Abstract—Due to its usefulness and wide deployment, IEEE 802.11 has been the subject of numerous studies but still lacks a complete analytical model. Hidden terminals are common in IEEE 802.11 and cause the degradation of throughput. Despite the importance of the hidden terminal problem, there have been a relatively small number of studies that consider the effect of hidden terminals on IEEE 802.11 throughput, and many are not accurate for a wide range of conditions. In this paper, we present an accurate new analytical saturation throughput model for the infrastructure case of IEEE 802.11 in the presence of hidden terminals. Simulation results show that our model is accurate in a wide variety of cases.

I. INTRODUCTION

IEEE 802.11 is one of the most widely adopted standards for Wireless Local Area Network since its development by the IEEE LAN/MAC Standards Committee in 1996. Many amendments have been standardized and widely deployed in a number of devices such as personal computers, laptops, mobile phones, home networks and other electronic devices that benefit from wireless networking due to their advantages such as low cost, high throughput and convenience. The standard has three MAC algorithms: CSMA/CA, RTS/CTS and PCF. This study is limited to the widely implemented CSMA/CA and RTS/CTS.

Hidden terminals are common in IEEE 802.11 because the communication and carrier-sense ranges of the access point and clients vary significantly due to obstacles, interference, transmission power, antenna gain, and location. Despite work toward alleviating the effects of hidden terminals, either by deployment or node design [12], in real deployments with the standard protocol, there will be hidden terminal situations. Hidden terminals effectively disable the carrier sense capability of the protocol and negatively affect the performance of system. Figure 1 presents simulation results for CSMA/CA of IEEE 802.11a at the data rate of 12Mbps: normalized saturation throughput and sequence traces of frames successfully received by the access point for two scenarios with an access point and two stations that are either exposed\(^1\) or hidden from each other. In the scenarios, stations are in the communication range of an access point and always have packets to send to the access point. Figure 1 (a) shows that the saturation throughput severely deteriorates in the hidden station scenario as the frame size increases. Because the collision probability increases with the frame size in the hidden station scenario, the resulting saturation throughput decreases. Figure 1 (b) shows that the hidden terminal scenario is not fair on the short-term time scale. This short-term unfairness negatively affects upper layer protocols such as TCP timeouts and high jitter for real-time audio and video streams [16].

In this paper, we present a new analytical saturation throughput model for CSMA/CA and RTS/CTS of IEEE 802.11 for common infrastructure scenarios with hidden stations. We then evaluate the accuracy of the model through extensive simulations. The simulation result shows that our model is very accurate for a wide range of conditions.

II. PROBLEM DEFINITION AND RELATED WORK

In this section, we define the problem tackled in this paper and briefly summarize the related work.

A. Problem Definition

Bianchi [2] presents a novel Markov chain model for the saturation throughput for IEEE 802.11 in the absence of hidden terminals. This study has been considered as one of the seminal papers for the throughput model of IEEE 802.11, and has been extended in many subsequent studies [2]–[5], [7], [8], [14]–[16], [19], [20], [24], [25]. A few of them, [4], [7], [8] extend Bianchi’s model to consider hidden terminals. In this paper, we call the similar models in [4], [7], [8] extended Bianchi’s models (although other papers extended Bianchi’s model in other directions, we are only interested in hidden terminal extensions). However, Bianchi’s model cannot accurately model hidden terminals as we will shortly explain.

\(^1\)Traditionally, “exposed nodes” refer to nodes in a situation where they are prevented from transmitting simultaneously due to carrier sense, although their simultaneous transmissions would be successful [13]. In this paper we do not consider such topologies, and we refer to nodes that can sense the carrier of each other as exposed nodes.
In Bianchi’s Markov chain model [2], the MAC state is represented by two variables: the current retransmission stage, and the remaining back-off time. Through the model, Bianchi obtains the transmission probability, $\tau$, that the station transmits a packet in a randomly chosen slot time using the collision probability of a transmitted packet, $p$. Finally, it calculates the saturation throughput using $\tau$ and $p$. A key approximation that enables this model is the assumption that the probability of collision, $p$, is constant and independent of the retransmission stage. Bianchi assumes that $p$ is the probability that at least one of the $n-1$ remaining stations transmits in a time slot, where $n$ is the number of stations. This assumption is reasonable in the exposed terminal scenario because the collision can occur only during the contention phase in the absence of hidden terminals. The resulting probability $p$ is [2]:

$$p = 1 - (1 - \tau)^{n-1}. \quad (1)$$

Extended Bianchi’s models [4], [7], [8] use the same original Markov chain model of [2], but obtain $p$ while taking into account a number of hidden terminals. When a frame is transmitted, the frame can collide with the frames transmitted by hidden terminals at any time during a vulnerable period, $T_v$, while it can collide with the frames transmitted by exposed terminals only during the contention. For a fixed frame size $t_{frame}$, the resulting probability of collision is:

$$p = 1 - (1 - \tau)^{N_E}((1 - \tau)^{N_H} T_v), \quad (2)$$

where $N_E$ is the number of exposed stations, $N_H$ is the number of hidden stations, and $\sigma$ is slot time. The vulnerable period, $T_v$, is the time period when another station can start a transmission that would collide with the current transmission. This time period spans the interval $(-t_{frame}, t_{frame} + SIFS + \delta)$ with respect to the start of the frame, i.e.:

$$T_v = 2t_{frame} + SIFS + \delta, \quad (3)$$

where $t_{frame}$ is the transmission time for the frame, and $\delta$ is the propagation delay. These models also assume that $p$ is constant regardless of the retransmission stage.

Figure 2 presents the normalized saturation throughput of CSMA/CA as a function of the frame size derived by the simulation and the extended Bianchi’s model when the number of stations are two and ten respectively, where IEEE802.11a is considered and the channel rate is 24Mbps. The figure shows that the extended Bianchi’s model is inaccurate.

There are several reasons for the inaccuracy of the extended Bianchi’s model. A key assumption when extending Bianchi’s model is that the state of the two stations is decoupled. As shown by the short-term unfairness in Fig. 1(b), that is not true in reality. Furthermore, Fig. 3 shows that the conditional
collision probability is not constant but rather increases with the retransmission stage in scenarios of Fig. 2. The variation of the conditional collision probability is especially severe when the number of stations is small.

In this paper, we present an accurate new Markov chain model reflecting the variation of the conditional collision probability as a function of the retransmission stage for the saturation throughput for IEEE 802.11 in the presence of hidden terminals for the general infrastructure cases. The new model takes into account the interactions between the two stations by jointly modeling the backoff stage of each of the two stations.

B. Related Work

Work in [4], [8], [20], [25] provides throughput models for IEEE 802.11 in the presence of hidden terminals. Work in [4] consider the infrastructure case, while [8], [20], [25] focus on the ad-hoc case. Ekici et al. [4] extend Malone’s model [19] to analyze IEEE 802.11 throughput without saturation for the infrastructure case with hidden nodes. This model assumes that the collision probability is constant regardless of the retransmission stage. Although the simulation result shows that the model is accurate in a scenario with two hidden stations, the model is accurate only when the offered load is small. As the load increases, the results of the model become inaccurate as shown in Fig. 2.

Work in [8], [20], [25] presents analytical models for multi-hop and ad-hoc networks. Hou et al. [8] analyze the throughput of the IEEE802.11 DCF scheme using the RTS/CTS access mechanism in multi-hop ad-hoc networks. The simulation results show that the model is accurate; however, if this model is applied to CSMA/CA, it becomes inaccurate, as it does not consider the retransmission stage for obtaining the collision probability. Work in [25] presents an analytical model for deriving saturation throughput in multi-hop ad hoc networks with nodes randomly placed according to a two dimensional Poisson distribution. In [20] the authors show that in saturated multihop networks hidden terminals result in high packet-loss rate, routing instability and unfairness problems. The authors shows that controlling the offered load at the sources can eliminate these problems. They also provide an analysis to estimate the optimal offered load that maximize the throughput of a multi-hop traffic flow. The paper provides both simulation results and experimental results with a real six-node multi-hop network.

The authors of [6] consider the twelve distinct cases of two contending flows, showing that the hidden terminal problem is equivalent to one of the symmetric cases with incomplete state (SIS). The authors also show that in this case severe short-term (on the order of seconds) unfairness may occur. The authors use an generalized Bianchi model to capture the behavior in some of those cases.

In [17] the authors clearly point to the causes of short-term unfairness in scenarios with hidden terminals, and, furthermore, quantify the unfairness by using a Markov chain model for the state of the system. The authors use a three variable state for the Markov chain with the retransmission stages of the two hidden stations as two of the variables and the state of the medium (e.g., idle, collision) as the third variable. They carefully compute the transition probabilities from each of these states and use them to quantify the short term unfairness present in these hidden terminal situations.

In [22] the authors extend the results of [2] by incorporating different transition probabilities in the two-dimensional Markov chain from [2] as a function of not only retransmission stage. The model in [22] employs the same states as the model in [2] (i.e., backoff state of a station), but the authors update the transition probabilities as a function of the positions of the interfering nodes with respect to the transmitting nodes (15 different cases in all) and taking into account the probabilities of occurrence of each of the cases. In contrast, the model proposed in this paper uses the retransmission stage of each station as system state. The authors further develop the model in [21] by noting the drawbacks of using Bianchi’s model for a hidden terminal situation, specifically the ambiguity of a variable-time slot when stations have incomplete knowledge of the channel state and solve it by adopting a fixed-size slot that is modeled by introducing additional states in Bianchi’s original model.

A similar approach employing fixed time slots and additional states in the Markov chain was used in [26]. In the same vein [9] use a discrete time slot for a relatively simple, but effective model employing a Markov chain that only models the state of the medium with four states (with nodes in region A active or not and nodes in region B active or not).

In [27] the authors developed an analytical model for the packet loss, delay and delay jitter for unsaturated networks and validated the model against simulations for low offered loads.

The work in [23] focuses on IEEE 802.11 networks with base stations and multiple terminals hidden from each other in a saturated environment. The state of proposed model is represented by the number of stations active, and the
in the saturation throughput. We assume that the payload size presented in Fig. 4. We assume that stations always have infrastructure scenarios in the presence of hidden terminals using a Markov chain model for IEEE 802.11 DCF for general

Fig. 4 (a) in case that both

infrastructure scenarios in the presence of hidden terminals

stations neatly separated into two groups

hidden from each other. and (b) Example of general infrastructure scenario, where there are 5 stations in the communication range of the access point, and the stations may irregularly be exposed or hidden from each other.

authors carefully compute the probabilities that each station can switch state from active to inactive. For computing the transition probabilities the authors consider each possible case of collision (destructive or not), success or idle slots, and compute the probability of occurrence of each of these events. In contrast, we initially focus on the (simpler) case with only two hidden station and use the retransmission stage of each station as system state.

III. Analytical Model

In this section, we present saturation throughput models using a Markov chain model for IEEE 802.11 DCF for general infrastructure scenarios in the presence of hidden terminals presented in Fig. 4. We assume that stations always have frames to transmit to the access point, as we are interested in the saturation throughput. We assume that the payload size is fixed. The access point does not transmit any data frame and only receives the frames from the stations and transmits ACKs to the stations. In Sec. III-A and III-B, we present a saturation throughput model for the infrastructure scenario in Fig. 4 (a) in case that both \( n_a \) and \( n_b \) are one. In Sec. III-C, we extend the model presented in Sec. III-B for the infrastructure scenario having many stations neatly separated into two groups as shown in Fig. 4 (a). Since the access point acts like a regular station when transmitting, this case also handles the case when the AP is transmitting data to the clients. Finally, in Sec. III-D, we model the saturation throughput of the general infrastructure scenarios with a mixture of hidden and exposed stations in Fig. 4 (b) using the model of Sec. III-C.

A. Simple Model for the IEEE 802.11 Network with Two Hidden Terminals

In this simple model, we consider two hidden terminal scenario, in which there are an access point and two stations, \( A \) and \( B \). The stations are exposed to the access point but they are hidden from each other. As a simplification, in this section we ignore the fact that the result of previous transmissions can affect the next transmissions in IEEE 802.11 to simplify the problem. Specifically, we make two simplifying assumptions that will be removed in Sec. III-B:

- we assume that when a successful transmission occurs, all stations choose new random back-off times instead of halting and restarting the previous back-off time in the next transmission attempt;
- we assume that when a collision occurs, all stations start their next transmission attempts at the same time, i.e., after the station that later transmits the lost frame ends its ACK timeout.

1) System State Model: Our system model is based on a two dimensional Markov chain as shown in Fig. 5. Although at the first glance this may look similar to Bianchi’s model [2], the two are fundamentally different: while Bianchi’s model represents the state of a node, with retransmission stage and back-off counters defining the state, our model represents the state of the entire system, by using the retransmission stage of each station as system state. The system changes state when the stations change their retransmission stage, as a result of either a successful transmission or a collision, i.e., after receiving an acknowledgment, or after an acknowledgment timeout.

In this section, we define and then compute the system state probability matrix \( X \). The elements of the probability matrix \( x_{ij} \) represent the probability that station \( A \) is in the retransmission stage \( i \) and station \( B \) is in the retransmission stage \( j \) for \( i, j \in \{0, m - 1\} \), where \( m \) is the maximum retransmission limit. The retransmission stages, \( i \) and \( j \), start from 0 at the first transmission and are increased by one every time a transmission results in a collision, up to \( m - 1 \). We construct the Markov chain model of the system state by using system states and transition probabilities between the states. By imposing the normalization condition to the system state model, we find the stationary system state probability \( X \).

Whenever a transmission occurs, the system can move from state \((i, j)\) to one of the following three states:

\[
(i, j) \rightarrow \begin{cases} 
(0, j), & i, j \in \{0, m - 1\}, \\
(i, 0), & i, j \in \{0, m - 1\}, \\
(i', j'), & i' = (i + 1) \mod m, \quad j' = (j + 1) \mod m.
\end{cases}
\] (4)

The first and second transitions represent the cases when successful transmissions occur. The first transition represents the case when station \( A \) transmits a frame successfully. Once a
station (e.g., A) transmits a frame successfully, the retransmission stage of the station goes back to zero for the next frame. This is the case when the sum of the backoff of station A and its vulnerable period $T_v$ is smaller than the backoff of station B. In this case B’s retransmission stage does not change. The vulnerable period, $T_v$ is:

$$T_v = \begin{cases} \frac{t_{frame} + \delta}{\sigma}, & \text{(if CSMA/CA is used)} \\ \frac{t_{RTS}}{\sigma}, & \text{(if RTS/CTS is used)} \end{cases}$$

(5)

where $t_{frame}$ is the transmission time for data frame, $t_{RTS}$ is the transmission time for RTS, $\delta$ is the propagation delay and $\sigma$ is the slot time. The second transition in (4) accounts for the case when station B transmits a frame successfully. The third transition in (4) represents the case when a collision occurs. A collision occurs when the difference between the backoffs of the hidden stations is smaller than $T_v$. When a collision occurs, the retransmission stages of both stations are incremented by one up to $m-1$. Once the retransmission stage reaches $m-1$ and a collision occurs, the retransmission stage is reset to zero for the transmission of the next frame. The Markov chain model for the system state with these transitions is shown in Fig. 5, in which the contention window size of the station increases exponentially from $CW_{min}$ to $CW_{max}$ as the retransmission stage increases. Therefore, when the system is in the state $(i, j)$, the contention window sizes of stations are:

$$CW_a = \min\{CW_{max}, (CW_{min} + 1)2^i - 1\},$$

$$CW_b = \min\{CW_{max}, (CW_{min} + 1)2^j - 1\},$$

(6)

where $CW_a$ is the contention window size of station A, and $CW_b$ is the contention window size of station B.

Based on this model, we can find the transition probabilities for the system states. Let $T$ denotes the transition probability matrix. Denote with $t_{ijkl}$, the transition probability that the system state moves from $(i, j)$ to $(k, l)$. Each state has three effective transition probabilities (i.e., $t_{ij0j}, t_{ij0i}$ and $t_{ijj'j'}$) (all other transition probabilities are zero). We first find the transition probability for a successful transmission (i.e., $t_{ij0j}$ and $t_{ij0i}$) and then that for a collision (i.e., $t_{ijj'j'}$).

The transition probability for a successful transmission will be computed first. First, let us calculate $t_{ij0j}$, the transition probability that station A transmits a frame successfully. The size of sample space is the product of $CW_a$ and $CW_b$. Station A can win the contention only when the back-off of station B is larger than the back-off of A by at least $T_v$; thus $t_{ij0j}$ is:

$$t_{ij0j} = \frac{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} CW_a CW_b}{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} 1} \;,$$

(7)

if the back-off of station A is larger than $CW_b - T_v$, A cannot win. Hence,

$$L_a = \min\{CW_a, CW_b - T_v\}.$$  

(8)

Similarly, $t_{ij0i}$, the transition probability that station B transmits a frame successfully is:

$$t_{ij0i} = \frac{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} CW_a CW_b}{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} 1},$$

(9)

where,

$$L_b = \min\{CW_b, CW_a - T_v\}.$$  

(10)

Due to the normalization condition of the transition probability, the transition probability of the collision, $t_{ijj'j'}$ is:

$$t_{ijj'j'} = 1 - t_{ij0j} - t_{ij0i}.$$  

(11)

By imposing the normalization condition of $X$, $\sum_{i=0}^{m-1} \sum_{j=0}^{m-1} x_{ij} = 1$, and using $X = TX$, we can compute the system state probability matrix $X$.

2) Time Model: We need to determine the time spent in each state in Fig. 5: the average back-off time matrix, $O$, the time for transition matrix, $E$, and the time for payload transmission matrix, $D$. When $t_{ijkl}$ is zero, the corresponding elements of the time matrices should also be zero; therefore we need to only take care of the elements of the time matrices corresponding to non-zero transitions in (4). Let $O$ denote the average back-off time matrix corresponding to each transition in the matrix $T$. Denote with $o_{ijkl}$, the average back-off time when the system moves from $(i, j)$ to $(k, l)$. First, let us calculate the average back-off time of the successful transmissions, $o_{ij0j}$ and $o_{ij0i}$. The total number of cases of back-off times for each transition is the same as the denominator of (8). The back-off time in each case is that of the station that transmits a frame successfully. This yields:

$$o_{ij0j} = \frac{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} \frac{CW_a}{\sigma}}{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} 1},$$

$$o_{ij0i} = \frac{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} \frac{CW_b}{\sigma}}{\sum_{b_a=0}^{L_a} \sum_{b_b=b_a+T_v}^{L_b} 1}.$$  

(12)

We have to calculate the average back-off time corresponding to a collision, $o_{ijj'j'}$. The total number of collisions is the difference between total number of cases and the number of cases of successful transmissions. A collision occurs when the difference of back-off times of stations is smaller than $T_v$. In each case, the maximum value between the back-off times of the stations should be considered when considering collisions by the second assumption (when a collision occurs,
Let $D$ denote the payload transmission time matrix. Denote with $d_{ijkl}$, the time to send the payload when in state $(i, j)$ and transitioning in state $(k, l)$. The payload is the data portion of the frame. When a collision occurs, $d_{ijkl'}$ is zero. The payload time for successful transmissions, $d_{ij0j}$, is the payload transmission time as it is. This yields:

$$d_{ij0j} = d_{ij0} = t_{\text{payload}}.$$  

3) Saturation Throughput Model: Let $S$ denote the normalized saturation system throughput, defined as the fraction of time the channel is used to successfully transmit payload bits. $X$ represents normalized time for each state and $T$ is the transition probability. The normalized saturation system throughput is equal to the total time spent in the payload transmission the payload transferred divided by the total time spent by the system. Thus:

$$S = \frac{\sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \sum_{k=0}^{m-1} \sum_{l=0}^{m-1} x_{ij}d_{ijkl}t_{ijkl}^s}{\sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \sum_{k=0}^{m-1} \sum_{l=0}^{m-1} (x_{ij}e_{ijkl}^c t_{ijkl} + x_{ij}e_{ijkl}^e t_{ijkl}')}$$

where $t_{ijkl}^s$ is the transition probability for a successful transmission, $e_{ijkl}^c$ is the time spent in successful transmission, $t_{ijkl}^e$ is the transition probability for a collision and $t_{ijkl}^e$ is the time spent in a collision.

B. An Enhanced Model for the IEEE 802.11 Network with Two Hidden Terminals

This model enhances the simple model in Sec. III-A to better match the behavior of IEEE 802.11. The enhanced model operates under the same assumptions presented in Section III, but removes the two additional assumptions introduced in Section III-A. The enhanced model accounts for the fact that in IEEE 802.11 the results of the previous transmission affect those of the next transmission. Specifically:

- when a successful transmission occurs, the hidden station that did not transmit, overhears the ACK frame, it halts its countdown and then resumes it with the remaining back-off time in the next transmission attempt. That is, the back-off time of the hidden station reduces as much as sum of $T_v$ and the winner’s back-off time after a successful transmission.
- when a collision occurs, the starting time instances of the next transmission attempts of the stations that collided (when they are hidden from each other) are different.

1) System State Model: In developing the extended model we recognize that the system model is not a Markov system, as the transition probabilities from the current state to the next state are also a function of the past states; however, the extended model we use is a Markov chain where the enhanced transition probabilities will also take into account the past transition probabilities will also take into account the past

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all stations start their next transmission attempts at the same time right after the station transmitting the lost frame later ends its timeout for ACK. Thus:

$$\alpha_{ijkl'} = \frac{DIFS + o_{ij0j} + t_{\text{frame}} + \text{SIFS} + \delta + t_{\text{ack}} + \delta,}{(\sum_{b_a=0}^{C_{W_a}} \sum_{b_b=0}^{C_{W_b}} + b_a + \sum_{b_a=0}^{C_{W_a}} \sum_{b_b=0}^{C_{W_b}} + b_a + T_v b_a T_a)\sigma}$$

Let $E$ denote the time matrix representing time spent in each transition in the matrix $T$. Denote with $e_{ijkl}$ the time the system spends in state $(i, j)$ given that it will transition in state $(k, l)$. The time spent in state $(i, j)$ for a successful transmissions, $e_{ij0j}$ is as shown in Fig. 6:

$$e_{ij0j} = \begin{cases} 
DIFS + o_{ij0j} + t_{\text{frame}} + \text{SIFS} + \delta + t_{\text{ack}} + \delta, \\
(\text{if CSMA/CA is used}) \\
DIFS + o_{ij0j} + t_{\text{rts}} + \text{SIFS} + \delta + t_{\text{cts}} + \text{SIFS} + \delta + t_{\text{frame}} + \text{SIFS} + \delta + t_{\text{ack}} + \delta, \\
(\text{if RTS/CTS is used}) 
\end{cases}$$

when a successful transmission occurs, the hidden station that did not transmit, overhears the ACK frame, it halts its countdown and then resumes it with the remaining back-off time in the next transmission attempt. That is, the back-off time of the hidden station reduces as much as sum of $T_v$ and the winner’s back-off time after a successful transmission.

When a collision occurs, the station waits for the timeout instead of receiving the ACK or CTS. The time for a collision, $e_{ijj'}$ is as shown in Fig. 6:

$$e_{ijj'} = \begin{cases} 
DIFS + o_{ijj'} + t_{\text{frame}} + \delta + t_{\text{timeout}}, \\
(\text{if CSMA/CA is used}) \\
DIFS + o_{ijj'} + t_{\text{rts}} + \delta + t_{\text{timeout}}, \\
(\text{if RTS/CTS is used}) 
\end{cases}$$

where $t_{\text{timeout}}$ is the expected time for receiving ACK or CTS packet. This yields:

$$t_{\text{timeout}} = \begin{cases} 
\text{SIFS} + t_{\text{ack}} + \delta, \\
(\text{if CSMA/CA is used}) \\
\text{SIFS} + t_{\text{cts}} + \delta. \\
(\text{if RTS/CTS is used}) 
\end{cases}$$

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<td>Time for collision for CSMA/CA (i.e., $e_{ij0}$)</td>
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result of previous transition. To obtain $t'_{ijkl}$, we consider two things. First, we consider the probability that the system state $(i, j)$ occurred from a transition from the system state $(p, q)$. We denote this probability with $P_{pqij}$. Thus:

$$P_{pqij} = \frac{t_{pqij}x_{pq}}{\sum_{p=0}^{m-1} \sum_{q=0}^{m-1} t_{pqij}x_{pq}}.$$  

(19)

Second, we consider the adjustment of the next transition probability by taking into account the previous transition (either a successful transmission or a collision). We denote this adjustment with $t''_{ijkl}$. These yield:

$$t'_{ijkl} = \sum_{p=0}^{m-1} \sum_{q=0}^{m-1} P_{pqij}t''_{ijkl}.$$  

(20)

Thus we can obtain the transition probability for successful transmissions (i.e., $t'_{ij0j}$ and $t'_{ij0i}$). The transition probability for collision, $t'_{ij'}$, is simply obtained by the normalization condition:

$$t'_{ij'Ij'} = 1 - t'_{ij0j} - t'_{ij0i}.$$  

(21)

The adjustment in the transition probability, $t''_{ijkl}$, depends on whether the previous transition was a successful transition or a collision. We first focus on how the next transition probability (i.e., $t''_{ijkl}$) should be adjusted when the previous transition was a successful transmission. Figure 7 shows the cases when a successful transmission occurred during the previous transition, where station B transmitted a frame successfully. Station A performs the back-off countdown, overhears the ACK frame and it halts its back-off countdown; it then resumes the remaining back-off countdown in the next transmission attempt; therefore the back-off time of the station that did not transmit a frame (i.e., $B$) reduces on average by $T_v$ from (5) and the average back-off time of the station that did transmit a frame is chosen from a smaller contention window:

$$t''_{ijkl} = \left\{ \begin{array}{ll}
 t_{ijkl}(CW_a, CW_b - n_rCW_r), & \text{(if } A \text{ transmitted a frame successfully)} \\
 t_{ijkl}(CW_a - n_rCW_r, CW_b), & \text{(if } B \text{ transmitted a frame successfully)}
\end{array} \right.$$  

(22)

where the expression $t_{ijkl}(a, b)$ represents the value of $t_{ijkl}$ as in (7), (9) and (11) with the contention window size of station $A$ being $a$ and that of station $B$ being $b$, $n_r$ is the average number of times of the contention window size to be reduced and $CW_r$ is the size of the reduction of the contention window after each successful transmission by the other station. $CW_r$ is determined, as shown in Fig. 7:

$$CW_r = \frac{\alpha_{pqij}}{\sigma} + T_v,$$  

(23)

The number of reductions in the contention window is random, depending on the value of the backoff chosen by the station that does not transmit; on the average, the expected number of reductions in the contention window, $n_r$, is half the number of times the reduction in the contention window fits in the contention window of the station that does not transmit:

$$n_r = \frac{l - 1}{2},$$  

(24)

where $l$ is:

$$l = \begin{cases}
 \frac{CW_a}{CW_r} & \text{if } A \text{ transmitted a frame successfully,} \\
 \frac{CW_b}{CW_r} & \text{if } B \text{ transmitted a frame successfully.}
\end{cases}$$  

(25)

The transition probability $t''_{ijkl}$ in (22) captures the probability of a chain of transitions where one station successfully sends a series of one or more packets during the backoff of the other station. During all these transitions except possibly the first one, the station sending packets is in backoff stage zero as it is successful in its transmission. For example, if the successful station is $B$ while $A$ is in backoff stage $\alpha$, the transitions modeled in (22) are $(\alpha, q) - (\alpha, 0) - ... - (\alpha, 0) - (k, 1)$. On the average there are $n_r - 1$ $(\alpha, 0)$ states between the first and last state in the sequence. Strictly speaking, in the sequence of state transitions above, only during the first state transition the value of the contention window of $A$ is decreased by $CW_r$ given in (23), while for the other transitions, the value of $CW_r$ for $q = 0$ should be used. However, due to the short term unfairness (illustrated in Fig. 1), $q = 0$ is the common case, and the approximation holds well. The previous transition was a successful transmission if $i \neq (p+1) \mod m$ or $j \neq (q+1) \mod m$. The retransmission stage of the station that transmits a frame successfully is reset to 0, thus $i=0$ means that $A$ transmitted a frame successfully, while $j=0$ means that $B$ transmitted a frame successfully. Thus, if $(i \neq (p+1) \mod m)$ or $(j \neq (q+1) \mod m)$ and $i(0)$, then $A$ transmitted a frame successfully. Similarly, if$(i \neq (p+1) \mod m)$ or $(j \neq (q+1) \mod m)$ and $j(0)$, then $B$ transmitted a frame successfully.

We now show how the next transition probability (i.e., $t''_{ijkl}$) should be adjusted when the previous transition was a
collision. Figure 8 shows the case when a collision occurred in the previous transition. The previous transition was a collision if \( i = ((p+1) \mod m) \) and \( j = ((q+1) \mod m) \).

Let us consider the transition probability (i.e., \( t'_{pqij} \)) that station A transmits a frame successfully in the next transmission attempt. First, let us consider the case when station A transmitted a frame faster than station B in the previous transmission as shown in Fig. 8. If station A chooses a back-off time smaller than \( T_a \) (\( 0 \leq b_a < T_a \)) in the next transmission, A cannot avoid a collision with the previous transmission of station B. The average difference between the back-off time and the difference of two contending stations when a collision occurs is\( \frac{T_a}{2} \). The average difference between the back-off times is a random variable equal to the difference of the minimum back-off time of the two back-off times of \( \sigma \) station A during the previous transmission minus the number of time slots taken by the timeout and DIFS. Under the assumption of a collision in the previous state, \( T_a \) is uniformly distributed, its average is:

\[
T_a = \max \{ 0, \frac{T_a}{2} - \frac{t_{\text{timeout}} + \text{DIFS}}{\sigma} \}. \quad (26)
\]

Therefore, \( 0 \leq b_a < T_a \) should be excluded in the calculation of the successful transition probability.

If station A chooses a back-off time such that \( T_a \leq b_a \leq CW_a \), in order for station A to transmit a frame successfully, station B should choose a back-off time larger than station A by at least \( T_a \): therefore the number of events when station A transmits a frame successfully is:

\[
E_{AA} = \sum_{T_a=0}^{T_{\text{max}}} \sum_{b_a=T_a}^{CW_a} \sum_{b_b=0}^{CW_b} 1, \quad (27)
\]

where \( T_{\text{max}} = T_a - t_{\text{timeout}} + \text{DIFS} \).

Second, let us consider the case when station B transmitted a frame faster than station A in the previous transmission (and collided) but A was successful in the next transmission. In the next transmission station B should choose a larger back-off time than station A by at least \( T_a + T_v \) in order for station A to transmit a frame successfully because of the assumption that the back-off of station B starts earlier than the back-off of station A by \( T_v \); therefore the number of events leading to a successful transmission by station A is:

\[
E_{AB} = \sum_{T_a=0}^{T_{\text{max}}} \sum_{b_a=0}^{CW_a} \sum_{b_b=b_a+T_a+T_v}^{CW_b} 1. \quad (28)
\]

We assume that the probability that A transmitted a frame faster than B in the previous transmission is equal to the probability that B transmitted faster than A; therefore the sample space of the all the cases leading to a successful transmission by A is \( 2T_{\text{max}}CW_aCW_b \). Thus \( t'_{pqij} \), the adjusted transition probability that station A transmits a frame successfully in the next transition, is:

\[
t'_{pqij} = \frac{E_{AA} + E_{AB}}{2T_{\text{max}}CW_aCW_b}. \quad (29)
\]

where \( i = ((p+1) \mod m) \) and \( j = ((q+1) \mod m) \).

In a similar way, \( t'_{ij0j} \), the adjusted transition probability that station B transmits a frame successfully in the next transition, is:

\[
t'_{ij0j} = \frac{E_{BA} + E_{BB}}{2T_{\text{max}}CW_aCW_b}. \quad (30)
\]

where \( E_{BA} \) and \( E_{BB} \) are:

\[
\begin{align*}
E_{BA} &= \sum_{T_a=0}^{T_{\text{max}}} \sum_{b_a=0}^{CW_a} \sum_{b_b=0}^{CW_b} \sum_{t_a=0}^{T_a-1} \sum_{t_b=0}^{T_b-1} b_{ij} \times 1, \\
E_{BB} &= \sum_{T_a=0}^{T_{\text{max}}} \sum_{b_a=0}^{CW_a} \sum_{b_b=0}^{CW_b} \sum_{t_a=0}^{T_a-1} \sum_{t_b=0}^{T_b-1} b_{ij} \times 1. 
\end{align*}
\quad (31)
\]

We can calculate \( t'_{ij0j} \) and \( t'_{ij0j} \) using (20), (22), (29) and (30) and \( t'_{ij0j} \) using (21). In result, we obtain the adjusted transition probability matrix, \( T' \), and by imposing the normalization condition, \( \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} b'_{ij} = 1 \), and using \( X' = T'X' \), we can obtain an enhanced stationary probability matrix \( X' \).

2) Time Model: The time for transition matrix, \( E \), and the time for payload matrix, \( D \), are similar to those of Sec. III-A (for matrix \( E \) the adjusted matrix \( O \) that is computed below will be used instead of the original one). The average back-off time matrix, \( O \) is also similar to that of Sec. III-A except the average back-off time corresponding to a collision, \( o_{ij0j} \).

In this model, we consider that the next transmission starts at the time when the station that transmitted a frame faster in the previous transmission ends (e.g., station A in Fig. 8) the time-out for ACK in the previous transmission as in Fig. 8; therefore the back-off time corresponding to a collision is the minimum back-off time of the two back-off times of the contending stations. Thus \( o_{ij0j} \) is:

\[
o_{ij0j} = \frac{1}{(\sum_{b_a=0}^{CW_a} \sum_{b_b=0}^{CW_b} b_{ij} = b_{ij} \times 1)} \times (\sum_{b_a=0}^{CW_a} \sum_{b_b=0}^{CW_b} b_{ij} = b_{ij} \times 1) \times (\sum_{b_a=0}^{CW_a} \sum_{b_b=0}^{CW_b} b_{ij} = b_{ij} \times 1). \quad (32)
\]
only two types of contending nodes: nodes that are exposed to itself and nodes that are hidden. As long as the models for the \(n_a\)-to-\(n_b\) case estimates the correct state (backoff, throughput, etc.) state for each station, the aggregate throughput should be reasonably accurately predicted by this extension.

As we will show in the results section, this extension works reasonably well even for relatively complex systems. The reasons for the relative accuracy of this approximation are well explained in [18]; in [18] the authors show that, as an approximation ("back of the envelope"), the throughput of an 802.11 node in a complex topology is primarily influenced by its direct neighbors and is largely independent of distant neighbors. From this perspective, the model in this section extends the simple model in [18] by taking into account the direct neighbors and their hidden neighbors.

For example, the scenario in Fig. 4 (b) consists of two stations in 4-to-1 scenario (B and C) and three stations in 3-to-2 scenario (A, D and E). We can calculate the normalized saturation throughput of the general hidden terminal scenario by calculating the weighted average of the normalized throughput of each \(n_a\)-to-\(n_b\) scenario using our model in Sec. III-C. Thus:

\[
S_g = \frac{\sum_{k=1}^{l} n_k S(n_{ak}, n_{bk})}{n}
\]

where \(S_g\) is the normalized saturation throughput of the general infrastructure scenario with hidden stations, \(n\) is the number of stations in the scenario, \(l\) is the number of different \(n_a\)-to-\(n_b\) scenarios in the general scenario, \(n_k\) is the number of stations in the \(k^{th}\) \(n_a\)-to-\(n_b\) scenario, and \(S(n_{ak}, n_{bk})\) is the normalized saturation throughput of the \(k^{th}\) \(n_a\)-to-\(n_b\) scenario. For example, for the topology in Fig. 4 (b):

\[
S_g = \frac{3S(4,1)+2S(3,2)}{5}
\]

E. IEEE 802.11a Physical Layer Model

The 802.11a standard operates in 5 GHz band and uses Orthogonal Frequency-Division Multiplexing (OFDM) modulation. It provides eight payload data rates (i.e., 6, 9, 12, 18, 24, 36, 48, 54Mbps) with different modulation schemes (i.e., BPSK, QPSK, 16-QAM and 64-QAM) and coding rates. We show how to obtain the transmission times of frame, payload, RTS, CTS and ACK for IEEE 802.11a. Refer to the IEEE 802.11a standard [11] for a more complete presentation.

In IEEE 802.11a standard, the MAC data frame (i.e., MAC Protocol Data Unit (MPDU)) consists of the MAC Header, payload (0 to 2312 bytes) and Frame Check Sequence (FCS). The MAC header and FCS together are 28 bytes, the RTS MPDU is 20 bytes, and the CTS and ACK MPDUs are 14 bytes long. During transmission, a PLCP preamble and a PLCP header are added to the MAC frame to create the physical layer frame (e.g., PLCP Protocol Data Unit (PPDU)). The PLCP preamble field, is composed of 10 repetitions of a short training sequence (0.8 \(\mu\)s) and two repetitions of a long training sequence (4 \(\mu\)s). The PLCP header except the SERVICE field constitutes a single OFDM symbol (4 \(\mu\)s). The 16-bit SERVICE field of the PLCP header and the MAC frame (along with six tail bits and pad bits) are transmitted at the data
rate specified in the RATE field. Therefore the transmission times of data frame, RTS, CTS and ACK is:

\[
\begin{align*}
t_{data} &= t_{PLCP\text{-preamble}} + t_{PLCP\text{-header}} + t_{SERVICE\text{-field}} \\
&\quad + t_{tail} + t_{MAC\text{-header}} + t_{FCS} + t_{payload}, \\
&= 16\mu s + 4\mu s + \frac{(16+6)+28+8+payload\text{-bit}}{data\text{-rate}(m)}, \\
t_{rts} &= t_{PLCP\text{-preamble}} + t_{PLCP\text{-header}} + t_{SERVICE\text{-field}} \\
&\quad + t_{tail} + t_{RTS\text{-MPDU}}, \\
&= 16\mu s + 4\mu s + \frac{(16+6)+20+8}{data\text{-rate}(m)}, \\
t_{cts} &= t_{PLCP\text{-preamble}} + t_{PLCP\text{-header}} + t_{SERVICE\text{-field}} \\
&\quad + t_{tail} + t_{CTS\text{-MPDU}}, \\
&= 16\mu s + 4\mu s + \frac{(16+6)+14+8}{data\text{-rate}(m)}, \\
t_{ack} &= t_{PLCP\text{-preamble}} + t_{PLCP\text{-header}} + t_{SERVICE\text{-field}} \\
&\quad + t_{tail} + t_{ACK\text{-MPDU}}, \\
&= 16\mu s + 4\mu s + \frac{(16+6)+14+8}{data\text{-rate}(m)}.
\end{align*}
\]

The frame formats of IEEE 802.11g [10] are the same as those of IEEE 802.11a; therefore the transmission times of data frame, RTS, CTS and ACK of IEEE 802.11g are identical to those of IEEE 802.11a.

IV. MODEL VALIDATION

We compare the result of our model with that of extended Bianchi’s model and the result of detailed simulations.

A. Simulation Setup

We use the discrete event simulation environment, OMNET++ [1], as the simulator and its mobility framework as the simulation model for IEEE 802.11. The mobility framework considers the signal-to-noise ratio and the bit-error to determine whether a frame is transmitted correctly or not. We consider neither the signal-to-noise ratio nor bit-errors in our model and disable these features. Thus, the receiver considers collisions the cases when it receives any other frames during the reception of a frame. OMNET++ mobility framework considers IEEE 802.11b. Because we are interested in IEEE 802.11a, we modify the parameters of the physical layer of IEEE 802.11 implementation of the mobility framework as shown in Sec. III-E. The parameters of IEEE 802.11a are shown in Table I.

We are interested in the saturation throughput; therefore the clients always have packets to send to the access point, and the access point does not transmit any data frame and only receives the data frames from the stations and replies with ACK frames.

To obtain the normalized system throughput, we observe the number of frames per second received by the access point, \( F_n \), and divide the product of \( F_n \) and the size of payload, \( P_r \), by the payload data rate, \( D_r \). Therefore the normalized system throughput \( S \) (also known as channel utilization) is:

\[
S = \frac{F_n P_r}{D_r}. \tag{36}
\]

We consider frame size and payload data rate as evaluation parameters. In each simulation, the system runs for 300 seconds.

- Frame size: As frame size increases, the collision probability also increases, and in result, the throughput decreases. In IEEE 802.11a, the maximum frame size at the MAC level is 2346 bytes, but we consider the range of 0 to 1500 bytes, because the access point is almost always connected to Ethernet in the infrastructure case, and the maximum frame size of Ethernet is 1500 bytes.
- Payload data rate: IEEE 802.11 provides eight payload data rates (6, 9, 12, 18, 24, 36, 48, and 54Mbps). We consider all of the data rates in our evaluations.

We consider three kinds of hidden terminal scenario: one-to-one, \( n_a\) to \( n_b \) and general scenarios presented in Fig. 4.

B. Simulation Results and Analysis

In this section, we present simulation result and analysis for one-to-one, \( n_a\) to \( n_b \) and general hidden terminal scenario respectively. We refer to the model developed in Sec III-A as “our simple model”, to the extended model developed in Sec. III-B as “our extended model”, and to the one in [4], [7], [8] as ”Extended Bianchi’s Model”.

1) One-to-one scenario: Figures 9 to 11 show the normalized saturation throughput as a function of the frame size. For the CSMA/CA (Figs. 9 and 10), for small frames, as the frame size increases, the normalized throughput also increases because the header induced overhead of the frame decreases.
After the normalized throughput reaches its maximum, it decreases as the frame size increases because the vulnerable period that causes collisions also increases. Note that the vulnerable period increases with the frame size as shown in (3). The normalized throughput of CSMA/CA may appear high in comparison with a pure ALOHA system in Figs. 9 and 10 (with a maximum value of about 0.35 to 0.4). This relatively high channel utilization is due to the short-term unfairness problem caused by the binary exponential back-off used in IEEE 802.11, in which if a station transmits a frame successfully, the station resets its contention window size to the minimum. As a result, the winning station has a smaller back-off time than the other stations, and monopolizes the channel (resulting in a decreased collision probability) until another station wins the contention. This effect deteriorates the short-term fairness but increases the throughput. Our simple model (described in Section III-A) estimates the performance of the system only in some situations, while showing a significant inaccuracy in other situations. This performance inconsistency of our simple model persists for a large range of parameters. To avoid cluttering the graphs, we will omit the results of the simple model from the rest of the evaluation section. Our extended model shows a good accuracy in Fig. 9 and 10. The results of our extended model are very close to those from the simulation, while the results of extended Bianchi’s model are relatively inaccurate. Due to the memory-less property of the Markov chain model, our model cannot exactly reflect the behavior of the real system; however, as shown in these figures, our enhanced model is very accurate. For RTS/CTS (Figs. 11), the throughput increases with the frame size because the size of the RTS frame is constant regardless of the frame size, resulting in a constant collision probability. For RTS/CTS, the results of extended Bianchi's model like those of our model match with those of the simulation because the size of RTS frame is too small to affect on the collision probability.

Figures 12 to 14 present the normalized saturation throughput varying payload data rate. For CSMA/CA (Figs. 12 and 13), the normalized throughput first increases with the data rate because the vulnerable period decreases with an increase in
data rate. After it peaks, the normalized throughput decreases because the physical layer header overhead of IEEE 802.11a is constant regardless of the data rate resulting a reduced channel utilization. The results of our model are exactly same as those of the simulation. For RTS/CTS (Figs. 14), the throughput decreases as the data rate increases because the physical layer header overhead of IEEE 802.11a is constant regardless of the data rate. For RTS/CTS, both the results of extended Bianchi’s model and those of our model match with those of the simulation.

2) The $n_a$-to-$n_b$ Scenario: Figure 15 shows the normalized saturation throughput for CSMA/CA as a function of the number of station for the hidden terminal scenario in Fig. 4 (a) fixing $n_a=1$ and varying $n_b$ from one to ten. The normalized throughput is almost constant regardless of $n_b$, the number of stations in the region $B$. The increased number of exposed stations in the region $B$ increases the contention in region $B$, but as the number of stations in the region $B$ increases, the frames from the station in region $A$ are more likely to collide with frames from stations in region $B$; as a result, the station in the region $A$ stays most of the time at a high retransmission stage. In the simulation, the number of frames the station in the region $A$ transmits successfully decreases as the number of stations in the region $B$ increases. On the other word, the number of frames stations in the region $B$ transmit successfully increases as the number of stations in the region $B$ does. In result, the normalized throughput is almost constant regardless of $n_b$, the number of stations in the region $B$. Figure 16 shows the normalized saturation throughput for CSMA/CA as a function of the number of stations for the scenario in Fig. 4 (a) when simultaneously varying both $n_a$ and $n_b$ from one to ten. The normalized throughput decreases as the number of stations increases because the increased number of exposed stations in both regions increases the contention, and the resulting collision probability increases. As shown in these figures, our model is very accurate.

3) General Scenario: We consider two general scenarios for the validation of our generalized model. The scenario in
Fig. 18. Normalized saturation throughput as a function of frame size for CSMA/CA for 3-to-2 scenario.

Fig. 19. Normalized saturation throughput as a function of frame size for CSMA/CA for the general scenario in Fig. 4 (b).

Fig. 20. Normalized saturation throughput as a function of frame size for RTS/CTS for the general scenario in Fig. 4 (b).

Fig. 4 (b) consists of two stations in 4-to-1 scenario (B and C), three stations in 3-to-2 scenario (A, D and E). Figures 17 and 18 show the normalized saturation throughput as a function of the frame size for CSMA/CA for each $n_a$-to-$n_b$ scenario in the general scenario in Figs. 4 (b). Our model is very accurate for both CSMA/CA and RTS/CTS. Finally, Figs. 19 and 20 show the normalized saturation throughput for CSMA/CA and RTS/CTS as a function of the frame size for the general infrastructure scenario in Fig. 4 (b). The “simulation mixed” results were obtained by using the simulation results for the $n_a$ to $n_b$ case in the weighted average formula for the general case (34). The main purpose of the simulation mixed results is to show whether the inaccuracy results from the $n_a$ to $n_b$ extension, or from the weighted average formula. The result of our model is very similar to the result of simulation mixed, but there are some differences between the result of our model and that of the simulation because the weighted average formula of the $n_a$-to-$n_b$ scenario cannot accurately capture the complex dynamics of the general scenario. However, as shown in these figures, our model is reasonably accurate for both CSMA/CA and RTS/CTS, while extended Bianchi’s model is accurate only for RTS/CTS.

Second, we consider the scenario with ten stations in Fig. 21, which consists of two stations in 5-to-5 scenario, one
station in 7-to-3 scenario, two stations in 8-to-2 scenario and five stations in 9-to-1 scenario. Figures 22 and 23 show the normalized saturation throughput for the general infrastructure scenario in Fig. 21. As shown in these figures, our model is reasonably accurate for both CSMA/CA and RTS/CTS, while extended Bianchi’s model is accurate only for RTS/CTS.

V. CONCLUSION

Hidden terminals are common in IEEE802.11, and performance degradations they cause have been widely known. Despite the importance of the hidden terminal, the studies that consider the effect of hidden terminals on IEEE 802.11 throughput are small and inaccurate. We present an accurate saturation throughput model for IEEE 802.11 for general infrastructure scenarios with hidden terminals. Simulation results show that our model provides a reasonable approximation for the behavior of these relatively complex systems. Although the main results in this paper are focused on the case with a single access point, we plan to extend the work to the more general case with multiple access points that is now common throughout enterprises and universities.

VI. ACKNOWLEDGMENTS

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