

Name: \_\_\_\_\_

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Underline the word circle if you are reading these directions. SHOW ALL YOUR WORK. WRITE NEATLY AND LARGE. CIRCLE FINAL ANSWER.

1. (10 points) Find  $f$ .

$$f'(t) = \sqrt{t}(3 - 7t^2), \quad f(1) = 2$$

2. (15 points) Use the definition of the integral, in terms of the infinite limit of sums, to evaluate the following integral.

$$\int_0^3 x^2 dx$$

(Note these identities:  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ ,  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ , and  $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$ .)

3. Determine whether the statement is true or false. If it is true, explain why. If it is false, give an example that disproves the statement.

(a) (5 points) If  $f$  and  $g$  are continuous on  $[a, b]$ , then

$$\int_a^b [f(x)g(x)]dx = \left( \int_a^b f(x)dx \right) \left( \int_a^b g(x)dx \right).$$

(b) (5 points) If  $f$  is continuous on  $[a, b]$ , then

$$\int_a^b xf(x)dx = x \int_a^b f(x)dx.$$

(c) (5 points) If  $f'$  is continuous on  $[a, b]$ , and  $F$  is an anti-derivative of  $f$ , then

$$\int_a^b f'(x)dx = F(b) - F(a).$$

4. (15 points) Find the anti-derivative and apply the Evaluation Theorem to evaluate the following integral.

$$\int_0^{1/5} 5^2 dv$$

5. (15 points) Apply the Fundamental Theorem of Calculus in order to find the derivative of the following function.

$$\int_{-x}^x \cos^2(u) du$$

6. (15 points) Apply the Substitution Rule in order to evaluate the following integral.

$$\int_0^2 y^2 e^{-y^3} dy$$

7. (15 points) Apply integration by parts in order to evaluate the following integral.

$$\int_1^{e^2} \frac{25}{4} x^{3/2} \ln(x) dx$$

8. **Bonus:** (15 points) If  $a$  and  $b$  are positive integers, show that the following identity is true by making the substitution  $u = 1 - x$ .

$$\int_0^1 x^a (1-x)^b dx = \int_0^1 x^b (1-x)^a dx$$