Asymptotic Stability of Congestion Control Systems with Multiple Sources

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Abstract

Congestion in computer networks is the main reason for reduced performance and poor quality of service; therefore, a good congestion control system is essential. The basic property of any control system is stability. We consider the problem of stability in computer network congestion control systems with multiple sources, which is the most common case in general purpose computer networks. The main result of the paper is the proof that for congestion control systems with linear controllers (e.g., ATM-ABR), the stability of the congestion control system with a single source is equivalent to the stability of the one with multiple sources. In other words, for the considered congestion control systems, if the system is stable for a single source it will be stable for an arbitrary large number of sources. The proof is based on a well-known necessary and sufficient stability test.

I. Introduction

Congestion control in data networks usually refers to the problem of controlling the sending rate of the source hosts such that the throughput of the network is maximized under some additional constraints (e.g., fairness). The importance of congestion control schemes cannot be overemphasized, as congestion can bring even a correctly operating network to its knees [1]. When congested, the routers in the network receive more packets than they can forward, their internal buffers overflow, and packets are dropped. If the congestion control scheme is not well designed, the sources will try to push even more packets through the network in response to packet drops, thus worsening the congestion.

Over the years, many congestion control protocols have been proposed, among them is also the one that forms the basis of the current TCP congestion control in the Internet [2]. In time, several improvements have been proposed and implemented [3]–[5]. In the last decade, a large number of papers took a rigorous, control theoretic approach to congestion control [6]–[8].

In these papers, either one or multiple data sources are throttled in such a way that congestion in the network is eliminated after it occurs or, even better, avoided altogether.

The main result of this work is that, for congestion control systems with linear controllers, the stability of the system with one source is equivalent to the stability of the system with multiple sources. The applicability of the result is potentially very broad. If stability can be shown for one source, the stability of the system with multiple sources follows automatically.

II. Model

Fig. 1 depicts a typical congestion control system in a data network with M sources transmitting data through a congested switch. The sources send data through a congested switch to their corresponding destinations (in reality, the circuits need not be distinct, e.g., a destination can get data from multiple sources; the separation is only conceptual). The congested switch sends congestion feedback information to the sources, and the sources may reduce or increase their data rate as a function of this feedback. The way in which the feedback is provided may be different from network to network. For example, in ATM networks [9], for available bit-rate flows, either a congested bit or an explicit rate request is sent back to the sources with the resource management (RM) cells on the return paths. In IP networks (e.g., the Internet), the transmission control protocol (TCP) uses an implicit feedback: if the packets are not acknowledged, it is assumed that they were dropped at the congested switch and the sending rate is reduced.

Some of the flows may be congested at different nodes. Depending on the particular feedback mechanism, the feedback of all congested switches can be cumulative (e.g., in IP networks), or only the most congested switch on the path of the flow can impose the transmission rate of the source of the flow (e.g., in ATM networks). The flows that are not controlled by the congested switch (e.g., ATM ABR flows congested elsewhere), will not be considered in the analysis, as they will not influence the stability congestion control system in any way (they will simply shift the equilibrium point by a constant value equal to their throughput).

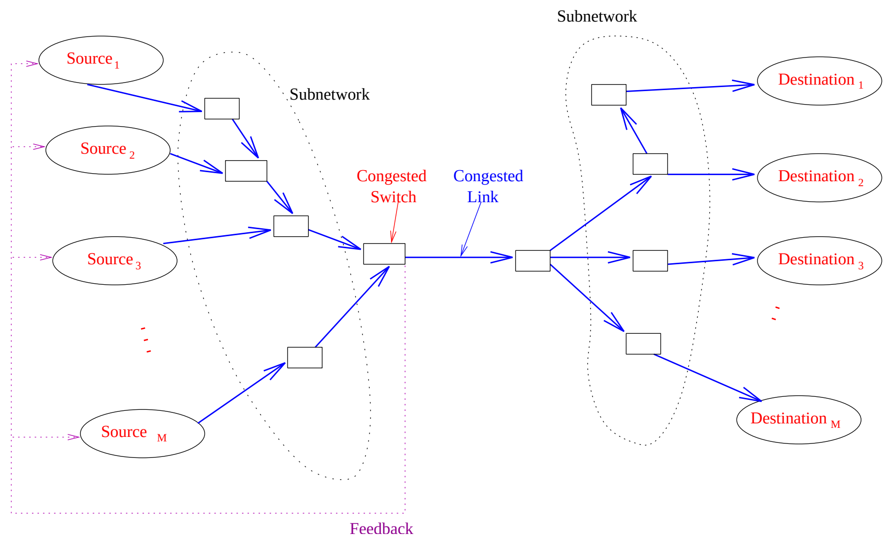

Fig. 1. A network with a congested node and multiple sources

Regardless of the feedback mechanism and the exact reaction of the sources to this feedback, the general feedback mechanism for multiple sources is similar to the one presented in Fig. 2.

In reality, the forward and return paths presented in Fig. 2 can be a single communication link (or a chain of links and switches) or not (it may be a separate channel altogether as in the case of TCP, which has implicit feedback); but qualitatively, they transport two different types of data. On the return paths, feedback information travels from the congested switch to the sources. On the forward paths, user data travels from the sources to the congested switch. Two different discrete time models were formulated [10]–[15] for the two quantities of interest: a “hold freshest sample”(HFS) model for the feedback information and a “variable bit-rate”(VBR) model for the user data volume.

The HFS model (depicted in Fig. 3) is a time-variant model for explicit feedback. We denote with $r(n)$ the rate computed at the congested node at time instant n, with $q(n)$ the explicit rate request received by the source and with $\bar{\tau}$ the maximum delay encountered on the feedback path between the congested switch and the source. In Fig. 3, at any one time only the switch $\alpha_j,\ 1 \leq j \leq \bar{\tau}$ corresponding to the instantaneous delay j at time n is closed, all the other ones

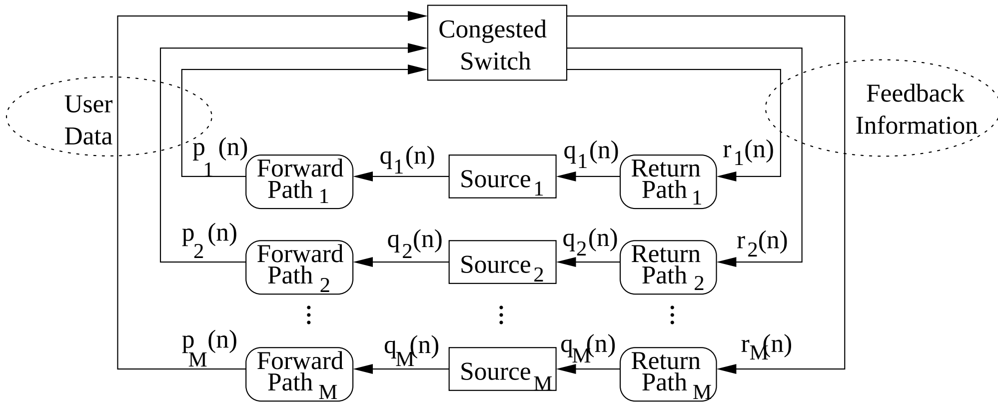

Fig. 2. The feedback loops of the congestion control system

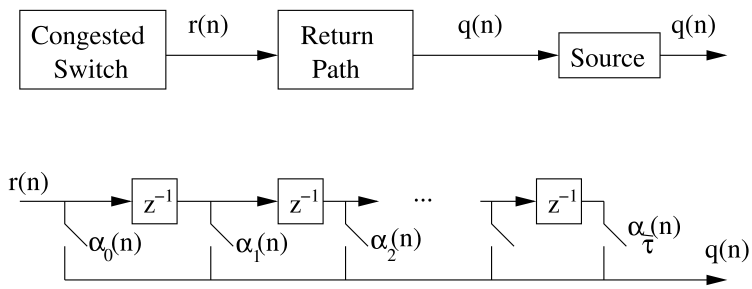

Fig. 3. A Hold Freshest Sample (HFS) delay model.

are open:

$$\alpha_j(n) = \begin{cases} 1 & \text{if } j = \tau(n) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Thus, $q(n) = r(n - \tau(n))$. We assume that the sources obey the explicit rate command from the switch and thus, the output data rate $q(n)$ is the same as the latest received rate request $q(n)$. If we assume that only the freshest sample is held (e.g., if a sequence number is used for $r(n)$ or if the network path is strictly first-in-first-out like in the case of ATM networks) additional constraints have to be imposed on the coefficients α_j [10]:

$$\alpha_j(n) = 1 \Rightarrow \alpha_k(n+1) = 0 \;\; \forall k > j + 1. \tag{2}$$

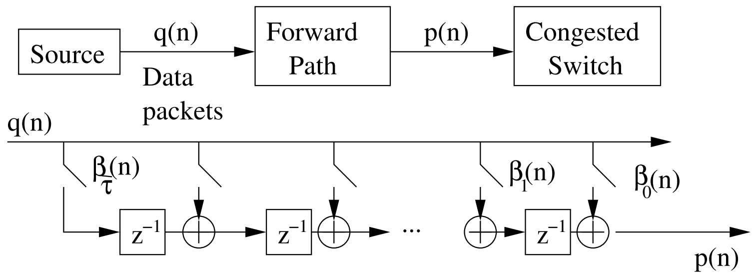

Fig. 4. A Variable Bit Rate (VBR) delay model.

The variable bit-rate (VBR) time-variant delay model (presented in Fig. 4) accurately models delays encountered by volumes of user data. Similarly to the HFS delay only the switch β_j corresponding to the current delay is closed, all the other ones are open (if the data sent at time instant n arrives in more than one time interval at the destination, several switches with weights summing up to one will be closed at any one time):

$$\beta_j(n) = \begin{cases} 1 & \text{if } j = \tau(n) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The model is capable to accurately model all possible events that may occur in a real network including out-of-order packet delivery, data clumping on the path to the congested switch, data loss (with $\beta_j < 1$), etc. [10], [15].

Using the time-variant delay models for the HFS and VBR delays, Fig. 5 depicts a detailed model that fits most congestion control systems for the case of multiple sources. We denote with M the maximum number of sources that may connect to the congested switch at any one time (we shall see that this is not a restriction, as M can be arbitrarily large). The sampling period T of the discrete time model can be chosen to be as small as necessary to capture the dynamics of the system. The focus of this paper is not on the system model. A detailed description of the model is available in [10], [11], [13], [15].

The weights $w_i(n)$ represent the "fair" share of the bandwidth allocated to source i. In ATM ABR networks, the weights are computed using a max-min fairness algorithm [16], [17], and are sent to the sources using the explicit rate (ER) field in the resource management (RM) cells. In TCP/IP networks, if a plain forwarding mechanism is used, all flows will receive an

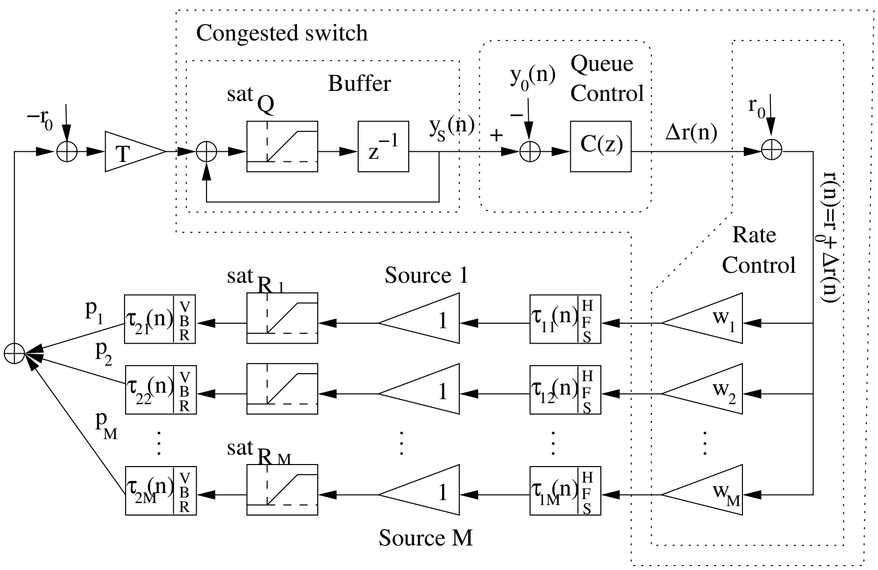

Fig. 5. System model for an ATM-ABR congestion control system [10], [11], [13], [15]

approximately equal share of the bandwidth (although the flows with shorter round-trip times will be favored). If a diffserv mechanism is implemented, different flows will receive different shares of the available bandwidth depending on their diffserv class. The weights $w_i(n)$ vary with time as sources connect or disconnect from the congested switch; however, they are piecewise constant, and their sum is equal to one at any one time instant:

$$\sum_{i=1}^{M} w_i(n) = 1. \tag{4}$$

The weights $w_i(n)$ change as new flows join or depart the congested server. It was shown [11], [13], [15], [18] that when the weights change, the system does not have an equilibrium point, and hence, there can be no question of system stability. Thus, an important assumption is that the weights are constant for relatively long periods of times (more than the time constant of the congestion control system).

The delays $\tau_{1,i}(n)$ and $\tau_{2,i}(n)$ correspond to the HFS and VBR models for the return paths and the forward path, respectively, where the index i denotes the source with $i = 1, ..., M$. We assume that the delays are bounded:

$$0 \;\; \leq \;\; \tau_{1,i}(n) \leq \bar{\tau}, \tag{5}$$

$$0 \quad \leq \quad \tau_{2,i}(n) \leq \bar{\tau}. \tag{6}$$

The constant rate r_0 is the depletion rate of the queue. For systems with flows congested elsewhere, r_0 is the capacity of the switch minus the throughput of those flows. $r(n)$ is the total desired rate as computed by the congested switch (more exactly equal to r_0 plus $\Delta_r(n)$ as computed by the controller $C(z)$). For the rate saturation nonlinearities, we can use a sector description for the rate saturation around a rate equilibrium point $q_{0,i}$:

$$sat_{R,i}(q_i) = R_i(n)(q_i - q_{0,i}) + q_{0,i}, \tag{7}$$

where $R_i(n) \in [R_{\min},\ 1]$. (This is not a linearization and models the full dynamic range of the nonlinearity [19]).

The model in Fig. 5 is fairly general and accurate considering the significant effects that can occur in congestion control systems. We will simplify it by representing the congested switch and the sources separately, as in Fig. 6.

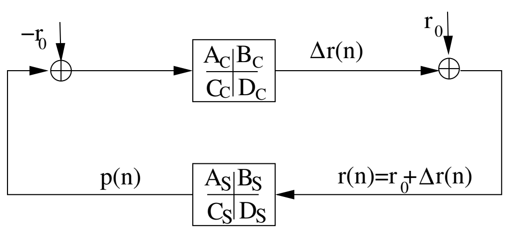

Fig. 6. Simplified model of the congestion control system in Fig. 5

Let us denote

$$p(n) = \sum_{i=1}^{M} p_i(n). \tag{8}$$

The sources can be represented in state space as:

$$x_s(n+1) \quad = \quad A_s(n)x_s(n) + B_s(n)r(n), \tag{9}$$

$$p(n) \quad = \quad C_s(n)x_s(n) + D_s(n)r(n). \tag{10}$$

We can represent the switch and rate controller in state space form:

$$x_c(n+1) \quad = \quad A_c(n)x_c(n) + B_c(n)\left(p(n) - r_0(n)\right) \tag{11}$$

$$\Delta r(n) \quad = \quad C_c(n)x_c(n) + D_c(n)\left(p(n) - r_0(n)\right), \tag{12}$$

where the state $x_c(n)$ includes both the queue of the switch as well as the state of the queue controller $C(z)$. The matrix $D_c(n) = 0$ regardless of the chosen controller (due to the presence of the queue in the switch).

For the sake of brevity in what follows, we will not explicitly show the dependence of $A_c, B_c, C_c, D_c, A_s, B_s, C_s, D_s, R_i$ and w_i on n. We can express the entire closed loop system in Fig. 6 in state space form, as follows:

$$x(n+1) = A(n)x(n) + B(n)r_0(n), \tag{13}$$

$$\Delta r(n) = C(n)x(n) + D(n)r_0(n), \tag{14}$$

where

$$A(n) = \begin{pmatrix} A_c + B_c D_s C_c & B_c C_s \\ B_s C_c & A_s \end{pmatrix}, \tag{15}$$

$$B(n) = \begin{pmatrix} B_c D_s - B_c \\ B_s \end{pmatrix}, \tag{16}$$

$$C(n) = \begin{pmatrix} C_c \\ 0 \end{pmatrix}, \tag{17}$$

$$D(n) = (0), \tag{18}$$

and the state $x(n)$ is

$$x(n) = \begin{pmatrix} x_c(n) \\ x_s(n) \end{pmatrix}. \tag{19}$$

The detailed model of the source for the single-source congestion control system is presented in Fig. 7.

For the single-source case, let us denote by a_s, b_s, c_s and d_s the matrices A_s, B_s, C_s and D_s from (9) and (10), respectively.

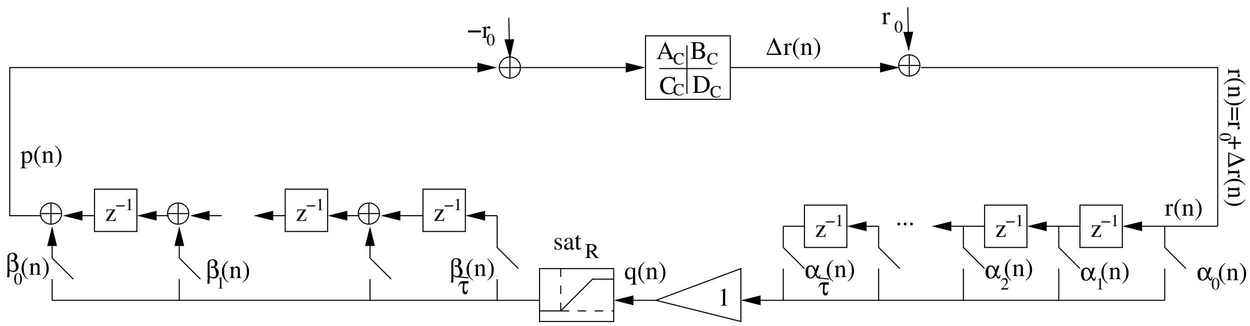

Fig. 7. Single-source congestion control system

$$a_s = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \dots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ R_1\beta_{\bar\tau}\alpha_1 & R_1\beta_{\bar\tau}\alpha_2 & R_1\beta_{\bar\tau}\alpha_3 & \dots & R_1\beta_{\bar\tau}\alpha_{\bar\tau-1} & R_1\beta_{\bar\tau}\alpha_{\bar\tau} & 0 & 0 & \dots & 0 & 0 \\ R_1\beta_{\bar\tau-1}\alpha_1 & R_1\beta_{\bar\tau-1}\alpha_2 & R_1\beta_{\bar\tau-1}\alpha_3 & \dots & R_1\beta_{\bar\tau-1}\alpha_{\bar\tau-1} & R_1\beta_{\bar\tau-1}\alpha_{\bar\tau} & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ R_1\beta_1\alpha_1 & R_1\beta_1\alpha_2 & R_1\beta_1\alpha_3 & \dots & R_1\beta_1\alpha_{\bar\tau-1} & R_1\beta_1\alpha_{\bar\tau} & 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \tag{20}$$

$$b_s = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ R_1\beta_{\bar\tau}\alpha_0 \\ \vdots \\ R_1\beta_1\alpha_0 \end{pmatrix}, \tag{21}$$

$$c_s = \begin{pmatrix} R_1\beta_0\alpha_1 & R_1\beta_0\alpha_2 & \dots & R_1\beta_0\alpha_{\bar\tau} & 0 & \dots & 0 & 1 \end{pmatrix}, \tag{22}$$

$$d_s = (R_1\beta_0\alpha_0). \tag{23}$$

For the case of multiple sources, we can represent the system in Fig. 5 as depicted in Fig. 8. Notice that the assumption of piecewise constant weights w_i allows us to interchange the constant weights and the unit delay blocks.

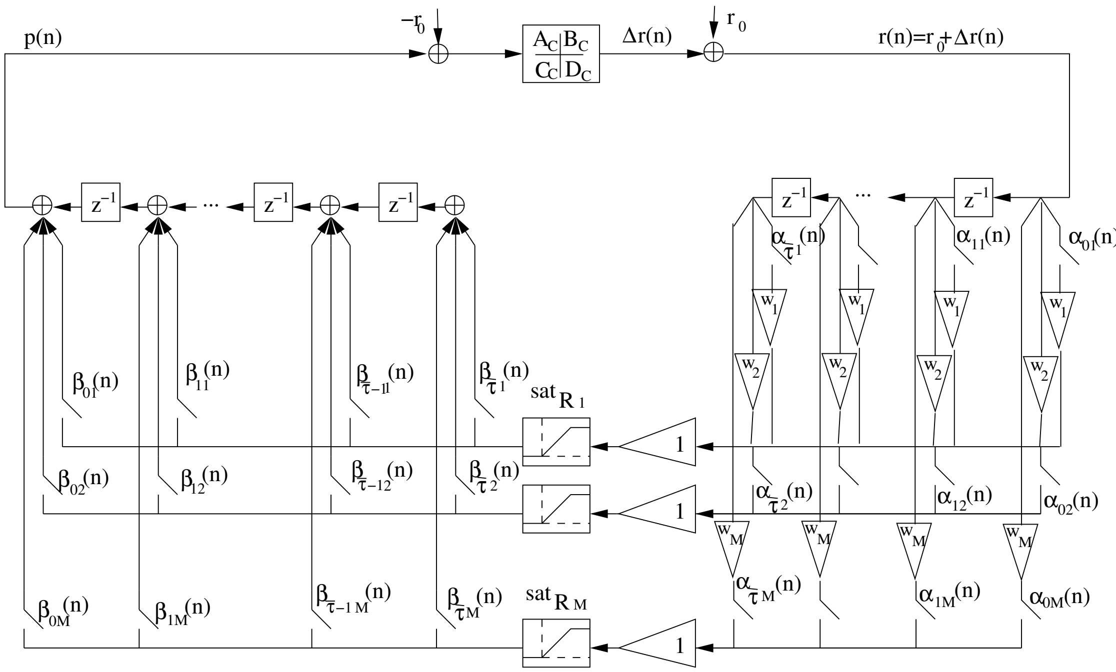

Fig. 8. More detailed model of the system in Fig. 5

III. Main Result

The main result of the paper is based on the following theorem [20], providing a necessary and sufficient condition for time-variant systems with polytopic uncertainties:

Theorem 1 [20]

The system

$$x(n+1) = F(n)x(n), \quad F(n) \in conv(F_1, \ldots, F_N), \tag{24}$$

is exponentially stable iff the system

$$\tilde{x}(n+1) = \tilde{F}(n)\tilde{x}(n), \quad \tilde{F}(n) \in \{F_1, \ldots, F_N\} \tag{25}$$

is exponentially stable, where by $conv(F_1, \ldots, F_N)$*, we denote the convex hull (convex matrix polyhedron) of the set of constant matrices* $\{F_1, \ldots, F_N\}$

In other words, the system (24) with $F(n)$ in the convex hull defined by the matrices $F_1, \ldots, F_N$ is stable iff the system with $\tilde{F}(n)$ that takes values only in the vertex matrices $F_1, \ldots, F_N$ is stable as well. The result is well known, and the proof is available in [20].

We are now in position to prove the main result of the paper.

Theorem 2

The system (13-14) with arbitrary number of sources M *is exponentially stable iff the system corresponding to the single source case (*$M = 1$*) is exponentially stable.*

Proof

The proof is based on Theorem 1. We will show that the matrices $A(n)$ corresponding to multiple-sources case are in the convex hull defined by the matrices of the single-source case.

For the single-source case, there are $(\bar{\tau}+1)^2$ possible instantaneous system matrices given by

$$\begin{aligned} a^{(j,k)}(n) &= \begin{pmatrix} A_c + B_c d_s^{(j,k)} C_c & B_c c_s^{(j,k)} \\ b_s^{(j,k)} C_c & a_s^{(j,k)} \end{pmatrix}, \\ j &= 0, \ldots, \bar{\tau}, \\ k &= 0, \ldots, \bar{\tau}, \end{aligned} \tag{26}$$

where $a_s^{(j,k)}, b_s^{(j,k)}, c_s^{(j,k)}$ and $d_s^{(j,k)}$ are given by (20-23). For example, $a_s^{(j,k)}$ corresponds to a_s in (20) with $\alpha_j = 1$ and $\beta_k = 1$, and $\alpha_\nu = 0, \forall \nu \neq j$, and $\beta_\nu = 0, \forall \nu \neq k$. $b_s^{(j,k)}, c_s^{(j,k)}$ and $d_s^{(j,k)}$ are defined similarly. (Note that the mappings

$$\begin{aligned} (j,k) &\rightarrow a_s^{(j,k)}, \\ (j,k) &\rightarrow b_s^{(j,k)}, \\ (j,k) &\rightarrow c_s^{(j,k)}, \\ (j,k) &\rightarrow d_s^{(j,k)} \end{aligned}$$

are not one to one!)

Let us consider M matrices $a^{(j_i,k_i)}$, $i = 1, \ldots M$ (not all of which have to be distinct) and formulate the convex combination of these matrices:

$$\begin{aligned} A(n) &= \sum_{i=1}^{M} w_i a^{(j_i,k_i)}(n), \\ (j_i,k_i) &\in \{0,1,\ldots,\bar{\tau}\}^2, \\ i &= 1,\ldots,M. \end{aligned} \tag{27}$$

Denote the rate sector corresponding to $a^{(j_i,k_i)}$ by $R_i(n)$ as in (7), i.e., $R_i(n) \in [R_{\min}, 1]$.

Using (26) and (4), (27) can be rewritten as:

$$A(n) = \begin{pmatrix} A_c + B_c\left(\sum_{i=1}^{M} w_i d_s^{(j_i,k_i)}\right) C_c & B_c\left(\sum_{i=1}^{M} w_i c_s^{(j_i,k_i)}\right) \\ \left(\sum_{i=1}^{M} w_i b_s^{(j_i,k_i)}\right) C_c & \sum_{i=1}^{M} w_i a_s^{(j_i,k_i)} \end{pmatrix}. \tag{28}$$

Equation (28) describes exactly the instantaneous system matrix $A(n)$ for the M source case and delay combinations $(j_i,k_i), i = 1,\ldots,M$, which is easily verified using, for example, Fig. 8.

By (27) the matrices $A(n)$ corresponding to the multiple-source case are, therefore, in the convex hull given by the instantaneous matrices $a^{(j,k)}(n)$ of the single-source case, i.e.,

$$A(n) \in conv\left(\left\{a^{(j,k)}(n) |\, (j,k) \in \{0,\ldots,\bar{\tau}\}^2\right\}\right). \tag{29}$$

In fact, for a given time instant $n = n_0$, the resulting multi-source matrix $A(n_0)$ is element of a convex polytope formed by maximally M vertex matrices taken from the entire set of $(\bar{\tau}+1)^2$ vertex matrices.

Therefore, by Theorem 1, the system (13-14) is exponentially stable iff the system corresponding to the single-source case is exponentially stable.

Comments:

- The result is *independent* of the actual number of sources as long as (4) holds, because the upper bound on the number of sources M can be made arbitrarily large.
- It was shown [11], [15] that if the delays in the forward path are time-varying, the congestion control system does not have an equilibrium point.
- The results may appear to be in contradiction with other results [21], [22] that show that, for time-invariant models, an unstable system with a single source with a large delay can be stabilized by the addition of another source with a small delay. However, such a model contradicts our assumption of time-variant, uncertain delays and the fact that all sources

have the same maximum delay $\bar{\tau}$ (5),(6). Therefore the time-invariant delay case is just a very special case in our model, like it is in a real network.

IV. EXAMPLE

In this section we will quantify the benefits of the main result of this paper. We will determine the computational complexity of a stability test for a system with M sources and of the same system with a single source.

It is shown that determining stability of a time-variant system is NP hard [20]. We will use the following theorem, a slight generalization of the results in [23], that provides a necessary and sufficient condition for time-variant systems with polytopic uncertainties:

Theorem 3 *The system,*

$$x(n+1) = A(n)x(n), \ \ A(n) \in conv(A_1, \ldots, A_N),$$

is exponentially stable iff there exists a sufficiently large integer k, such that

$$\|A_{i_1} \cdot \ldots \cdot A_{i_k}\| \leq \gamma < 1 \ \ \forall (i_1, \ldots, i_k) \in \{1, \ldots, N\}^k,$$

where $\| \cdot \|$ is any vector induced matrix norm.

Consider the ATM congestion control system depicted in Fig. 5. If no saturation nonlinearities are considered, the number of vertex matrices in the polytope is:

$$N_{M,\bar{\tau}} = (\bar{\tau}+1)^{2M} \, . \tag{30}$$

Obviously, the number of vertex matrices of the polytope for the single-source system is $N_{1,\bar{\tau}} = (\bar{\tau}+1)^2$.

The practical implementation of the stability test in Theorem 3 implies testing the norms of all the combinations $A_{i_1} \cdot \ldots \cdot A_{i_k}$. The number of combinations to be tested is

$$C_{M,\bar{\tau},k} = N_{M,\bar{\tau}}^k,$$

which limits the test usability for large values of k. (It is shown that the complexity of all existing stability tests increases exponentially with the number of vertex matrices [20].)

Consider a system with $M = 10$ sources, with a maximum delay in the forward and return paths of $\bar{\tau} = 5$. Assume that stability can be shown using Theorem 3 with $k = 5$. In this case the size of the polytope is $N_{10,5} = 3.65 \times 10^{15}$, and the number of vertex matrices combinations to be tested is $C_{10,5,4} \approx 6.5 \times 10^{77}$.

Using Theorem 2 the size of the polytope is reduced to $N_{1,5} = 36$, and the number of combinations to be tested becomes $C_{1,5,5} = 6 \times 10^7$, i.e., a factor of $(\bar{\tau}+1)^{2(M-1)} \approx 1.08 \times 10^{70}$ fewer matrix combinations have to be tested.

V. Conclusion

A detailed model of a class of congestion control systems is considered. For the considered systems, we prove a theorem that, simply put, states that, for computer congestion control systems with linear controllers, the stability of the system with a single source is equivalent to the stability of the system with multiple sources. The proof is based on a well-known result on stability of time-variant systems. The usefulness of the theoretical result is accentuated by the NP hard nature of stability tests for time-variant systems.

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