1 (30 pts) A company makes cans for soda with a volume of $175\pi$ cm$^3$. To cut costs, the company wants to minimize the amount of material needed to make the can. In other words, the objective is to minimize the surface area of the can.

A (5 pts) Sketch the can.
B (5 pts) Find the objective function.
C (5 pts) Find the constraint function.
D (5 pts) Simplify and minimize the objective function.
E (5 pts) Check that the point found is a minimum.
F (5 pts) Find all dimensions (height and radius) of the can.

2 (25 pts) The size of a butterfly population $P(t) = P_0e^{kt}$, where the population is 500 at time 0. After 10 days the population size has grown to 750.

A (5 pts) What is $P_0$?
B (5 pts) What is $k$?
C (5 pts) Write the equation for $P(t)$ using the values for $P_0$ and $k$ predicted above.
D (5 pts) Find the doubling time for the population?
E (5 pts) What is the population after 30 days?

3 (25 pts): Integrate the following functions

A (5 pts)
$$\int (x^4 - 6x^2 + 2x + 1) \, dx$$

B (5 pts)
$$\int \left( \frac{1}{\sqrt{x}} + \sqrt{x} \right) \, dx$$

C (5 pts)
$$\int (3e^{3x} - e^{-x}) \, dx$$

D (5 pts)
$$\int_0^{\pi} (\cos(4x) + \sin(2x)) \, dx$$

E (5 pts)
$$\int \left( \frac{1}{x + 3} + (3x + 4)^2 \right) \, dx$$
4 (25 pts): Let \( f(x) = \sqrt{x} \), the objective is to integrate this function for \( 2 \leq x \leq 4 \) using Riemann sums and average values.

A (5 pts) Assume that 4 intervals is needed. Draw the intervals and calculate \( \Delta x \).
B (5 pts) Calculate the Riemann sum using the midpoint rule.
C (5 pts) Compare your result with direct integration.
D (5 pts) For the same function calculate the average value over the same interval.
E (5 pts) Prove the formula for the average value using the fundamental theorem of calculus.

5 (25 pts): Let \( f_1(x) = x^2 + 2 \) and \( f_2(x) = 6 \)

A (5 pts) Sketch the functions \( f_1 \) and \( f_2 \).
B (5 pts) How many sub-areas are spanned for \( -3 \leq x \leq 3 \).
C (5 pts) Calculate the area between the functions \( f_1 \) and \( f_2 \) for \( -3 \leq x \leq 3 \).
D (5 pts) For \( f_1(x) \) calculate the solid of revolution for \( 0 \leq x \leq 1 \).
E (5 pts) Sketch the solid of revolution.