1. (10 points) Find $f$.

$$f''(s) = \pi^3, \quad f(0) = \frac{\pi}{2}, \quad f'(\pi) = \pi^4$$

2. (15 points) Use the definition of the integral, in terms of the infinite limit of sums, to evaluate the following integral.

$$\int_0^3 (2x^3 - 9x) \, dx$$

(Note these identities: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{i=1}^n i^3 = \left( \frac{n(n+1)}{2} \right)^2$.)
3. (15 points) Find where the function \( g(x) \) obtains its global maximum value if

\[
g(x) = \int_0^x \left( \frac{1}{\sqrt{t}} - 2\sqrt{t} \right) e^{-t} dt.
\]

4. (15 points) Find the anti-derivative and apply the Evaluation Theorem to evaluate the following integral.

\[
\int_0^1 t \left( 5 + \sqrt{t} \right) dt
\]
5. (15 points) Apply the Fundamental Theorem of Calculus in order to find the derivative of the following function.

\[ \int_{1}^{e^{x^2}} \ln(y) \, dy \]

6. (15 points) Apply the Substitution Rule in order to evaluate the following integral.

\[ \int_{0}^{\pi/4} \cos(2\theta) \, d\theta \]
7. (15 points) Apply integration by parts in order to evaluate the following integral.

\[ \int_{1}^{e} \ln(x) \, dx \]

8. **Bonus:** (15 points) If \( a \) and \( b \) are positive integers, show that the following identity is true by making the substitution \( u = 1 - x \).

\[ \int_{0}^{1} x^{a} (1 - x)^{b} \, dx = \int_{0}^{1} x^{b} (1 - x)^{a} \, dx \]