

# Optimal Max-min Fair Resource Allocation in Multihop Relay-enhanced WiMAX Networks

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**Abstract**—In this paper, an optimal resource allocation scheme is proposed for WiMAX networks enhanced with non-transparent relay stations (RSs). We first focus on two-hop relaying networks, since this scenario has the largest throughput gain. Furthermore, many researchers have proposed different resource allocation schemes in two-hop relaying networks, and, thus, we can compare the performance of our proposed scheme with two well-known schemes, namely, *orthogonal* and *overlapped* in terms of cell throughput, outage rate, and computational time. We then explore an extension of the proposed scheme to a general multihop relaying scenario by taking into account spatial reuse in both the access and relay zone periods. The numerical results show that the highest cell throughput can be achieved by the proposed optimal scheme while maintaining the fairness and lower outage performance of the orthogonal scheme at a higher computational cost. Moreover, the cell throughput degradation from increasing the number of hops from two to three hops, under proposed optimal scheduling scheme, is only 12% and 19% when six RSs and nine RSs are deployed respectively.

## I. INTRODUCTION

In the last decade, multihop wireless networks emerged from academic and niche (mostly military) applications into the mainstream wireless landscape. WiFi wireless mesh networks have been proven to be viable alternatives to the traditional WiFi deployments, especially in outdoor environments. The WiMAX specification was recently amended [1] to include a multihop relays. The LTE specifications are currently undergoing a similar extension [2]. For all wireless technologies multi-hop relay stations (RSs) enable a tradeoff between the performance and cost of providing wireless coverage, in many cases reducing the capital expenditure costs of new networks when compared with the versions without relays.

The WiMAX multihop relay amendment allows for two different types of RSs; *transparent* relays are added to existing WiMAX cells in order to boost the throughput of the cell (by increasing the signal-to-noise (SNR) ratio), however do not allow for coverage extensions. In contrast, *non-transparent* relays can be used to extend the range of the cells as well as increase the throughput in these extended cells. In this paper we focus on non-transparent RSs that transmit on the same frequency as the base station (BS). Both types of relays can serve unmodified subscriber stations (SSs) (i.e., the SSs do not distinguish between genuine BSs and RSs).

While multihop relays can considerably reduce the deployment costs of WiMAX networks as well as increase their

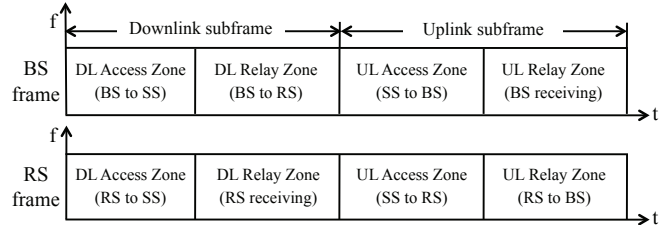


Fig. 1. Non-transparent mode frame structure.

performance, they also significantly complicate the scheduling of the resources in the network.

WiMAX uses schedules to allocate slots in both uplink (UL) and downlink (DL) directions by using separate UL and DL subframes. As shown in Fig. 1, within one of these subframes, the 802.16j amendment allows for an *access zone* and a *relay zone*. During the access zone, the SSs communicate directly with the BS and the RSs. During the relay zone, the RSs communicate with the BS. Given a setup with a BS, several RSs and a few SSs, each transmission in each of the zones has to be planned (the schedules can be different for different frames). While (as usual) the standard allows for great flexibility in accommodating a schedule, (also as usual) the standard does not offer any procedures for determining those schedules. One of the drawbacks of using RSs is a considerably more complicated scheduling of transmissions. Due to interference, each transmission potentially reduces the SNR to all other stations in the cell (and thus reducing the achievable data rate), therefore the scheduling algorithm has to take into account a large number of interactions in the cell.

Two existing scheduling approaches simplify the problem by considering two extreme solutions to the problem. On one hand, *orthogonal* allocation [3] assume that only one RS or the BS transmit at any one time (thus eliminating interference from multiple service nodes, but disallowing frequency reuse). On the other hand, *overlapped* schedules [3] assume that all service nodes transmit at the same time (thus maximizing frequency reuse, but increasing the interference to each node).

In this paper we propose an optimal scheduling scheme that maximizes throughput for all users while preserving fairness in the cell. We consider the cases of two and three relaying hops. We compare our optimal scheme with the existing (orthogonal and overlapped) schemes and show that the optimal scheme achieves a considerably higher throughput than either of the existing approaches while preserving the low

outage probability of the orthogonal scheme.

The rest of this paper is organized as follows. In the next section, we discuss related work. In Section III, we present the system model including SINR analysis. In Section IV, we present three resource allocation schemes, orthogonal, overlapped, and optimal schemes in two-hop relaying networks. The extended multihop relaying optimal scheme is presented in Section V. Numerical results and analysis are shown in Section VI. Finally, Section VII concludes the paper.

## II. RELATED WORK

A number of papers have quantified the benefits of using relays in multihop WiMAX systems. The work by Hoymann *et al.* [4] shows an analytical approach for dimensioning cellular multihop WiMAX networks and analyzes the network capacity by placing RSs at the border of the BS's transmission range for coverage extension. Velez *et al.* [5] show that deployment with RSs can be less expensive than using the BS alone by analyzing the impact of using RSs on costs and revenues. The work by Ge *et al.* [6] also studies the selection of optimal position for RSs, which can increase the expected end-to-end capacity for individual SSs. However, the scheduling problem was not considered in [4]–[6].

Many researchers have recently proposed scheduling schemes for OFDMA based WiMAX networks. The simplest scheduling scheme is fixed assignment scheduling [7], in which a BS allocates a fixed amount of bandwidth for every RSs; in this simple scheduling scheme the system throughput is significantly degraded due to inefficient resource utilization. To enhance network performance, most recent scheduling schemes allocate resources in a dynamic manner. Sayenko *et al.* [8] present a round-robin based scheduling solution for the 802.16 BS that ensures the QoS requirements of SSs in the uplink and downlink directions. Work by Deb *et al.* [9] studies the proportional fair scheduling problem, while taking into account frequency selectivity and multiuser diversity. However, the previously mentioned scheduling schemes restrict the transmission opportunity to one node at a time and, hence they do not optimally utilize the capacity of the network.

In order to take frequency reuse into account for the non-transparent RS mode, Park *et al.* [3] present two scheduling schemes named *orthogonal* and *overlapped* and compare the performance of the two schemes. Chang *et al.* [10] also propose an efficient scheduling algorithm allowing spatial reuse and reducing the overall transmission time. The work by Yang *et al.* [11] introduces an RS grouping algorithm to overcome frequent handoffs and low spectrum utilization problems. However, load balancing and dynamic boundary selection between access and relay zone problems were not considered in [3], [10], [11]. Due to the potential for frequency reuse in non-transparent RS mode, it is more challenging to explore how well-known fairness schemes can be used in 802.16j systems. Fairness has been extensively studied in various wireless network areas [12]–[14]. Max-min fairness is the most common fairness definition in both wired and wireless communication networks. In our previous paper [15],

TABLE I  
SIMULATION PARAMETERS

System Parameters	
Operating Frequency	3.5 GHz
Duplex	TDD
Channel Bandwidth	10 MHz
BS/RS Height	50 m
SS Height	1.5 m
BS/RS Antenna Gain	17 dBi
SS Antenna Gain	0 dBi
BS/RS Power	20 W
SS Power	200 mW
BS/RS Noise Figure	3 dB
SS Noise Figure	7 dB
Reuse Factor	7
Sectorization Factor	1
Terrain Type	A
OFDMA Parameters	
FFT Size	1024
Sub-carrier Frequency Spacing	10.94 kHz
Useful Symbol Time	91.4 $\mu$ s
Guard Time	11.4 $\mu$ s
OFDM Symbol Duration	102.9 $\mu$ s
Data Sub-carriers(DL / UL)	720 / 560
Pilot Sub-carriers(DL / UL)	120 / 280
Null Sub-carriers(DL / UL)	184 / 184
Sub-channels(DL / UL)	30 / 35

we show how well-known fairness schemes such as max-min and proportional fairness can be implemented in 802.16j systems. We also propose an optimal scheduling scheme to maximize cell throughput under a max-min fairness constraint in two-hop relaying networks [16].

In this paper, we extend the proposed optimal scheme to a general multihop relaying scenario by exploiting frequency reuse in both access and relay zone periods. Thus, the path selection and the boundary between the access and relay zone intervals will be determined optimally based on the traffic load. Although we consider only the downlink analysis, our scheme can be easily extended to include the uplink.

## III. SYSTEM MODEL

We consider a WiMAX network enhanced with non-transparent RSs and each cell consists of a BS, RSs, and SSs. We assume that every node has a single omni-directional antenna, hence, no terminal can transmit and receive simultaneously. The cell radius, 1200m, is determined by the condition that the cell coverage probability under Rayleigh fading channel is greater than 90% [17], [18]. The parameters used for the analysis are listed in Table I.

The Erceg-Greenstein model [17] is used to model path loss (this is also the model recommended by the IEEE 802.16 working group). The Erceg-Greenstein model has three variants, depending upon terrain type, namely A, B, and C. Type A has the highest path loss and is applicable to hilly terrains with moderate to heavy tree densities. Type C has the lowest path loss and applies to flat terrains with light tree densities. Type B is suitable for intermediate terrains. We assume terrain type A for the general scenario in this work, but the impact of terrain type on the cell throughput is also explored in the numerical result section.

For a given the transmission power  $P_t$ , the received signal power  $P_r$  is given by  $P_r = \frac{G_t G_r P_t}{L}$ , where  $G_t$ ,  $G_r$ , and  $L$

TABLE II  
SINR THRESHOLD SET

MCS	Downlink Data Rate $d_m$ [Mbps]	Spectral Efficiency $E_m$ [bps/Hz]	SINR Threshold $\bar{\gamma}_m$ [dB]
QPSK 1/2	5.25	1.0	9.1
QPSK 3/4	7.87	1.5	11.73
16 QAM 1/2	10.49	2.0	13.87
16 QAM 3/4	15.74	3.0	17.55
64 QAM 2/3	20.99	4.0	20.86
64 QAM 3/4	23.61	4.5	22.45
64 QAM 5/6	26.23	5.0	24.02

represent the transmitting antenna gain, receiving antenna gain, and the path loss of the channel respectively. The received signal to interference and noise ratio (SINR) is determined by  $SINR = \frac{P_r}{P_N + \beta P_I}$ , where  $\beta$  is the number of co-channel cells of the first tier,  $P_N$  is the noise power, and  $P_I$  is the interference signal power from a neighboring cell on the same frequency as the current cell.

Depending on the link quality, a variety of modulation and coding schemes (MCS) are supported in WiMAX networks. Table II shows the achievable data rates denoted as  $d_1, d_2, \dots, d_7$  and the corresponding MCS; the last column represents the minimum required threshold values of signal to interference and noise ratio (SINR),  $\bar{\gamma}_m$ , computed by bit error rate expression for M-QAM [19] when bit error rate is  $10^{-6}$ . This approximation provides a closed-form expression for the link spectral efficiency of M-QAM as a function of the SINR and bit error rate:

$$E_m = \log_2 \left( 1 + \frac{1.5\gamma}{-\ln 5P_b} \right), \quad (1)$$

where  $E_m$  is the spectral efficiency,  $\gamma$  is SINR, and  $P_b$  is the bit error rate. With the assumption of a Rayleigh fading channel, the received SINR,  $\gamma$ , is an exponential random variable [20]. Therefore, the probability that a transmitter can achieve data rate  $d_m$  can be expressed as:

$$p(d_m) = \int_{\bar{\gamma}_m}^{\bar{\gamma}_m+1} \frac{1}{\gamma^*} \exp\left(-\frac{\gamma}{\gamma^*}\right) d\gamma, \quad (2)$$

where  $\gamma^*$  is the average SINR. Consequently, the average achievable data rate,  $d_s$ , can be computed by:

$$d_s = \sum_{m=1}^7 d_m \cdot p(d_m). \quad (3)$$

The relay data rate (BS-RS-SS) is influenced by the link capacities of both hops involved. In order to utilize channel bandwidth efficiently, i.e., avoid wasting resource and overflowing data, the incoming and outgoing data at the relays should be equal:

$$d_{BS-RS} \cdot t_A = d_{RS-SS} \cdot t_B, \quad (4)$$

where  $d_{BS-RS}$  and  $d_{RS-SS}$  are the capacities of BS to RS and RS to SS links respectively when each link is given the whole bandwidth, and  $t_A$  and  $t_B$  are the durations of BS to RS and RS to SS link allocations respectively. The average

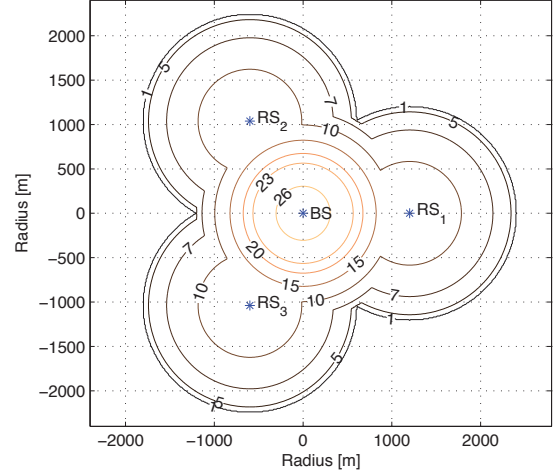


Fig. 2. A coverage extension scenario with three RSs and the achievable data rates in the cell.

data rate of an SS using a relay is equal to the amount of data received divided by the time required to receive it:

$$d_{BS-SS} = \frac{d_{BS-RS} \cdot t_A}{t_A + t_B}, \quad (5)$$

as the RS cannot receive from the BS while transmitting to the SS. Using (4), (5) can be rewritten as:

$$\frac{1}{d_{BS-SS}} = \frac{1}{d_{BS-RS}} + \frac{1}{d_{RS-SS}}. \quad (6)$$

Figure 2 shows a coverage extension scenario with three RSs: the BS is located at the center of the cell and three RSs are deployed at the edge of the BS's transmission range to extend the cell coverage. Thus, the SSs can receive data either directly from the BS or through one of the RSs according to link capacity and scheduling scheme. All RSs and the BS are referred to as *service nodes* in the rest of this paper, the one-hop links between service nodes are referred to as *relay links* (BS to RS and RS to RS), and links between one service node and its associated SSs as *access links* (BS to SS and RS to SS). Each contour line in Fig. 2 represents the achievable average data rate of an SS according to its location inside a cell. Usually, the SSs located near the BS can achieve higher data rate compared to the SSs located far away from the BS. The maximum data rate 26.23 Mbps can be achieved through a direct link from the BS, whereas the achievable maximum data rate of an SS via an RS is 13.12Mbps (from (6)). However, even for SSs located near an RS, the average achievable data rate is only 10.67Mbps since the average achievable data rate of an RS from the BS is 17.97Mbps. The data rate of an SS varies according to the current SINR value as fading channels are assumed. We assume that all channels have flat fading over one frame interval such that the channel gains remain fixed over a frame interval but change independently from one frame interval to the next.

#### IV. SCHEDULING SCHEMES IN TWO-HOP SCENARIO

In this section we present three scheduling schemes:

TABLE III  
NOTATIONS USED

Notation	Description
$\mathcal{R}$	Set of RSs
$\mathcal{R}^+$	Set of service nodes (BS and RSs)
$\mathcal{U}$	Set of possible transmission subsets of service nodes
$\mathcal{V}$	Set of possible transmission subsets of relay links
$\mathcal{L}$	Set of possible relay links ( $l_{ij} \in \mathcal{L}$ )
$l_{ij}$	Relay link between service node $i$ and $j$ ( $i \in \mathcal{R}^+, j \in \mathcal{R}$ )
$\mathcal{S}$	Set of SSs
$\mathcal{S}_b$	Set of SSs associated with BS
$\mathcal{S}_R$	Set of SSs associated with RSs
$\mathcal{S}_r^u$	Set of SSs associated with RS $r \in \mathcal{R}$ in the subset $u$
$\mathcal{S}_{r^+}^u$	Set of SSs associated with $r^+ \in \mathcal{R}^+$ in the subset $u$
$d_s^u$	Achievable data rate of SS $s \in \mathcal{S}$ in the subset $u$
$d_r$	Achievable data rate of RS $r \in \mathcal{R}$ from the BS
$d_{ij}^v$	Achievable data rate for a relay link $l_{ij}$ in the subset $v$
$\lambda_s^u$	Time fraction allocated to SS $s \in \mathcal{S}$ in the subset $u$
$\lambda_r$	Time fraction allocated to RS $r \in \mathcal{R}$ in relay zone
$\lambda^u$	Time fraction of subset $u \in \mathcal{U}$ in access zone
$\lambda^v$	Time fraction of subset $v \in \mathcal{V}$ in relay zone
$\lambda_{ij}^v$	Time fraction allocated for a relay link $l_{ij}$ in the subset $v$
$T_s$	Throughput of SS $s \in \mathcal{S}$

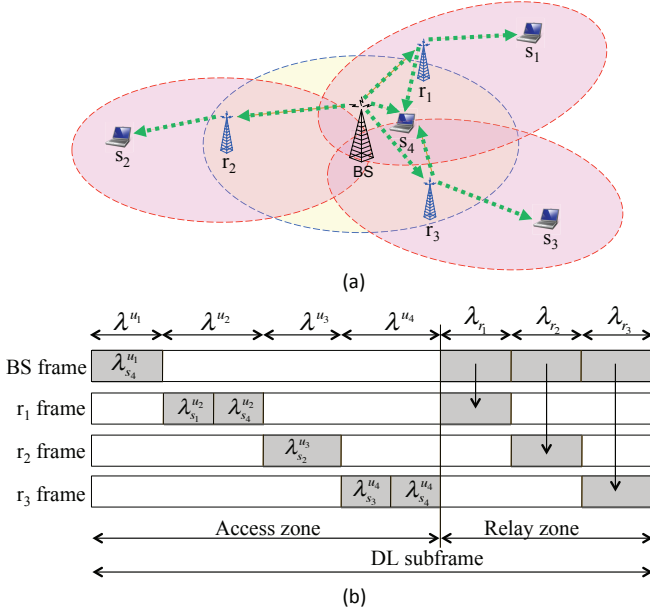


Fig. 3. (a) A sample scenario with three RSs and four SSs. (b) a possible orthogonal resource allocation in the sample scenario (a).

- orthogonal,
- overlapped, and
- optimal

in a two-hop relaying scenario under the max-min fairness constraint. We denote these schemes by OrTH, OvTH, and OpTH respectively. Although the key features of OrTH and OvTH were introduced in [3], the boundary selection problem was not considered. In [3], the boundary between access and relay zones was not dynamically selected according to the traffic load but statistically determined for each scenario. To account for the dynamic boundary selection problem, as well as fairness between the subscriber stations, we formulate the optimization problem for each scheduling scheme such that the cell throughput is maximized. The proposed OpTH scheme, which combines the advantages of OrTH and OvTH is presented at the end of this section.

Due to the fact that the original tile (two-dimensional time $\times$ frequency) scheduling problem is NP-hard [9], we shall not deal with multiuser resource allocation over the frequency domain. In other words, we do not consider frequency selectivity, thus the entire spectrum is allocated to each node whenever they are allowed to transmit, i.e., scheduling is done by assigning time slots to every node.

#### A. Orthogonal Scheme (OrTH)

The essential consideration in the orthogonal scheduling scheme is to avoid interference between access links by restricting transmission opportunities to one service node at a time during the access zone period. However, by precluding frequency reuse, the radio resource efficiency can be significantly reduced. We formulate the optimization problem for the orthogonal scheme in order to maximize cell throughput by determining the time duration of the transmissions for each SS and RS in the access and relay zones under max-min

fairness constraints [13]. The principle of max-min fairness states that none of active SSs can achieve more throughput than other SSs without decreasing the throughput of other SSs. Since for the orthogonal scheme it is impossible to increase the throughput of an SS without decreasing that of other SSs, for the orthogonal scheme the max-min fairness is equivalent to absolute fairness. Let  $\mathcal{R}$  and  $\mathcal{S}$  be the sets of RSs and SSs respectively, and  $\mathcal{R}^+$  be the set of the service nodes (i.e.,  $|\mathcal{R}^+| = |\mathcal{R}| + 1$ ). We use the notation  $\mathcal{U}$  for the set of possible simultaneously active service nodes during the access zone period. Each element  $u \in \mathcal{U}$  represents one subset of  $\mathcal{R}^+$  with all the service nodes in that subset able to concurrently transmit to their associated SSs. For the orthogonal scheme in the sample scenario as shown in Fig. 3, only four transmission subsets ( $\mathcal{U} = \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}\}$ ) are possible since only one service node is allowed to be active at a time (i.e.,  $\mathcal{R}^+ = \mathcal{U}$ ).

#### Scheduling objective:

Since every SS will achieve an equal throughput due to the equivalence of the absolute and max-min fairness constraint for the orthogonal scheme, our goal of maximizing cell throughput corresponds to maximizing the throughput,  $T_s$ , of any subscriber  $s \in \mathcal{S}$ :

$$\max_{s \in \mathcal{S}} T_s. \quad (7)$$

An active SS can be associated with more than one service node at different times during the access zone period. For example,  $s_4$  in Fig. 3(a) can be served by BS,  $r_1$ , or  $r_2$  at different times. The achievable data rate of an SS,  $d_s$ , in each transmission subset  $u \in \mathcal{U}$  is denoted by  $d_s^u$ . Also, the time fraction allocated to an SS,  $\lambda_s$ , in each transmission subset  $u$  is denoted by  $\lambda_s^u$ . Thus, the throughput of an SS,  $T_s$ , during the current DL subframe, is the summation of throughputs received by  $s$  in each transmission subset  $u$  when  $s$  was

allocated the time fraction  $\lambda_s^u$ :

$$T_s = \sum_{u \in \mathcal{U}} d_s^u \lambda_s^u, \quad \forall s \in \mathcal{S}. \quad (8)$$

### Scheduling constraints:

The constraints that the schedule should satisfy are as follows:

**1. Max-min fairness constraints:** for the orthogonal scheme this constraint ensures that every active SS in a cell achieves an equal throughput:

$$T_{s_1} = T_{s_2}, \quad \forall s_1, s_2 \in \mathcal{S} (s_1 \neq s_2). \quad (9)$$

**2. Relaying constraints:** let  $d_r$  and  $\lambda_r$  be the achievable data rate and time fraction allocated for transmission from the BS to RS  $r \in \mathcal{R}$  respectively. The set of SSs associated with an RS  $r$  is denoted as  $\mathcal{S}_r$ . To ensure that there is no data loss at the RSs, the data transferred from BS to RS  $r \in \mathcal{R}$  is equal to the data transferred from RS  $r$  to the associated SSs:

$$d_r \lambda_r = \sum_{s \in \mathcal{S}_r} d_s \lambda_s, \quad \forall r \in \mathcal{R}. \quad (10)$$

**3. Time sharing constraints 1:** this constraint ensures that resources within the duration of each transmission subset  $u$  are fully utilized by the associated SSs. The set of SSs associated with one of the service nodes  $r^+ \in \mathcal{R}^+$ ,  $\mathcal{S}_{r^+}$ , when the  $r^+$  is active during the transmission subset  $u$  is denoted as  $\mathcal{S}_{r^+}^u$ . Thus, the time fraction of each transmission subset  $u$ ,  $\lambda^u$ , is equal to the summation of time fractions allocated to SSs associated with  $r^+$  when  $r^+$  is the element of subset  $u$ :

$$\lambda^u = \sum_{s \in \mathcal{S}_{r^+}^u} \lambda_s^u, \quad \forall r^+ \in \mathcal{R}^+, \forall u \in \mathcal{U}, |\mathcal{S}_{r^+}^u| > 0. \quad (11)$$

**4. Time sharing constraints 2:** this constraint captures the fact that the DL subframe consists of an access zone and a relay zone. The summation of time fractions of every transmission subset will be equivalent to access zone time fraction, and the summation of time fractions allocated to RSs is the same as the relay zone time fraction. The sum of access and relay zone time fractions should be less than or equal to 1:

$$\sum_{u \in \mathcal{U}} \lambda^u + \sum_{r \in \mathcal{R}} \lambda_r \leq 1. \quad (12)$$

**5. Lower and upper bound constraints:** finally, the lower and upper bounds of each element should be set up to solve the optimization problem by using linear programming. The time fractions allocated to SSs and RSs must be positive and smaller than one:

$$0 \leq \lambda_s^u, \lambda_r \leq 1, \quad \forall u \in \mathcal{U}, \forall s \in \mathcal{S}, \forall r \in \mathcal{R}. \quad (13)$$

By using scheduling constraints (9)-(13), the objective function (7) can be maximized. The solution produced by solving the linear programming is always feasible. Once the throughput of an SS (7) is maximized, the cell throughput can be computed by:

$$\text{Cell Throughput} = \sum_{s \in \mathcal{S}} T_s. \quad (14)$$

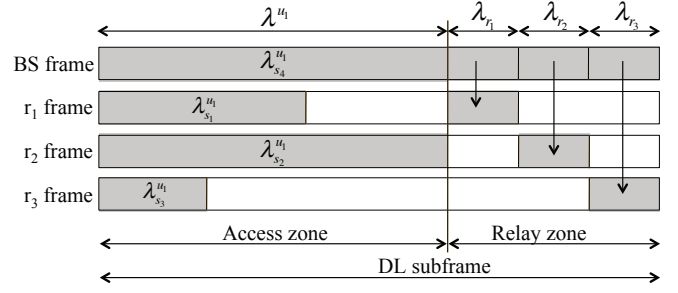


Fig. 4. Overlapped resource allocation example for the sample scenario in Fig. 3(a).

### B. Overlapped Scheme (OvTH)

In contrast to the orthogonal scheme, the focus of the overlapped scheme is to fully reuse radio resources during the access zone interval [3]. On one hand, the cell throughput can be increased, but, on the other hand, outage events are increased due to significant interference. In general, when there are many SSs distributed uniformly throughout the cell, all service nodes can be active during the entire access zone interval to maximize frequency reuse. However, when there are few SSs, or when they are non-uniformly distributed in a cell, not every service node has to be active because some of the service nodes do not have associated SSs to serve, hence, they only introduce interference. Unlike the orthogonal schemes, the overlapped scheme will consider only one transmission subset of the active service nodes (i.e., one subset of  $\mathcal{R}^+$ ). In other words, once the subset of active service nodes is determined at the beginning of a frame, it will last for the entire current frame duration.

In order to compute the maximum cell throughput for the overlapped scheme, we can also formulate the optimization problem as in Section IV-A by considering only one subset  $u \in \mathcal{U}$  instead of the set  $\mathcal{U}$ . By using the same example scenario in Fig. 3(a) with an assumption that every SS has different achievable data rate from its service node ( $d_{s_4} > d_{s_3} > d_{s_1} > d_{s_2}$ ), one possible resource allocation for overlapped scheme is shown in Fig. 4. Every service node can be active simultaneously to serve associated SSs during the access zone period. Therefore, an SS can be served only by the service node that has the strongest link capacity to that SS. Moreover, one unique aspect of the overlapped scheme is that there may be wasted resources due to the fairness constraints. As shown in Fig. 4, when the achievable data rate of  $s_3$  from  $r_3$  is higher than that of  $s_1$  and  $s_2$  from  $r_1$  and  $r_2$  respectively, a smaller time fraction will be allocated for  $s_3$  to preserve fairness. In other words, subscribers with low achievable data rates are allocated large fractions of time while subscribers with high link capacities may have smaller fractions. Hence, the absolute fairness still holds for the SSs associated with RSs, but the SSs associated with the BS directly may achieve a higher throughput without decreasing the throughput of the rest of SSs associated with the RSs thus preserving the max-min fairness of the cell. Consequently, the equivalence of the absolute and max-min fairness constraint does not hold for the overlapped scheme. Details on fairness schemes are expanded

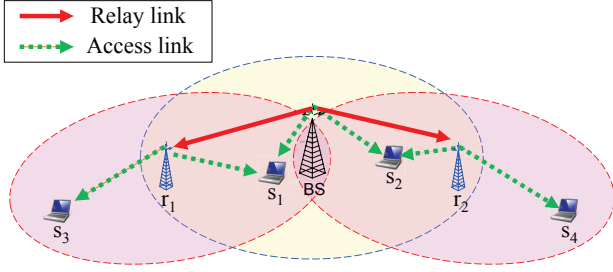


Fig. 5. Example of one WiMAX cell with two RSs and four SSs.

in our previous paper [15]. As a result, the scheduling objective of an overlapped scheme is to maximize cell throughput:

$$\max \sum_{s \in \mathcal{S}} T_s. \quad (15)$$

Due to the fact that an SS can be associated with only one service node during the access zone period in the overlapped scheme, the set  $\mathcal{S}$  can be divided into two disjoint subsets  $\mathcal{S}_b$  and  $\mathcal{S}_R$ . The set  $\mathcal{S}_b$  represents the SSs served by the BS and  $\mathcal{S}_R$  represents the SSs served by RSs. The max-min fairness constraints ensure that every SS achieves an equal throughput in each subset  $\mathcal{S}_b$  and  $\mathcal{S}_R$ . However, the throughput of an SS in  $\mathcal{S}_b$  could be higher than the throughput of an SS in  $\mathcal{S}_R$ . Therefore, the max-min fairness constraints for the overlapped scheme can be expressed as:

$$\begin{aligned} T_{s_1} &= T_{s_2}, & \forall s_1, s_2 \in \mathcal{S}_b (s_1 \neq s_2) \\ T_{s_3} &= T_{s_4}, & \forall s_3, s_4 \in \mathcal{S}_R (s_3 \neq s_4) \\ T_{s_5} &\geq T_{s_6}, & \forall s_5 \in \mathcal{S}_b, \forall s_6 \in \mathcal{S}_R. \end{aligned} \quad (16)$$

Since the overlapped scheme considers only one subset of  $\mathcal{R}^+$  and there may be wasted resources in the access zone due to fairness, the relaying constraint for overlapped scheme will be the same as (10) but the time sharing constraints (11), (12) are modified as:

$$\lambda^u \geq \sum_{s \in \mathcal{S}_{r^+}^u} \lambda_s^u, \quad \forall r^+ \in \mathcal{U}, \forall u \in \mathcal{U}, |\mathcal{S}_{r^+}^u| > 0 \quad (17)$$

$$\lambda^u + \sum_{r \in \mathcal{R}} \lambda_r \leq 1. \quad (18)$$

The cell throughput for the overlapped scheme can be maximized by solving linear programming with the objective (15) under constraints (10), (13), (16), (17), (18).

Another interesting problem for the overlapped scheme is determining the subset of active service nodes at the beginning of a frame. In addition to the traffic distribution of SSs in a cell, determining service nodes can be affected by additional *subset selection objectives*. We introduce three subset selection objectives:

- maximizing cell throughput,
- maximizing the number of active service nodes, and
- maximizing the number of served SSs.

We describe these objectives through an example:

**Example.** Consider a simple relay network with one BS, two RSs ( $r_1, r_2$ ), and four SSs ( $s_1, s_2, s_3, s_4$ ) as shown in Fig.

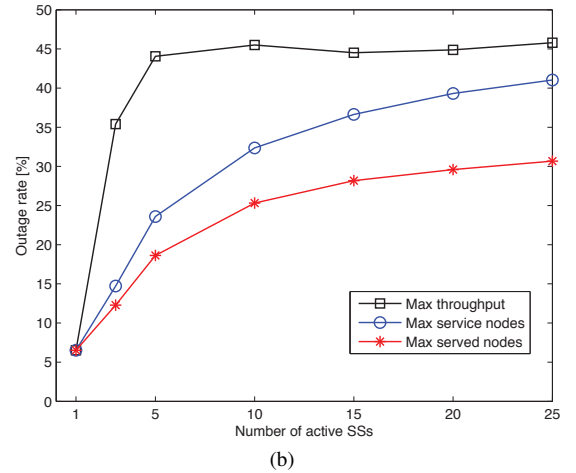
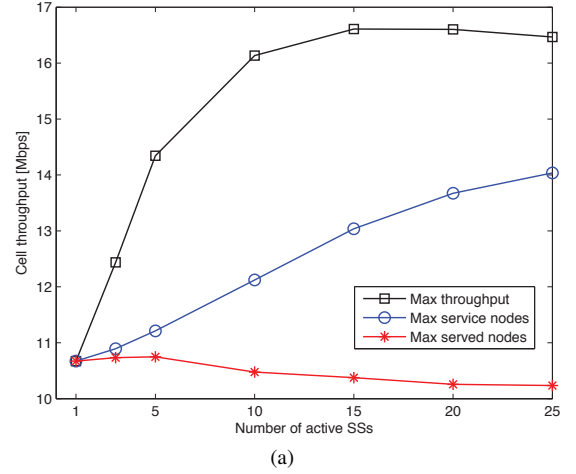


Fig. 6. (a) Cell throughput and (b) outage probability as a function of the number of active SSs within a cell for different subset selection objectives for the overlapped scheme.

5. Subscribers  $s_3$  and  $s_4$  can be served only via  $r_1$  and  $r_2$  respectively, whereas the  $s_1$  and  $s_2$  can also be served by the BS directly. Assume that the link BS to  $s_1$  has the highest capacity compared to the rest of the links, hence,  $s_1$  can be served by the BS when the BS and  $r_1$  are active at the same time, while  $s_2$  can neither be served by BS nor by  $r_2$  when the BS and  $r_2$  are concurrently active.

If we consider every possible transmission subset of the service nodes in this example, the number of subsets of  $\mathcal{R}^+$  is 7 (i.e.,  $\mathcal{U} = \{\{BS\}, \{r_1\}, \{r_2\}, \{BS, r_1\}, \dots, \{BS, r_1, r_2\}\}$ ). Any subset could be chosen at the beginning of a frame and the selected service nodes in that subset will be optimally scheduled to maximize cell throughput by using the optimization problem formulated above. However, the resulting cell throughputs from the optimization problem can be significantly different from each other according to the subset chosen. In this example, when we choose the subset  $\{BS\}$ , only  $s_1$  and  $s_2$  are served by the BS with high throughputs but  $s_3$  and  $s_4$  can not be served. When another subset  $\{BS, r_1, r_2\}$  is used,  $s_2$  can not be served due to interference but the achievable throughput of each SS is lower than the previous case. If we choose the subset  $\{r_1, r_2\}$ , every SS is served but the throughput of each SS will be lower than in the other

two cases. Therefore, with the subset selection objective of maximizing cell throughput, the subset  $\{BS\}$  will be chosen, when maximizing the number of service node objective, the chosen subset is  $\{BS, r_1, r_2\}$ , and  $\{r_1, r_2\}$  is selected for maximizing the number of served SSs.

Figure 6 shows the cell throughput and outage rate as a function of the number of active SSs for different subset selection objectives for the overlapped scheme. The  $N$  SSs are randomly placed in the cell with a uniform distribution. The maximum throughput can be achieved in the max throughput subset selection objective case, but, at the same time, the outage rate for this case is much higher than in the other cases. Similarly, the lowest outage rate is achieved by max served nodes subset selection objective case, but the cell throughput of that case is the lowest. From a service provider's customer satisfaction perspective, we consider the max served nodes case as the most realistic subset selection objective for the overlapped scheme.

### C. Optimal Scheme (OptH)

The key task of an optimal scheduling scheme is to maximize throughput via frequency reuse under max-min fairness constraint, while avoiding outage due to interference. With the overlapped scheme, it is shown that some areas of the cell may not be serviced when multiple service nodes are transmitting simultaneously, and wasting resources is inevitable. To eliminate these problems, we need to consider all possible transmission subsets of service nodes during the access zone period as depicted in Fig. 7, i.e., the set  $\mathcal{U}$  is the power set of  $\mathcal{R}^+$  excluding the empty set in the optimal scheme scenario ( $|\mathcal{U}| = 2^{|\mathcal{R}^+|} - 1$ ). The achievable data rate of an SS,  $d_s^u$ , varies for each subset of active service nodes because intra cell interference changes according to the number of active service nodes in each subset.

There should be no wasted resources under an optimal scheme and none of the active SSs can achieve more throughput without decreasing the throughput of other SSs, thus the equivalence of the absolute and max-min fairness holds for the optimal scheme. The objective of the optimal scheme is the same as that of the orthogonal scheme (7), and the max-min fairness constraints are also the same as (9). In the orthogonal and overlapped schemes, the set  $\mathcal{S}_r$  does not change over one DL subframe interval, since an RS  $r$  can be active as part of only one subset  $u$ . However, in the optimal scheme, an RS  $r$  can be active more than once as part of different subsets. Let  $\mathcal{S}_r^u$  be the set of SSs associated with RS  $r \in \mathcal{R}$  in each transmission subset  $u$ . Thus, the relaying constraint for optimal scheme can be expressed as:

$$d_r \lambda_r = \sum_{u \in \mathcal{U}} \sum_{s \in \mathcal{S}_r^u} d_s^u \lambda_s^u, \quad \forall r \in \mathcal{R}. \quad (19)$$

When there are more than two active service nodes in the subset  $u$ , the first time sharing constraint ensures that every service node fully utilizes the resources. For example, when  $r_1^+$  and  $r_2^+$  are active service nodes in the subset  $u$ , the summation of time fractions allocated to SSs associated with  $r_1^+$  should

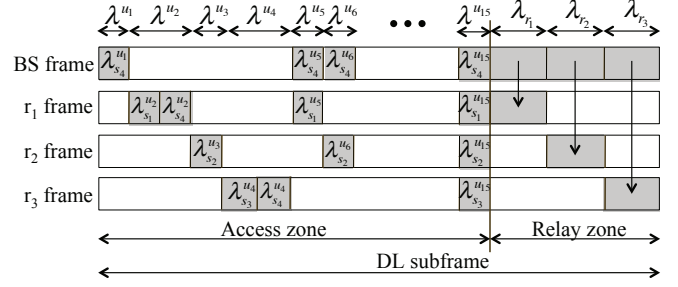


Fig. 7. Optimal resource allocation example for the sample scenario in Fig. 3(a).

be equal to the summation of time fractions allocated to SSs associated with  $r_2^+$ :

$$\lambda^u = \sum_{s \in \mathcal{S}_{r_1^+}^u} \lambda_s^u = \sum_{s \in \mathcal{S}_{r_2^+}^u} \lambda_s^u, \quad (20)$$

$$\forall r_1^+, r_2^+ \in u, \forall u \in \mathcal{U}, |\mathcal{S}_{r_1^+}^u| > 0, |\mathcal{S}_{r_2^+}^u| > 0.$$

The second time sharing constraint and lower and upper bound constraint for optimal scheme are the same as (12), (13). Therefore, the cell throughput for the optimal scheme can be maximized by solving linear programming with the objective (7) under constraints (9), (12), (13), (19), (20).

## V. SCHEDULING SCHEMES IN MULTIHOP SCENARIO

In this section, we explore the impact of an increased number of relay hops on the optimal scheduling scheme. Although the two-hop scenario analyzed in Section IV is technically a multihop scenario, we will use the term “multihop” to refer to scenarios with more than one relay tier. There has been much research in evaluating the performance of using RSs in two-hop relay networks, while little work has thoroughly examined the performance of more than two-hop relay networks. Many researchers believe that more than two-hop relay scenario is not preferable in practice for two reasons. First, more hops per connection cause a greater delay, therefore real time applications will be affected. Second, the bandwidth efficiency suffers with increasing number of hops since additional hops consume extra bandwidth leading to throughput degradation. Nevertheless, the cell coverage will be significantly extended by increasing the number of hops. Therefore, It is important to evaluate the performance trade-off between coverage extension and throughput degradation. Notably, it is of interest to explore how multihop relaying can affect the network throughput under an optimal scheduling scheme since the optimal scheme can maximize cell throughput while minimizing outage performance. The main differences between two-hop and multihop scenarios are in the relay zone period of a frame. The RSs do not communicate with each other in a two-hop scenario, whereas the RSs are allowed to relay data to/from another RSs in a multihop scenario. Moreover, the resources are always orthogonally shared by RSs during the relay zone period of the DL subframe in a two-hop scenario, while frequency reuse is possible in the relay zone of a multihop scenario. To evaluate the performance of frequency reuse in the relay zone, we

present two schemes, namely orthogonal in the relay zone for multihop (OrMH) and optimal scheme for multihop (OpMH) in order to analyze the effect of frequency reuse in the relay zone. Both schemes are modified from the proposed scheme, OpTH, in Section IV-C.

#### A. Orthogonal in Relay Zone for Multihop (OrMH)

When an SS is receiving data from the BS through multiple RSs, there are multiple paths to the SS. To maximize cell throughput under the max-min fairness constraints, the optimal path should be selected by the scheduling scheme. If we assume that resources are orthogonally shared by RSs in the relay zone for a multihop scenario, the optimal path to each SS will be corresponding to the path that has the highest achievable relay data rate of the SS, i.e., a higher capacity link is always preferable for RSs to relay data during relay zone period. To modify the proposed optimal scheduling scheme in Section IV-C for the multihop scenario, we need to consider every possible link between RSs as a relay link. The RSs receive data only from the BS in the two-hop scenario, thus at most, one time fraction of the DL subframe is allocated to each RS during the relay zone period. In a multihop scenario, however, more than one time fraction can be allocated to the RSs. Let  $l_{ij}$  be a relay link from the service node  $i \in \mathcal{R}^+$  to  $j \in \mathcal{R}$ , and let  $\mathcal{L}$  be the set of all possible relay links (i.e.,  $l_{ij} \in \mathcal{L}$ ). Also, let  $d_{ij}$  and  $\lambda_{ij}$  be the achievable data rate and time fraction allocated for a relay link  $l_{ij}$  respectively. To formulate the optimization problem for the OrMH scheme, the relaying and second time sharing constraints from the OpTH scheme should be modified. To simplify the notation, we denote with  $T_r$  the total amount of data transferred from an RS  $r$  to the associated SSs during the DL subframe interval:

$$T_r = \sum_{u \in \mathcal{U}} \sum_{s \in \mathcal{S}_r^u} d_s^u \lambda_s^u, \quad \forall r \in \mathcal{R}. \quad (21)$$

To ensure that there is no data loss at the RSs, the summation of the incoming data transferred from the other service nodes including the BS to an RS  $j \in \mathcal{R}$  should be equal to the summation of the outgoing data transferred to the other RSs and the associated SSs from the RS  $j$  as follows:

$$\sum_{l_{ij} \in \mathcal{L}} d_{ij} \lambda_{ij} = \sum_{l_{jk} \in \mathcal{L}} d_{jk} \lambda_{jk} + T_j, \quad \forall i \in \mathcal{R}^+, \forall j, k \in \mathcal{R}. \quad (22)$$

Due to the orthogonality in the relay zone under OrMH scheme, the summation of every time fraction allocated to each relay link  $l_{ij} \in \mathcal{L}$  is equivalent to relay zone time fraction, thus the second time sharing constraint (12) is rewritten as:

$$\sum_{u \in \mathcal{U}} \lambda^u + \sum_{l_{ij} \in \mathcal{L}} \lambda_{ij} \leq 1, \quad \forall i, j \mid l_{ij} \in \mathcal{L}. \quad (23)$$

Therefore, the cell throughput for the OrMH scheme can be maximized by solving linear programming with the objective (7) under constraints (9), (13), (20), (22), (23).

#### B. Optimal Scheme for Multihop (OpMH)

Unlike in the OrMH scheme, multiple relay links can be active simultaneously during the relay zone period in the OpMH scheme, and the optimal path from the BS to each SS via multiple RSs does not have to be the path that has the highest achievable relay data rate of the SS. In other words, although it is clear that a higher capacity relay link is preferable, a lower capacity relay link can also be scheduled for RSs to relay data to maximize the efficiency of frequency reuse. Thus, throughput degradation due to the increase of number of hops can be minimized by maximizing the frequency reuse efficiency in the relay zone. In order to take into account every possible simultaneous transmissions between relay links, we define the set  $\mathcal{V}$  as the power set of  $\mathcal{L}$  (the relay links) excluding the empty set and any sets of links that cannot be active at the same time. Thus, each element  $v \in \mathcal{V}$  is a subset of relay links that can be active at the same time. The achievable data rate  $d_{ij}$  and time fraction  $\lambda_{ij}$  allocated for a relay link  $l_{ij} \in \mathcal{L}$  can vary according to the simultaneous transmission subset  $v$ . Therefore, we denote with  $d_{ij}^v$  and  $\lambda_{ij}^v$  the achievable data rate and time fraction allocated to  $l_{ij}$  when the relay links in the subset  $v$  are active, i.e., when a relay link  $l_{ij}$  is an element of subset  $v$  and  $v$  has more than two elements, the value of achievable data rate  $d_{ij}^v$  will be less than or equal to  $d_{ij}$  due to interference. To formulate optimization problem for the OpMH scheme, the relaying constraint (22) is further modified to consider simultaneous transmission subsets:

$$\sum_{v \in \mathcal{V}} \sum_{l_{ij} \in v} d_{ij}^v \lambda_{ij}^v = \sum_{v \in \mathcal{V}} \sum_{l_{jk} \in v} d_{jk}^v \lambda_{jk}^v + T_j, \quad (24)$$

$$\forall i \in \mathcal{R}^+, \forall j, k \in \mathcal{R},$$

Also, the summation of every time fraction of subset  $\lambda^v$  is equivalent to the relay zone time fraction, thus the second time sharing constraint (23) is rewritten as:

$$\sum_{u \in \mathcal{U}} \lambda^u + \sum_{v \in \mathcal{V}} \lambda^v \leq 1. \quad (25)$$

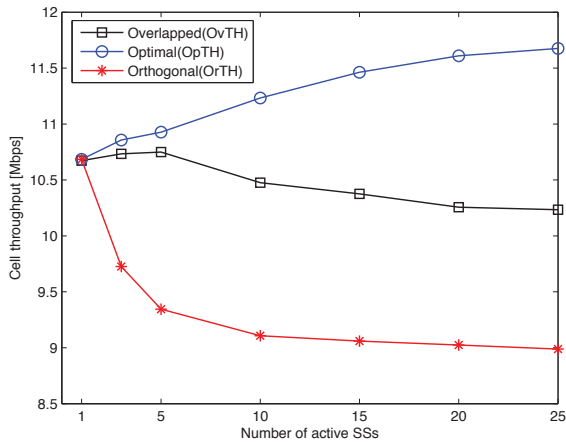
Moreover, to ensure that resources within the duration of each transmission subset  $v$  are fully utilized by the associated relay links, another time sharing constraint is needed to OpMH scheme. For example, when  $l_1$  and  $l_2$  are relay links in the subset  $v$ , the time fraction of subset  $v$  should be equal to the time fractions allocated to  $l_1$  and  $l_2$ :

$$\lambda^v = \lambda_{l_1}^v = \lambda_{l_2}^v, \quad \forall l_1, l_2 \in v, \forall v \in \mathcal{V}. \quad (26)$$

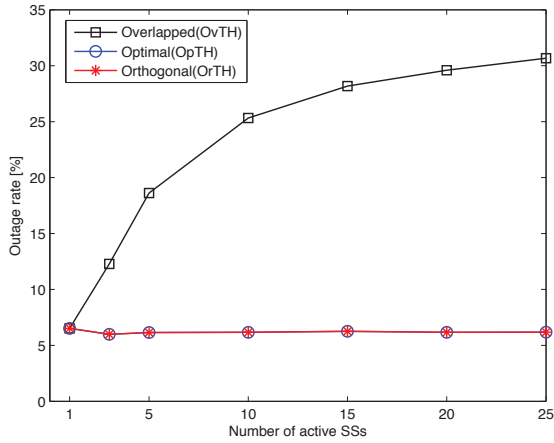
Therefore, the cell throughput for the OpMH scheme can be maximized by solving linear programming with the objective (7) under constraints (9), (13), (20), (24), (25), (26).

## VI. NUMERICAL RESULTS AND ANALYSIS

In this section we first evaluate the performance of the proposed optimal resource allocation scheme for the two-hop scenario (OpTH) by comparing it with the orthogonal (OrTH) and overlapped schemes (OvTH). We then consider the multihop case with six and nine relays and compare its performance with the two-hop case with three relays.



(a)



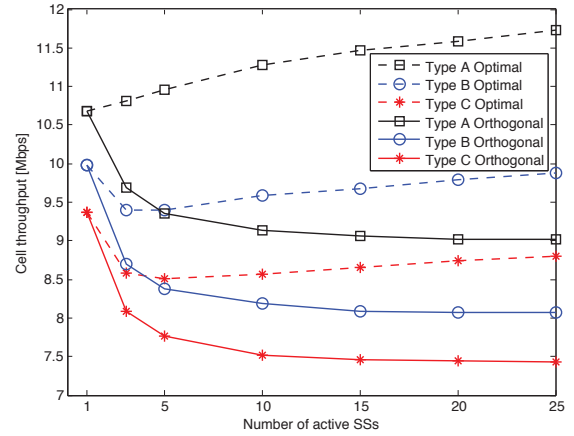
(b)

Fig. 8. (a) Cell throughput and (b) outage probability as a function of the number of active SSs within a cell for different two-hop scheduling schemes.

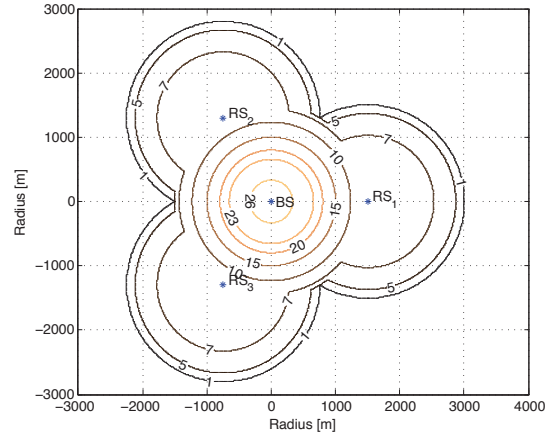
### A. Two-hop Scenario

We analyze the cell throughput and outage rate as a function of the number of active SSs in a cell. From the optimization problems presented in Section IV, the cell throughput for each scheduling scheme can be computed by solving the corresponding linear programming problems. Fig. 8(a) shows cell throughput results from the three different scheduling schemes. To obtain the average cell throughput value, the simulation is repeated 10,000 times for each scenario with  $N$  active SSs randomly placed within a cell. We computed the 95% confidence intervals, but do not show them as they are very small and would clutter the graphs.

When there is only one active SS in a cell, hence no frequency reuse and co-channel interference, the average throughput of the SS under different scheduling schemes is equal to 10.67Mbps. As the number of active SSs increases, the cell throughput achieved by OrTH decreases because it is more likely to have SSs with low link capacities consuming large fractions of the time in order to preserve fairness. In contrast, the cell throughput for OpTH grows as the number of active SSs increases since the optimal scheme maximizes frequency reuse, while minimizing co-channel interference in order to avoid outage. The cell throughput for OvTH with max



(a)



(b)

Fig. 9. (a) Cell throughput under different terrain types and (b) contour graph for terrain type B.

served nodes set selection objective is lower than that of the proposed OpTH. Fig. 8(b) shows the outage rate as a function of the number of active SSs within a cell for the three different scheduling schemes. The outage rate from proposed OpTH is identical to the result from OrTH. Although there is no interference between service nodes in the OrTH scheme, about 6% of active SSs still encounter outage due to the Rayleigh fading channels. This result shows that the cell coverage probability under Rayleigh fading channel was improved from 90% when not using RSs to 94% when using three RSs in addition to coverage extension. In contrast, the outage rate for the OvTH continues to rise significantly as more SSs join the cell because the number of active service nodes also increases leading to an insufficient SINR for many SSs. Therefore, the cell throughput can be significantly enhanced by using the proposed OpTH compare to OrTH scheme while preserving the low outage performance as OrTH scheme and fairness.

Figure 9(a) shows the cell throughput of OpTH and OrTH for the three different terrain types in [17]. Since terrain type B and C have a lower path loss than type A, the cell radius for types B and C are determined to be 1500m and 1700m respectively by the condition that the cell coverage probability under Rayleigh fading channel is greater than 90%.

The three RSs are assumed to be deployed at the edge of the transmission range of the BS in each terrain type scenario. The cell throughput achieved by OrTH decreases as terrain type changes from A to C because the percentage of the cell area that has a lower achievable data rate increases in scenarios with a lower path loss and thus is more likely to have users with a lower achievable data rate. In Fig. 9(b), the contour graph of terrain type B scenario shows that the percentage of the cell area that has an achievable data rate less than 10Mbps is increased compared to the type A contour graph in Fig. 2. Similarly, the cell throughput achieved by OpTH also decreases in the lower path loss scenario. Moreover, when the number of active SSs is changing from 1 to 3 and 5 in terrain type B and C, the capacity of the network decreases, while for terrain type A increases. This interesting effect occurs because for terrain types B and C the average achievable data rate of the link between BS to RS decreases due to the higher co-channel interferences caused by the lower path loss scenarios. For example, the average achievable data rate of the BS to RS link under terrain type A is 17.97Mbps, while for the types B and C, the average achievable data rates are 13.72Mbps and 11.31Mbps respectively. As a result, the effectiveness of frequency reuse is diminished in the scenarios with lower path loss. Accordingly, an effective way of enhancing cell throughput under terrain type B or C is to increase the BS to RS link capacity by reducing the distance between the BS and RSs.

### B. Complexity

In this subsection we explore the computational time for each scheduling scheme in the two-hop scenario. Every scheme uses the linear programming to solve optimization problem, that is, every time fraction of active SSs under each transmission subset of active service nodes will be optimally determined through linear programming. In general, the number of variables in LP function is the primary factor that affects the computation time. If we compare the number of variables in each scheduling scheme, given  $N$  SSs and  $M$  RSs in a cell, the number of variables (i.e., time fractions) for OvTH, OrTH, and OpTH schemes are  $N$ ,  $N \cdot M$ , and  $N \cdot 2^M$  respectively (in the worst case). Solving a linear programming with  $n$  variables takes  $O(n^3)$  steps using the simplex method [21]. We used MATLAB's solver *linprog*, which is based on a variant of Mehrotra's predictor-corrector algorithm.

To examine the computational time of each scheduling scheme, we focus on the time consumed by LP function in each scheme. Fig. 10 shows the computational times as a function of the number of active SSs and RSs respectively for different scheduling schemes. The computational time for OpTH is significantly higher than OrTH and OvTH, especially, when the number of RSs increases, the computational time of OpTH is exponentially increased since every possible combination of the active service nodes are considered in the optimal scheme. The OrTH case computation time is always higher than OvTH case since only one transmission subset is considered in the overlapped scenario and we exclude

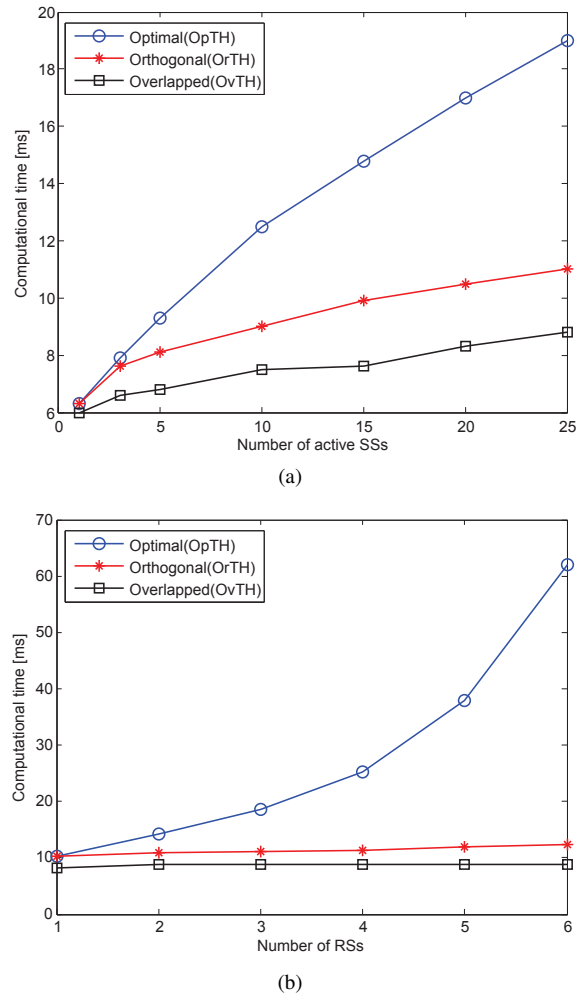


Fig. 10. (a) Computation time as a function of the number of active SSs and (b) computation time as a function of the number of active RSs in two-hop scenario.

the additional computation time for selecting the appropriate transmission subset of service node for the overlapped scheme.

### C. Multihop Scenario

To evaluate the performance of the proposed optimal scheme for the multihop scenario (OpMH), we use two sample coverage extension scenarios with six RSs and nine RSs to allow more than two-hop relaying. Fig. 11(a) shows not only how are RSs deployed for the multihop scenarios but also how much the coverage can be extended by RSs. Although we do not assume a hexagonal cell shape in this work, we show it in Fig. 11(a) to demonstrate coverage enhancement. For instance, when we deploy three RSs at the edge of the cell as shown in Fig. 11(a)(ii) the total cell coverage will be three times larger than the cell without RSs. Similarly, the coverage of multihop scenarios with six and nine RSs can be five and seven times larger than that of a cell with no RSs.

The locations of RSs and the achievable average data rate of an SS in the considered multihop scenarios with six RSs and nine RSs are shown in Fig. 11(b) and (c). When an SS is located near the RSs and receiving data from the BS via

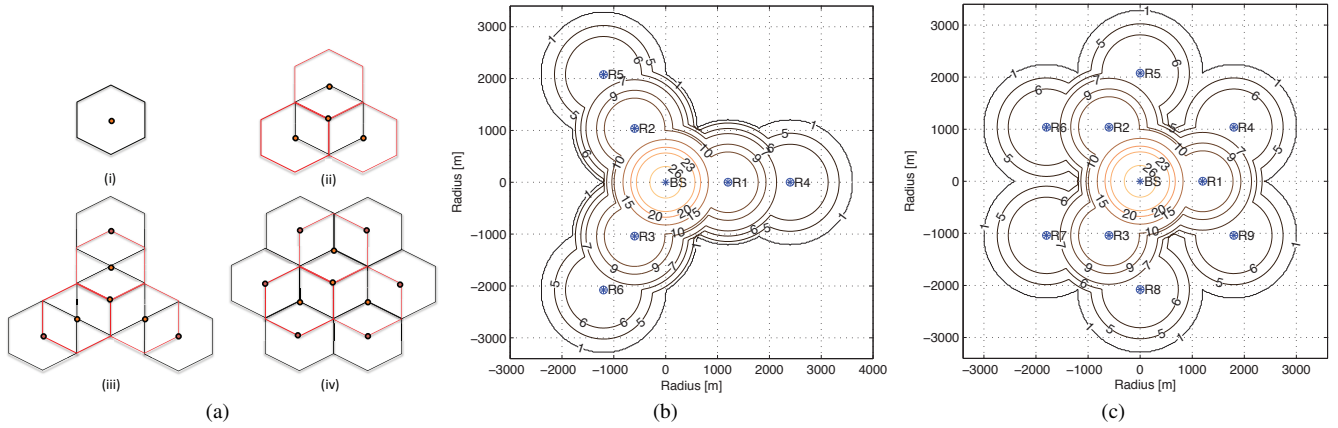


Fig. 11. (a) A coverage extension scenario with (i) no RS, (ii) three RSs, (iii) six RSs, (iv) nine RSs. (b) Six RSs and (c) nine RSs contour graphs in multiplex scenario.

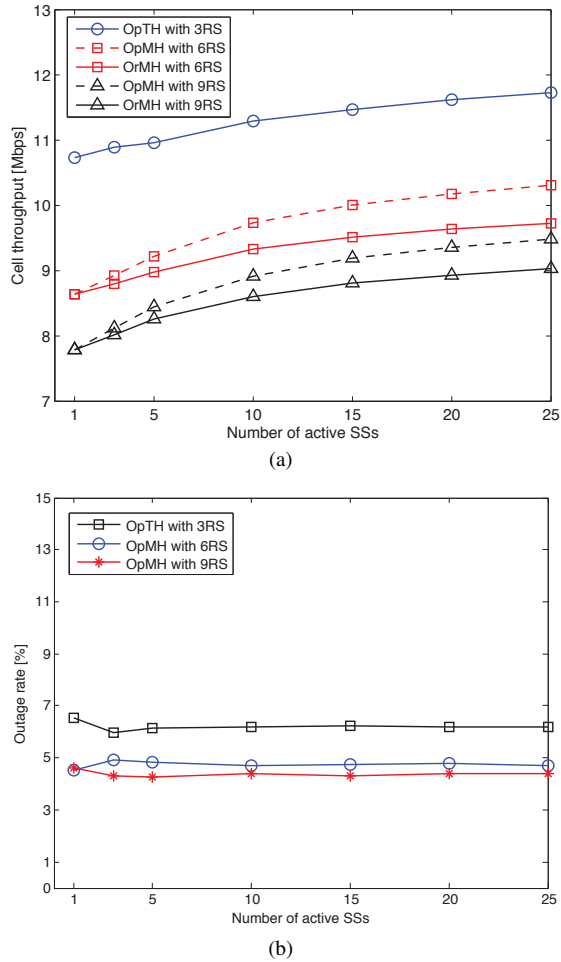


Fig. 12. (a) Cell throughput and (b) outage probability as a function of the number of active SSs within a cell for different multihop scenarios.

two RSs, the average achievable data rate of an SS is only 6.69Mbps. To investigate the efficiency of the frequency reuse in the relay zone, we compare the performance of the OpMH with the OrMH scheme in terms of the cell throughput as a function of the number of active SSs in a cell. Fig. 12(a) shows the performance results of OrMH and OpMH when those

schemes are used in both multihop scenarios with six RSs and nine RSs respectively. The cell throughputs from the OpTH scheme with three RSs are also plotted in this figure to quantify the cell throughput degradation from increasing the number of relay hops from two to three. As the number of active SSs increases, the cell throughput achieved by OrMH in both six RSs and nine RSs cases is growing in a similar manner, and the throughput degradation from the increase in the number of hops is approximately 17% and 23% respectively. As a result of the frequency reuse in the relay zone, the cell throughput achieved by OpMH is always higher than OrMH scheme, thus the throughput degradation is relatively small (12% and 19% respectively). Fig. 12(b) shows the outage rates for the OpMH scheme in both six and nine RSs scenarios. The outage rates from OrMH scheme are not plotted in this figure since those are identical to the results of OpMH scheme. We also plot the outage rate from the OpTH scheme with three RSs to highlight the difference between two-hop with three RSs and three-hop with six and nine RSs scenarios. The overall outage rates on both two-hop and three-hop scenarios are stable and do not depend on the number of active SSs. However, the outage rates from OpMH are slightly lower than the two-hop relaying case because the additional relays increase the chance of at least one good link between an SS and one of the RSs.

## VII. CONCLUSION

In this paper we studied the resource allocation problem in 802.16j based networks enhanced with non-transparent relays. We first proposed an optimal scheme for two-hop relaying networks by maximizing the frequency reuse efficiency under max-min fairness constraints. We then explore an extension of the proposed scheme to a general multihop relaying scenario by taking into account frequency reuse in both access and relay zone periods. We formulated the optimization problem and solutions by using linear programming. We evaluated the performance of our proposed optimal scheme for a two-hop scenario by comparing it with well-known scheduling schemes such as the orthogonal and overlapped schemes. Our

results showed that the highest cell throughput can be achieved with the proposed optimal scheme, while, at the same time, maintaining the fairness and lower outage performance of the orthogonal scheme although the complexity of the optimal scheme is higher than that of the orthogonal and overlapped schemes. Moreover, the cell throughput degradation from increasing the number of hops from two to three hops, under proposed optimal scheduling scheme, is only 12% and 19% when six RSs and nine RSs are deployed respectively.

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