

Name: \_\_\_\_\_

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Underline the word circle if you are reading these directions. SHOW ALL YOUR WORK. WRITE NEATLY AND LARGE. CIRCLE FINAL ANSWER.

1. (10 points) Find  $f$ .

$$f''(u) = 2e^u - \cos(u), \quad f(0) = 1, \quad f'(0) = 1$$

2. (15 points) Use the definition of the integral, in terms of the infinite limit of sums, to evaluate the following integral.

$$\int_{-1}^2 (1 - 2x) dx$$

(Note these identities:  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ ,  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ , and  $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$ .)

3. (15 points) One can show that

$$\int_0^1 \frac{4}{1+x^2} dx = \pi.$$

What is the formula for the Riemann sum approximating this integral where the rectangles use right endpoints as sample points (i.e., what is  $R_n$ )? Estimate  $\pi$  by evaluating the formula with  $n = 2$  rectangles.

4. (15 points) Find the anti-derivative and apply the Evaluation Theorem to evaluate the following integral.

$$\int_1^e \left( \frac{1}{s^2} + \frac{1}{s} \right) ds$$

5. (15 points) Apply the Fundamental Theorem of Calculus in order to find the derivative of the following function.

$$\int_x^{10} s^3 ds$$

6. (15 points) Apply the Substitution Rule in order to evaluate the following integral.

$$\int_0^1 \frac{e^t}{e^t + 1} dt$$

7. (15 points) Apply integration by parts in order to evaluate the following integral.

$$\int_1^4 \frac{\ln(x)}{x^2} dx$$

8. **Bonus:** (15 points) If  $a$  and  $b$  are positive integers, show that the following identity is true by making the substitution  $u = 1 - x$ .

$$\int_0^1 x^a (1-x)^b dx = \int_0^1 x^b (1-x)^a dx$$