1. (10 points) Find $f$. 

$$f'(t) = \sqrt{t} \left( 3 - 7t^2 \right), \quad f(1) = 2$$

2. (15 points) Use the definition of the integral, in terms of the infinite limit of sums, to evaluate the following integral.

$$\int_0^3 x^2 \, dx$$

(Note these identities: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{i=1}^n i^3 = \left( \frac{n(n+1)}{2} \right)^2$.)
3. Determine whether the statement is true or false. If it is true, explain why. If it is false, give an example that disproves the statement.

(a) (5 points) If \( f \) and \( g \) are continuous on \([a, b]\), then
\[
\int_{a}^{b} [f(x)g(x)]dx = \left( \int_{a}^{b} f(x)dx \right) \left( \int_{a}^{b} g(x)dx \right).
\]

(b) (5 points) If \( f \) is continuous on \([a, b]\), then
\[
\int_{a}^{b} xf(x)dx = x \int_{a}^{b} f(x)dx.
\]

(c) (5 points) If \( f' \) is continuous on \([a, b]\), and \( F \) is an anti-derivative of \( f \), then
\[
\int_{a}^{b} f'(x)dx = F(b) - F(a).
\]

4. (15 points) Find the anti-derivative and apply the Evaluation Theorem to evaluate the following integral.
\[
\int_{0}^{1/5} 5^2dv
\]
5. (15 points) Apply the Fundamental Theorem of Calculus in order to find the derivative of the following function.

\[ \int_{-x}^{x} \cos^2(u)du \]

6. (15 points) Apply the Substitution Rule in order to evaluate the following integral.

\[ \int_{0}^{2} y^2 e^{-y^3} dy \]
7. (15 points) Apply integration by parts in order to evaluate the following integral.

\[
\int_{1}^{\infty} \frac{25}{4} x^{3/2} \ln(x) \, dx
\]

8. **Bonus:** (15 points) If \( a \) and \( b \) are positive integers, show that the following identity is true by making the substitution \( u = 1 - x \).

\[
\int_{0}^{1} x^a \ (1 - x)^b \, dx = \int_{0}^{1} x^b \ (1 - x)^a \, dx
\]