Compartments
Cardiovascular Compartments
Cardiovascular compartment equation

\[ V - V_{\text{unstressed}} = C(p - p_{\text{atm}}) \implies \frac{dV}{dt} = C \frac{dp}{dt} \]

\[ \frac{dp}{dt} = \frac{(q_{\text{in}} - q_{\text{out}})}{C} \]

\[ \frac{dp}{dt} = \left( \frac{p_{\text{in}} - p}{R_{\text{in}}} - \frac{p - p_{\text{out}}}{R_{\text{out}}} \right) / C \]
Compartments
Left and Right Ventricles

Cervical Ventricle (Cvc)

Ventricles:
- Right Ventricle
- Left Ventricle

Lungs:
- PA, gas

Ventricular Pressures:
- pD1, g
- pD2, g
- pD3, g

Systemic Tissue:
- cB, g

Cerebral Tissue:
- cB, g

Head:
- Head
- Vp, Vp

Thorax:
- Thorax
- Vp, Vp

Body:
- Body
- Vp, Vp

Other Components:
- Rv, Vrv
- Vap, Cap
- Rp, Vp
- Vvs, Cvs
- Vrs, Vrv
- Vvs, Cvs
- Vrs, Vrv
- Vrs, Vrv
- Vrs, Vrv
- Vrs, Vrv

Quantities:
- qv, qvc
- qc, qp
- Rs, qS
- VD, pD1, g, pD2, g, pD3, g
- pexp, CO2
- pac, pvp, pac
Compartments
Left and Right Ventricles

Right Ventricle
- $cv,g$
- $qv$
- $Rpv$
- $Erv$
- $Vrv$

Left Ventricle
- $qvp$
- $Rmv$
- $ca,g$
- $plv$
- $qa$
- $Rav$
- $Elv$
- $Vlv$
Model
Heart Equation

\[ E(t) = \begin{cases} 
  E_d + \frac{1}{2}(E_s - E_d)(1 - \cos\left(\frac{\pi}{T_s} t\right)) & 0 \leq t \leq T_s \\
  E_d + \frac{1}{2}(E_s - E_d)(\cos\left(\frac{\pi}{T_s}(t - T_s)\right) + 1) & T_s \leq t < T_s + T_r \\
  E_d & T_s + T_r \leq t \leq T 
\end{cases} \]

\[ p_{lv} = E(t)(V_{lv} - 10) \]
Respiratory Compartments

- Respiratory compartments represent
  - Gas exchange from breathing

- Gases metabolized in tissue
Compartments
Tissue Gas Exchange

Rc  qc  MB,g
Cerebral Tissue  cB,g
MB,g

Rs  qs  MS,g
Systemic Tissue  cS,g
MS,g
Tissue Gas equations

\[ V \frac{dc}{dt} = R + q(c_{in} - c_{out}) \]

- \( V \), volume of tissue
- \( c_{in}, c_{out} \), concentration of gas entering/leaving
- \( q \), blood flow through respiratory compartment
- \( R \), metabolitic rate
Compartments

Lungs

- \( p_{A,gas} \)
- \( V_D \)
- \( p_{D1,g} \)
- \( p_{D2,g} \)
- \( p_{D3,g} \)
- \( p_{L,g} \)
- \( p_{exp,CO2} \)

\( R_p \)
Breathing

When breathing, air flows through 3 “anatomical dead space" compartments

\[ V_D \frac{dp_{D,g}}{dt} = \frac{dV_A}{dt} (p_{in,g} - p_{out,g}) \]

- \( V_D \), volume of dead space compartment \( D \)
- \( p_{D,g} \), partial pressure of gas \( g \) in compartment \( D \)
- \( p_{in,g}, p_{out,g} \), partial pressure of gas entering/leaving compartment \( D \)
- \( \frac{dV_A}{dt} \), airflow rate
Concentrations change in two ways

- Inhalation
- Transport via blood flow

\[ V_A \frac{dp_{A,g}}{dt} = \frac{dV_A}{dt} (p_{D,g} - p_{A,g}) + 1.21 \cdot 713q_p (c_{v,g} - c_{A,g}) \]
Air flows in opposite direction, so dead space compartments are reversed
Only one term for lung equation

\[ V_A \frac{dp_{A,g}}{dt} = 1.21 \cdot 713q_p(c_v,g - c_{A,g}) \]

Switch from inhalation to exhalation based on data
Dissociation Constants

- Dissociation laws convert alveolar partial pressures to blood gas concentrations
- \( \text{O}_2 \) and \( \text{CO}_2 \) are absorbed into the blood stream differently
- \( \text{CO}_2 \) dissociation is approximately linear
  \[
  c_{\text{CO}_2} = K_{\text{CO}_2} p_{\text{CO}_2} + k_{\text{CO}_2}
  \]
- \( \text{O}_2 \) dissociation is approximately
  \[
  c_{\text{O}_2} = k_{\text{O}_2} \left( 1 - e^{-K_{\text{O}_2} p_{\text{O}_2}} \right)^2
  \]
- \( K_{\text{CO}_2}, k_{\text{CO}_2}, K_{\text{O}_2}, \) and \( k_{\text{O}_2} \) are dissociation parameters
Decoupled System

- A respiratory cycle takes 4 to 5 seconds
- A cardiac cycle is approximately 1 second
- Solving the coupled cardio-respiratory ODE system takes hours
- Multiple ODE solutions are necessary for a single optimization step
- The cardiovascular states do not depend on the respiratory states
- The respiratory states only need the blood flow from the cardiovascular states
Subset Selection

- Relative sensitivities used for first cut
# Cardiovascular Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Optimized Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>4.27</td>
<td>2.54 [mmHg·sec/ml]</td>
</tr>
<tr>
<td>$R_s$</td>
<td>1.08</td>
<td>1.28 [mmHg·sec/ml]</td>
</tr>
<tr>
<td>$C_a$</td>
<td>2.40</td>
<td>0.536 [ml/mmHg]</td>
</tr>
<tr>
<td>$C_{ac}$</td>
<td>0.358</td>
<td>2.96 [ml/mmHg]</td>
</tr>
<tr>
<td>$T_{s,frac}$</td>
<td>0.350</td>
<td>0.139</td>
</tr>
<tr>
<td>$E_{s,l}$</td>
<td>0.848</td>
<td>0.721 [mmHg/ml]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value [ml/sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_s$</td>
<td>Mean systemic flow</td>
<td>55.0</td>
</tr>
<tr>
<td>$q_c$</td>
<td>Mean cerebral flow</td>
<td>26.9</td>
</tr>
<tr>
<td>$q_p$</td>
<td>Mean pulmonary flow</td>
<td>82.0</td>
</tr>
<tr>
<td>Parameter</td>
<td>Initial Value</td>
<td>Optimized Value</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$K_{CO_2}$</td>
<td>$2.28 \times 10^{-3}$</td>
<td>$6.88 \times 10^{-3}$ [mmHg]</td>
</tr>
<tr>
<td>$k_{CO_2}$</td>
<td>$9.76 \times 10^{-1}$</td>
<td>$2.61 \times 10^{-1}$ [mmHg]</td>
</tr>
<tr>
<td>$M_{S,CO_2}$</td>
<td>1.11</td>
<td>4.20 [mmHg]</td>
</tr>
<tr>
<td>$V_D$</td>
<td>67.8</td>
<td>151 [mL]</td>
</tr>
</tbody>
</table>