Plasticity

(Ch. 3 – sections : 3-1 to 3-6; 3-8)

Multiaxial Loading - Tresca and von-Mises yield criteria

- Plastic flow under uniaxial loading : **σ_o is the uniaxial yield strength**

3-1 to 3-3 : true σ vs ε : note – during plastic flow volume is conserved (Δ = 0) :

$$\Delta = \frac{\Delta V}{V} = \varepsilon_1+\varepsilon_2+\varepsilon_3 = \varepsilon_x+\varepsilon_y+\varepsilon_z = 0 \quad \text{(Eq. 3-5)}$$ ---note contrast with elasticity---

recall from Hooke's law : $\Delta = \varepsilon_x+\varepsilon_y+\varepsilon_z = \frac{1-2\upsilon}{E}(\sigma_x+\sigma_y+\sigma_z)$ so for Δ=0, ν=0.5 here.

----- constancy of volume implies $Al = A_o l_o$ so that $\varepsilon = \ln(\frac{l}{l_o}) = \ln(\frac{A_o}{A})$

3.4 yield criteria – multiaxial loading (σ_{ij}) :

(a) von Mises (distortion criteria)

define $\sigma_{eff} = \frac{1}{\sqrt{2}}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}^{1/2}$ (Eq. 3-12)

if $\sigma_{eff} = \sigma_o$, yield occurs – these are principal stresses, and in terms of all 6 components –

Eq. 3-13 : $\sigma_{eff} = \frac{1}{\sqrt{2}}\left\{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right\}^{1/2}$

--- *distortion energy, leading to change of shape, reaches a critical value*

(b) Tresca Criterion (maximum shear stress criterion)

recall Mohr's circle - $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = \frac{\sigma_{max}-\sigma_{min}}{2}$. so $\tau_{max}=\tau_o$ for yielding

Since τ = σ/2, define $\sigma_{eff} = \sigma_1 - \sigma_3 = \sigma_{max} - \sigma_{min}$; if = σ_o yielding takes place.

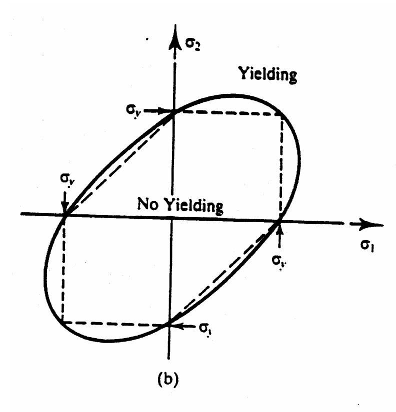
--- Tresca is more conservative than von-Mises ---

means predicts yielding even when von-Mises does not (Fig. 3-5)

Combined Stress States and Yield Locus (3-5 and 3-6)
- yield surface and normality (3-8) (Fig. 3-8)

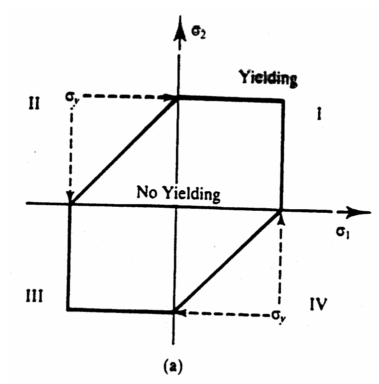
Yield Locus : Plane Stress Condition ---

Von-Mises – ellipse (Fig. 3-5) –
major and minor axes related to σ_o



note different loading conditions - yield occurs if the stress state is above the locus

Tresca – Prism – yield takes place when any stress = σ_o (Fig. 3-5)- note



(a) σ_o same in tension and compression

(b) τ_o for pure shear = $\sigma_o/2$

(c) Tresca locus is inside von-Mises implies Tresca is more <u>conservative</u>

Summary :

(a) experiments yield closer to von-Mises condition

(b) both von-Mises and Tresca – same at uniaxial and equi-biaxial loadings

(c) Hydrostatic stresses – no yielding

(d) Deviatoric stresses – yielding and plasticity

(e) Tresca more conservative – predicts yielding at lower stresses

(f) 2-D : yield locus; 3-D : yield surface (Fig. 3-7)

--- in both cases, σ_{eff}, σ_g, $\overline{\sigma}$ are used

--- recall $\sigma_{eff\ Tresca} = \tau_{max} = \sigma_{max} - \sigma_{min}$ and

$$\sigma_{eff\ von\text{-}Mises} = \frac{1}{\sqrt{2}}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}^{1/2}$$

Octahedral Stresses and Strains :

Check von-Mises for uniaxial --- $\sigma_{eff} = \sigma_o$ ---

In-Class Work:

Given the stress state σ_{ij} (MPa) = $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.

1. Find the principal stresses, σ_1, σ_2 and σ_3 (none of them are zero).

2. Show that the *trace* is invariant.

(Try at home)

(a) Evaluate σ'_{ij} for a 60^{o} CCW rotation around z-axis, and check the answer by finding the trace.

(b) Evaluate the hydrostatic and deviatoric components of the stress tensor (check your answers).

Given the uniaxial yield stress is 5.5 MPa,

(a) Does Tresca criterion predict yielding?

(b) Does von-Mises criterion predict yielding?