1 (25 pts): Integrate the following functions (5 pts per problem).

1. \[\int (6x^3 - 2x^2 - 4x + 1) \, dx\]

**Solution:** Use the power rule \(\int x^n \, dx = 1/(n+1)x^{n+1} + C\) to get

\[6 \cdot 4x^4 - 2 \cdot 3x^3 - 4 \cdot 2x^2 + x + C = 24x^4 - 6x^3 - 8x + x + C\]

2. \[\int \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right) \, dx\]

**Solution:** Rewrite the function and use the power rule \(\int x^n \, dx = 1/(n+1)x^{n+1} + C\) to get

\[\int \left(x^{-1/2} + x^{1/2}\right) \, dx = \frac{1}{1 - 1/2}x^{1/2} + \frac{1}{1 + 1/2}x^{3/2} + C\]

\[= 2x^{1/2} + \frac{2}{3}x^{3/2} + C\]

3. Use the exponential rule \(\int e^{kx} \, dx = 1/k \cdot e^{kx} + C\) to get

\[\int_0^1 (3e^{3x} - e^{-x}) \, dx\]

**Solution:**

\[\left[3 \cdot 1/3 \cdot e^{3x} - (-e^{-x})\right]_0^1 = \left[e^{3x} + e^{-x}\right]_0^1 = e^3 + e^{-1} - 2\]

4. \[\int_0^\pi (\sin(2x) - \cos(3x)) \, dx\]

**Solution:** Use \(\int \sin(x) \, dx = -\cos(x) + C\) and \(\int \cos(x) \, dx = \sin(x) + C\)

\[\left[-\frac{1}{2} \cos(2x) - \frac{1}{3} \sin(3x)\right]_0^\pi = -\frac{1}{2} - \frac{1}{2} = 0\]

5. \[\int \left(\frac{1}{2x} + \frac{1}{3x^2}\right) \, dx\]

**Solution:** Rewrite the function and use the power rule \(\int x^n \, dx = 1/(n+1)x^{n+1} + C\) and that \(\int 1/x \, dx = \ln|x| + C\) to get

\[\int \left(\frac{1}{2} \cdot x^{-1} + \frac{1}{3} \cdot x^{-2}\right) \, dx = 1/2 \cdot \ln|x| + 1/3 \cdot (-1) \cdot x^{-1} + C\]

\[= 1/2 \cdot \ln|x| - 1/3 \cdot x^{-1} + C\]
2 (20 pts) Use substitution to evaluate the integrals

1. \[ \int (x^2 - x + 3)^3(4x - 2) \, dx \]

**Solution:** Let \( u = x^2 - x + 3 \) giving \( \frac{du}{dx} = 2x - 1 \) \( \Leftrightarrow dx = 1/(2x - 1)du \) the rewrite

\[
\int u^32(2x-1)\frac{1}{2x-1} \, du = \int 2u^3 \, du = \frac{1}{2} \cdot \frac{1}{4}u^4 + C = \frac{1}{2} \cdot (x^2 - x + 3)^2 + C
\]

2. \[ \int_0^{\sqrt{\pi}} x \cos(x^2) \, dx \]

**Solution:** Let \( u = x^2 \) giving \( \frac{du}{dx} = 2x \) \( \Leftrightarrow dx = 1/(2x)du \), note \( x = 0 \Rightarrow u = 0 \) and \( x = \sqrt{\pi} \Rightarrow u = \pi \) the rewrite

\[
\int_0^{\pi} x \cos(u) \frac{1}{2x} \, du = \frac{1}{2} \int_0^{\pi} \cos(u) \, du = \frac{1}{2} \cdot [\sin(u)]_0^\pi = 0
\]

3 (20 pts): Let \( f_1(x) = x^3 - 4x \) and \( f_2(x) = x^3/3 - x \).

1. (5 pts) Find maxima and minima for each of the two functions, \(-3 \leq x \leq 3\). Find both \( x \) values and corresponding function values.

**Solution:** Find local min/max and check end-points. First, we find local min/max by solving \( f'(x) = 0 \)

\[
f'_1(x) = 3x^2 - 4 = 0 \Leftrightarrow x^2 = \frac{4}{3} \Leftrightarrow x = \pm \frac{2}{\sqrt{3}}
\]

\[
f'_2(x) = 3x^2/3 - 1 = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1
\]

The 4 points for the two functions include

\[
\begin{align*}
f_1(-3) &= -15 \text{ abs min} & f_2(-3) &= -6 \text{ abs min} \\
f_1(3) &= 15 \text{ abs max} & f_2(3) &= 6 \text{ abs max} \\
f_1\left(-\frac{2}{\sqrt{3}}\right) &= 3.0792 & f_2(-1) &= \frac{2}{3} \\
f_1\left(\frac{2}{\sqrt{3}}\right) &= -3.0792 & f_2(1) &= -\frac{1}{3}
\end{align*}
\]

2. (5 pts) Find points of intersection for the two functions.

**Solution:** Solve \( f_1(x) = f_2(x) \)

\[
x^3 - 4x = \frac{1}{3}x^3 - x \Leftrightarrow \frac{2}{3}x^3 - 3x = 0 \Leftrightarrow x \left(\frac{2}{3}x^2 - 3\right) = 0 \Leftrightarrow x = 0 \text{ or } x = \pm \frac{3}{\sqrt{2}}
\]
3. (5 pts) Use the values found above to sketch the two functions.

4. (5 pts) Calculate the area between the two functions.

\[
A = \int_{-\sqrt{2}}^{0} (f_1 - f_2) \, dx + \int_{0}^{\sqrt{2}} (f_2 - f_1) \, dx \\
= \int_{-\sqrt{2}}^{0} x^3 - 4x - \left(\frac{x^3}{3} - x\right) \, dx + \int_{0}^{\sqrt{2}} \frac{x^3}{3} - x - \left(\frac{x^3}{3} - 4x\right) \, dx \\
= \int_{-\sqrt{2}}^{0} \frac{2}{3} x^3 - 3x \, dx + \int_{0}^{\sqrt{2}} \frac{2}{3} x^3 + 3x \, dx \\
= \left[\frac{1}{6} x^4 - \frac{3}{2} x^2\right]_{-\sqrt{2}}^{0} + \left[-\frac{1}{6} x^4 + \frac{3}{2} x^2\right]_{0}^{\sqrt{2}} = 3.375 + 3.375 = 6.75
\]

4 (20 pts): Use integration by parts to rewrite integrals as

1. (10 pts)

\[
\int_{1}^{2} x^4 \ln(x^2) \, dx
\]

**Solution:** Let \( f = \ln(x^2) \Leftrightarrow f' = \frac{1}{x^2} 2x = \frac{2}{x} \) and \( g = x^4 \Leftrightarrow G = \frac{x^5}{5} \). Use integration by parts to get

\[
\int_{1}^{2} f g \, dx = [f G]_{1}^{b} - \int_{1}^{b} f' G \, dx \\
\left[\frac{x^5}{5} \ln(x^2)\right]_{1}^{2} - \int_{1}^{2} \frac{2}{x} x^5 \, dx = \left[\frac{x^5}{5} \ln(x^2)\right]_{1}^{2} - \frac{2}{5} \int_{1}^{2} x^4 \, dx \\
= \left[\frac{x^5}{5} \ln(x^2)\right]_{1}^{2} - \frac{2}{5} \left[\frac{x^5}{5}\right]_{1}^{2} \\
= 8.87 - (2.56 - 0.08) = 6.39
\]

5 (20 pts): A new animal protection program that has been in effect for 25 years has led to an increase in the population of bears. At the start of the program the projected bear enclosure had 10 bears, and
during this period the population was $P(t) = P_0e^{0.002t}$. To judge the effectiveness of the program the world's nature organization wanted to know the total population today (10 pts) as well as the average population over the timespan of the program (10 pts).

**Solution:** The population today can be found by evaluating $P(t)$ at $t = 25$ using $P_0 = 10$

$$P(10) = 10e^{0.002 \cdot 25} = 10.5127$$

The average population is given by

$$\frac{1}{25 - 0} \int_0^2 510e^{0.002t} \, dt = \frac{10}{25} \left[ \frac{1}{0.002} e^{0.002t} \right]_0^5 = 210.2542 - 200 = 10.2542$$