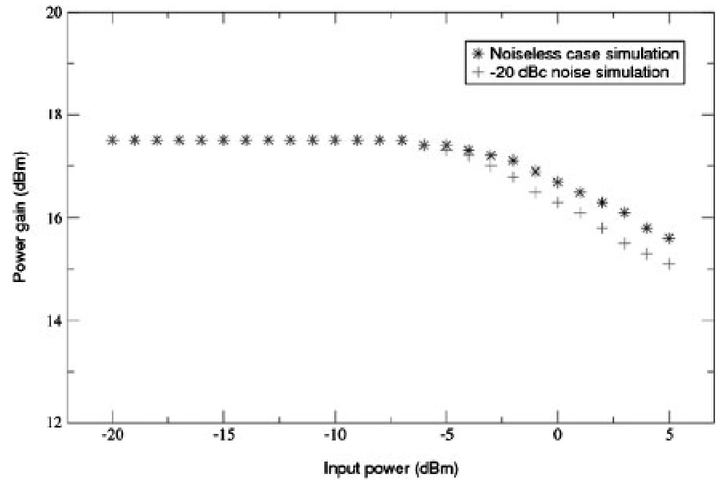

Figure 7. Comparison between simulated curves of gain with no input noise and noise maintained at −20 dBc.

Table I. MESFET model parameters for X-band MMIC.

Device 1			
a0 = 0.09910	a1 = 0.08541	a2 = −0.0203	a3 = −0.015
beta = 0.01865	gama = 0.8293	vds0 = 6.494	vt0 = −1.2
vbi = 0.8	cgd0 = 3f	cgs0 = 528.2f	is = 3e − 12
nr = 1.2	t = 1e − 12	vbd = 12	kf = 1e − 9
Device 2			
a0 = 0.1321	a1 = 0.1085	a2 = −0.04804	a3 = −0.03821
beta = 0.03141	gama = 0.7946	vds0 = 5.892	vt0 = −1.2
vbi = 1.5	cgd0 = 4e − 15	cgs0 = 695.2f	is = 4e − 12
n = 1.2	t = 1e − 12	vbd = 12	kf = 1e − 9

A similar plot of simulated power gain *versus* input with no input noise is compared with the plot of simulated power gain with noise level maintained at −20 dBc and is shown in Figure 7. Just as in the case of MATLAB, the white-noise sequence was generated in *f* REEDA™ using the Ziggurat technique and is represented in the simulator as a white-noise-generating source that feeds the input of the first amplifier. pHEMTs in MMIC were modelled in *f*REEDA™ using the Curtice Cubic model [18] and the parameters for the polynomial coefficients for the model were obtained from Filtronic Corporation.‡‡ These parameters are listed in Table I. The transistor-level noise models have been developed and validated as presented in [16, 19].

To observe the degradative effect of noise on the purity of the output waveform, a snapshot of the transient output is shown in Figure 8 with noise level maintained at −20 dBc. This figure

‡‡ http://www.filtronic.co.uk.

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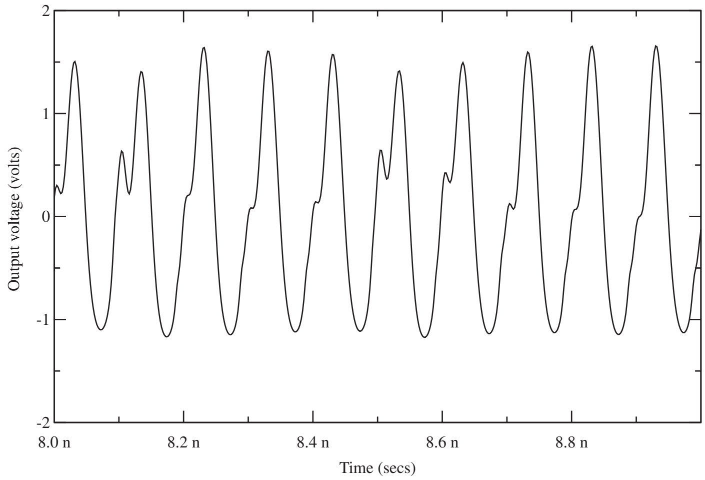

Figure 8. Degradation of the output sinusoid due to noise.

shows distortion in the amplitude and shape of the output sinusoid with the input power level set at 0 dBm. It is important to mention that this distortion is due to both saturation effects and noise interference.

Although the simulated gain curve in the high-noise level case does show some compression compared with the noiseless case, it can be seen that the amount of compression is not as much as in the measured results of Figure 5. The device models for the amplifiers have a limited ability to capture higher-order nonlinear effects and signal and noise interactions, which could explain the imperfect agreement between the gain and measurement and also between simulations of the noiseless case and the noisy case. It is important to be able to try and uncover the reason for this discrepancy. A summary of the procedures used is presented in the following section with the reader being warned that it results in an open ending and perhaps opens an avenue for further research.

5. FURTHER INVESTIGATIONS

As seen in the previous section, simulated curves of power gain *versus* input power do not show as much compression at high power levels in the presence of noise as do measured curves. This section presents a summary of the attempts made to find a resolution to this problem.

The first attempt focuses on improvements to the Curtice Cubic model. Since the release of the Curtice Cubic model, there have been several newer models for MESFET that have appeared in the literature that aim to have better large signal behavior in the saturation and cutoff regions and better model behavior in the knee regions of the large signal characteristics. One example is the Parker–Skellern (PS) model that takes a different approach compared with the Curtice Cubic model in the sense that it models drain current as a power-law function of effective gate and drain voltages [20]. The capacitance model used is the one proposed in [21]. The PS model was substituted for the Curtice Cubic model in the simulation. An example set of parameters for this model is provided in [20] and although the simulation does not exactly

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match the output characteristics of the circuit while using the Curtice model, the difference in the peak voltage output is less than 10%. With the new PS model, the results both with and without noise unfortunately do not indicate any changes when compared with simulations with the original Curtice model. In other words, the model does not seem to predict any noticeable reduction in gain when the circuit is fed with noise as compared with when the circuit is fed with sinusoidal input alone for higher levels of input power. In a qualitative sense, both the Curtice Cubic and the PS models show similar characteristics.

The *f*REEDA^{TM} framework makes it possible to handle time delays inside the model of an element. The Curtice Cubic model in *f*REEDA^{TM} has a parameter that allows the user to set a specific value of time delay in the simulation netlist. This time delay value, τ, makes the Curtice model compute the output current as a function of voltage that is τ time units in the past. Choosing the appropriate time delay depends on the channel transit time of the active devices in the circuit and the elements surrounding it. Although it is possible to obtain a fairly accurate estimate of the time delay [22], a first attempt at selecting an appropriate value was made with a trial-and-error approach. For values of time delay of less than 20 ps, there was no appreciable decrease in gain between the noiseless and noisy cases but there was a noticeable difference with delay set at 20 ps for both amplifiers. A plot showing the drop in gain with input power varying from 0 to 5 dBm with the delay set at 20 ps was compared with the measured gain characteristics in the presence of noise as shown in Figure 5 and the simulated gain characteristics in the presence of noise as shown in Figure 7. This plot is shown in Figure 9. As seen in this figure, the power gain dropoff obtained with a delay of 20 ps is quite sharp. At 0 dBm, it is comparable with the gain value of the simulated case with no delay but at 5 dBm it is comparable with the measured result. At powers lower than 0 dBm, simulations using this delay value produce results than can differ quite significantly from the measured results. This might lead one to suggest that the effects of delay mechanisms, if validated, are dependent on input power that is not unreasonable and it was originally reported by Curtice [18]. A snapshot of the output voltage taken with the input power set at 0 dBm shows considerably more distortion compared with the

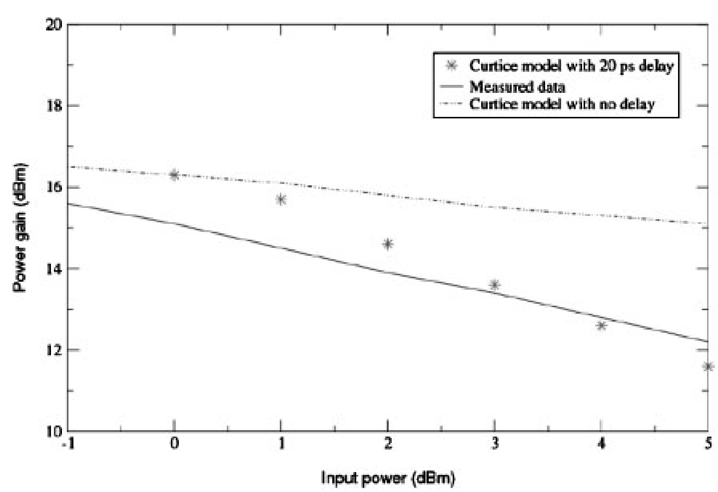

Figure 9. Comparing simulated gain obtained with a 20 ps delay and no delay with measured gain.

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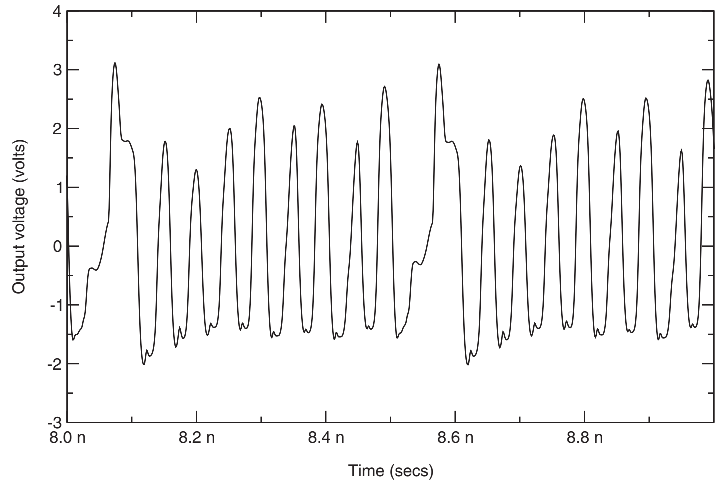

Figure 10. Distorted output voltage with a 20 ps delay.

output of Figure 8 and this is shown in Figure 10. It is known that power amplifiers can exhibit strong delays and this can have serious effects on performance, particularly at high input powers as measured in [23], quantified in [24] and modelled in [25]. Although the investigations in this section are rudimentary, they present an interesting avenue for future research in the understanding of the effects of interactions of noise with large input signals on the RF performance of amplifiers.

6. CONCLUSIONS

The major result of this work is the verification that effects of high levels of noise can be modelled in a transient circuit simulator. The requirement is to include deterministic and stochastic signals at each node in a circuit. Then at each node instead of performing the solution of an ODE, the solution of an SDE, discretized in the conventional manner, is required. As stochastic signals have very small variations relative to the expected deterministic signal levels, it is crucial that the transient simulator achieve high dynamic range.

The approach is verified by using nonlinear elements for the circuit devices in a high-dynamic-range simulator. Measured and simulated plots for the gain of the amplifier *versus* input power level are compared in the cases of no input noise and high-level noise. The results also demonstrate an increased level of compression in the presence of larger levels of noise and this effect has also been captured by simulation, albeit not to the same extent as measurement. Being able to model the deep compression region accurately would likely require more advanced nonlinear models for the amplifier with voltage-dependent values of time delay and a simulator that can handle an analysis with variable time delays. Regardless, these results demonstrate that the effects of having non-negligible levels of noise at the input of high-power amplifier circuits can be reproduced and modelled.

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REFERENCES

1. Luniya S, Steer MB, Christoffersen C. High dynamic range transient simulation of microwave circuits. *IEEE Radio and Wireless Conference*, Atlanta, GA, U.S.A., 2004; 487–490.
2. Christoffersen CE, Mughal UA, Steer MB. Object oriented microwave circuit simulation. *International Journal of RF and Microwave Computer-Aided Engineering* 2000; **10**(3):164–182.
3. Itô K. Stochastic integral. *Proceedings of the Imperial Academy*, *Tokyo* 1944; **20**(8):519–524.
4. Oksendal B. *Stochastic Differential Equations*. Springer: Berlin, 2003.
5. Stratonovich RL. A new representation for stochastic integrals and equations. *SIAM Journal on Control* 1996; **4**(2):362–371.
6. West BJ, Bulsara AR, Lindenberg K, Seshadri V, Shuler KE. Stochastic processes with non-additive fluctuations—I. Itô and Stratonovich calculus and the effects of correlations. *Physica A*: *Statistical Mechanics and its Applications* 1979; **97**(2):211–233.
7. Uhlenbeck GE, Ornstein LS. On the theory of the Brownian motion. *Physical Review* 1930; **36**(5): 823–841.
8. Kloeden PE, Platen E. *Numerical Solution of Stochastic Differential Equations*. Springer: Berlin, 1999.
9. Mehrotra A, Sangiovanni-Vincentelli AL. Noise analysis of non-autonomous radio frequency circuits. *IEEE International Conference on Computer-Aided Design* 1999; 55–60.
10. Keshner MS. $1/f$ noise. *Proceedings of the IEEE* 1982; **70**(3):212–218.
11. Bulsara AR, Lindenberg K, Seshadri V, Shuler KE, West BJ. Stochastic processes with non-additive fluctuations—II. Some applications of Itô and Stratonovich calculus. *Physica A*: *Statistical Mechanics and its Applications* 1979; **97**(2):234–243.
12. Smythe J, Moss F, McClintock VE, Clarkson D. Ito versus Stratonovich revisited. *Physics Letters A* 1983; **97**(3): 95–98.
13. Booch G. *Object-oriented Analysis and Design with Applications*. Addison-Wesley: Reading, MA, 1993.
14. Equational-based process modeling. http://ascend.cheme.cmu.edu/ftp/pdfPapersRptsSlides/processModeling.pdf [22 January 2008].
15. Steer MB, Christoffersen CE. Generalized circuit formulation for the transient simulation of circuits using wavelet, convolution and time-marching techniques. *Proceedings of the 15th European Conference on Circuit Theory and Design*, Helsinki, Finland, 2001; 205–208.
16. Kriplani NM. Modelling colored noise under large-signal conditions. *Ph.D. Thesis*, North Carolina State University, 2005.
17. Marsaglia G, Tsang WW. The Ziggurat method for generating random variables. *Journal of Statistical Software* 2000; **5**(8):1–7.
18. Curtice WR, Ettenberg M. A nonlinear GaAs FET model for use in the design of output circuits for power amplifiers. *IEEE Transactions on Microwave Theory and Techniques* 1985; **33**(12):1383–1394.
19. Kriplani NM, Victor A, Steer MB. Time-domain modelling of phase noise in an oscillator. *IEEE European Microwave Conference*, Manchester, England, 2006; 514–517.
20. Parker AE, Skellern DJ. A realistic large-signal MESFET model for SPICE. *IEEE Transactions on Microwave Theory and Techniques* 1997; **45**(9):1563–1571.
21. Statz H, Newman P, Smith IW, Pucel RA, Haus HA. GaAs FET device and circuit simulation in SPICE. *IEEE Transactions on Electron Devices* 1987; **34**(2):160–169.
22. Heiter GL. Characterization of nonlinearities in microwave devices and systems. *IEEE Transactions on Microwave Theory and Techniques* 1973; **21**(12):797–805.
23. Vuolevi JHK, Rahkonen T, Manninen JPA. Measurement technique for characterizing memory effects in RF power amplifiers. *IEEE Transactions on Microwave Theory and Techniques* 2001; **49**(8):1383–1389.
24. Ku H, McKinley HD, Kenney JS. Quantifying memory effects in RF power amplifiers. *IEEE Transactions on Microwave Theory and Techniques* 2002; **50**(12):2843–2849.
25. Ku H, Kenney JS. Behavioral modeling of nonlinear RF power amplifiers considering memory effects. *IEEE Transactions on Microwave Theory and Techniques* 2003; **51**(12):2495–2504.

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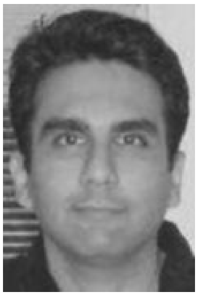
AUTHORS' BIOGRAPHIES



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