Fairness Schemes in 802.16j Mobile Multihop Relay Networks

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***Abstract*—Multihop relaying in WiMAX networks is considered a cost effective way to extend the coverage of the base stations, increase cell capacity, or both. In this paper, we consider deploying non-transparent relay stations for coverage extension and study the issue of fairness schemes in such networks. Since there are access and relay zones in both downlink and uplink frames and simultaneous transmissions are possible in the access zone, it is challenging to schedule resources optimally for serving subscriber stations in a fair manner. We evaluate the performance of well known fairness schemes such as max-min and proportional fairness in such networks. We also propose a new scheduling scheme, named subsection fairness that can achieve better throughput than traditional fairness schemes by maximizing bandwidth utilization. Our numerical results evaluate the performance of each scheduling scheme in terms of cell throughput and fairness.**

I. INTRODUCTION

Most broadband wireless service providers are planning to provide mobile users high speed data service regardless of their location and mobility. Due to the high cost of dense base stations (BS) deploying, using low cost relay stations (RS) is foreseen as a cost-effective solution. The relay enhanced WiMAX (IEEE 802.16j) is recently receiving a great deal of attention as a potential primary technology for broadband wireless access. According to the IEEE 802.16j standard [1], RSs are classified into two categories, transparent mode and non-transparent mode. Transparent RSs cannot provide coverage extension and aim to enhance the throughput within the BS cell coverage, while non-transparent RSs are appropriate for coverage extension and/or throughput improvement. In this paper, we focus on deploying non-transparent RSs for coverage extension. The frame structure specified in [1] is divided into access and relay zones. The BS and the RSs are allowed to transmit to their associated subscriber stations (SS) simultaneously during the access zone period.

The scheduling problem, i.e., assigning transmission opportunities to each link in the network in order to maximize a certain objective function is an important factor in determining network performance. Since the standard does not specify any particular scheduling scheme, it is of interest to explore how the well known fairness schemes such as max-min and proportional fairness can be implemented in 802.16j networks especially in the non-transparent RS mode. We consider scheduling schemes such as max-flow, absolute fairness, and subsection fairness in order to show that there is a trade-off between throughput and fairness, and also compare those scheduling schemes by varying the number of active SSs in a cell. Moreover, we provide closed-form expressions for computing the cell throughput for each scheduling scheme.

The rest of this paper is organized as follows. In the next section, we discuss related work. In Section III, we present the system model including SINR analysis and fading channel. In Section IV, we present fairness schemes and provide closed-form expressions for the corresponding cell throughput. Numerical results and analysis are shown in Section V. Section VI concludes the paper.

II. RELATED WORK

Recent research efforts have been devoted to quantifying the benefits of using relays in multihop WiMAX systems. In [2], the authors show an analytical approach for dimensioning cellular multihop WiMAX networks and analyze the network capacity by placing RSs at the border of the BS's transmission range for coverage extension. However the scheduling problem was not considered. Several studies have considered the problem of scheduling for OFDMA based WiMAX networks [3], [4]. The authors in [3] present a round-robin based scheduling solution for the 802.16 BS to ensure the QoS requirements of SSs in the uplink (UL) and downlink (DL) directions, and work in [4] studies the proportional fair scheduling problem while taking into account frequency selectivity and multiuser diversity. However, the scheduling schemes in [3], [4] restrict the transmission opportunity to one node at a time and, hence they do not effectively utilize the capacity of the network. According to the standard [1], the BS and RSs can transmit simultaneously for the non-transparent RS mode.

Fairness has been extensively studied in various wireless network areas [5]–[7]. Max-min and proportional fairness are the most common fairness definitions in both wired and wireless communication networks (e.g. 802.11 essentially implements max-min fairness). Unfortunately, those fairness schemes have not been examined as thoroughly in 802.16j based networks with non-transparent RSs. In this paper, we show how the max-min and proportional fairness can be achieved in such networks, while considering the most relevant characteristics of IEEE 802.16j systems. We also present max-flow and absolute fairness as extreme choices to be compared with other well known fairness schemes. Moreover, we provide another fairness scheme; subsection fairness, which can

TABLE I
Simulation Parameters

System Parameters	
Operating Frequency	3.5 GHz
Duplex	TDD
Channel Bandwidth	10 MHz
BS/RS Height	50 m
SS Height	1.5 m
BS/RS Antenna Gain	17 dBi
SS Antenna Gain	0 dBi
BS/RS Power	20 W
SS Power	200 mW
BS/RS Noise Figure	3 dB
SS Noise Figure	7 dB
OFDMA Parameters	
FFT Size	1024
Sub-carrier Frequency Spacing	10.94 kHz
Useful Symbol Time	91.4 μs
Guard Time	11.4 μs
OFDM Symbol Duration	102.9 μs
Data Sub-carriers(DL / UL)	720 / 560
Pilot Sub-carriers(DL / UL)	120 / 280
Null Sub-carriers(DL / UL)	184 / 184
Sub-channels(DL / UL)	30 / 35

achieve more throughput than the well known fairness schemes by maximizing bandwidth utilization. Although we consider only the downlink analysis, all of our schemes can be easily extended to include the uplink.

III. System Model

We consider a WiMAX network enhanced with non-transparent RSs for coverage extension. In each cell, as shown in Figure 1, the BS is located at the center of the cell, and three RSs are deployed to extend the cell coverage; the SSs are distributed uniformly in the coverage area of the cell. We assume that every node in the network is equipped with a single omni-directional antenna, and transmissions at all nodes are constrained to the time-division half-duplex mode, i.e., no terminal can transmit and receive at the same time. Although the standard does not place a limit on the number of hops from the BS to the SSs, in this paper the maximum number of hops is limited to two since an increased number of hops causes an increased delay and decreased bandwidth efficiency. The parameters used for analysis are listed in Table I.

The Erceg-Greenstein model [8] is used to model path loss (this is also the model recommended by the IEEE 802.16 working group). For a given transmission power P_t, the received signal power P_r is given by $P_r = \frac{G_t G_r P_t}{L}$, where G_t, G_r, and L represent the transmitting antenna gain, receiving antenna gain, and the path loss of the channel respectively. The received signal to interference and noise ratio (SINR) is determined by $SINR = \frac{P_r}{P_N + \beta P_I}$, where β is the number of co-channel cells of the first tier, P_N is the noise power, and P_I is the interference signal power from a neighboring cell on the same frequency as the current cell.

Depending on the link quality, a variety of modulation and coding schemes (MCS) are supported in WiMAX networks. Table II shows the achievable data rates denoted as $C_1, C_2, ..., C_7$ according to MCS, and the last column gives the minimum required SINR, $\overline{\gamma}_m$, computed by bit error rate expression for M-QAM [9] when bit error rate is 10^{-6}.

TABLE II
SINR Threshold Set

MCS	**Downlink Data Rate** C_m**[Mbps]**	**Spectral Efficiency [bps/Hz]**	**Threshold** $\overline{\gamma}_m$**[dB]**
QPSK 1/2	5.25	1.0	9.1
QPSK 3/4	7.87	1.5	11.73
16 QAM 1/2	10.49	2.0	13.87
16 QAM 3/4	15.74	3.0	17.55
64 QAM 2/3	20.99	4.0	20.86
64 QAM 3/4	23.61	4.5	22.45
64 QAM 5/6	26.23	5.0	24.02

With the assumption of a Rayleigh fading channel, the received SINR, γ, is an exponential random variable [10]. Therefore, the probability that a transmitter can achieve data rate C_m can be expressed as:

$$p(C_m) = \int_{\overline{\gamma}_m}^{\overline{\gamma}_{m+1}} \frac{1}{\gamma^*} \exp\left(-\frac{\gamma}{\gamma^*}\right) d\gamma, \tag{1}$$

where γ^* is the average SINR. Consequently, the average achievable data rate, C_{ss}, can be computed by:

$$C_{ss} = \sum_{m=1}^{7} C_m \cdot p(C_m). \tag{2}$$

IV. Scheduling Schemes

In this section we present several scheduling schemes that can be used in 802.16j systems for non-transparent RS systems and compare those scheduling schemes in terms of cell throughput and fairness. We assume that the entire spectrum is allocated to each node whenever they are allowed to transmit, i.e., scheduling is done by assigning time slots to every node. A key task of scheduling scheme is to determine the time duration of the access zone and the time durations allocated to each RS in the relay zone.

In addition to parameters in Table I, we assume a reuse factor of 7, one sector per cell, and hilly terrains with heavy tree densities (i.e., terrain type A [8]). The cell radius, 1500m, is determined by the condition that the SINR value at an SS located at the cell edge is greater than the minimum threshold value in Table II, i.e., 9.1dB. For the general case, when R RSs ($RS_1, ..., RS_R$) are deployed and N SSs are receiving data from the BS within the same DL frame duration, i.e., there are N active SSs in a cell, the cell area is divided into $R+1$ subsections denoted as S_0, S_1, ..., S_R, and the number of active SSs in each subsection is denoted as n_0, n_1, ..., n_R respectively. Thus, the following equation is satisfied:

$$N = n_0 + n_1 + ... + n_R. \tag{3}$$

The SSs in subsection S_0 can receive data directly from the BS, while the SSs in subsection S_i ($i = 1, ..., R$) should receive data from the BS via RS_i ($i = 1, ..., R$). We denote with λ_0 and λ_1, ..., λ_R the fraction of time in the frame allocated to the access zone and fractions of a frame allocated to RS_1, RS_2, ..., RS_R in the relay zone respectively. Thus, the following equation is satisfied:

$$\lambda_0 + \lambda_1 + \cdots + \lambda_R = 1, \tag{4}$$

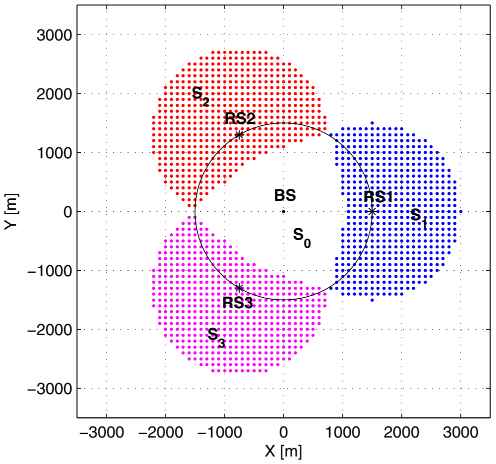


Fig. 1. A coverage extension scenario with three RSs.

as we assume that there is no overhead in the frame. Let us also denote with C_j^i and t_j^i the link capacity of the j^{th} active SS ($j = 1, ..., n_i$) with its parent node and the time fraction allocated to the j^{th} active SS in subsection S_i ($i = 0, ..., R$) respectively, and let C_{BR_1}, C_{BR_2}, ..., C_{BR_R} denote the link capacities between BS and RS_1, RS_2, ..., RS_R respectively. The SSs in each subsection may use part or all of the access zone fraction of the λ_0:

$$\lambda_0 \geq t_1^i + t_2^i + ... + t_{n_i}^i, \quad i \in \{0, 1, ..., R\}. \tag{5}$$

When the access zone is fully utilized, inequality (5) becomes an equality.

A. Max Flow

The authors in [11] use a *conflict graph* to compute the optimal throughput that the multihop wireless network can support between the sources and the sinks. The conflict graph indicates which groups of links cannot be active simultaneously. By using this conflict graph model, the cell's maximum throughput can be computed by solving a linear programming problem. However, the drawback of using this approach for maximizing the cell throughput is that when a subscriber link has a low capacity, it will not be scheduled. In other words, only the SSs connected with highest link capacities are scheduled to transmit resulting in the starvation of the SSs with small capacity links. Therefore, severe starvation is common with this conflict graph scheme.

B. Absolute Fairness (AF)

In an absolute fairness scheme, every active SS achieves equal throughput. With this approach, more time will be scheduled for SSs connected with lower link capacities, thereby decreasing the overall cell throughput. In the following, we provide a closed form expression for computing the cell throughput with this scheduling scheme.

In each subsection S_i, due to absolute fairness, the throughput of all nodes should be equal:

$$C_1^i t_1^i = C_2^i t_2^i = \; ... \; = C_{n_i}^i t_{n_i}^i, \quad i \in \{0, 1, ..., R\}. \tag{6}$$

However, the throughput per node of each subsection could be different from the others since each subsection could have a different number of active SSs as well as link capacities. It is likely that the throughput of each node in the subsection with the highest number of active SSs will be the smallest. The throughput of the other nodes is also restricted to achieve equal throughput throughout the cell. We define the average data rate (H_i) of subsection S_i as the sum of the throughput of active nodes in S_i divided by the total time duration of access zone:

$$H_i = \frac{n_i C_1^i t_1^i}{t_1^i + t_2^i + ... + t_{n_i}^i}, \quad i \in \{0, 1, ..., R\}. \tag{7}$$

Using (6), we can eliminate times t_j^i in (7):

$$H_i = \frac{n_i}{\frac{1}{C_1^i} + \frac{1}{C_2^i} + \cdots + \frac{1}{C_{n_i}^i}}, \quad i \in \{0, 1, ..., R\}. \tag{8}$$

The data rate H_i is the harmonic mean value of active node's link capacities in S_i. To ensure that every active SS can achieve the same amount of throughput regardless of their associated link capacities, it is necessary to control the time fractions allocated to RSs. Therefore, only the subsection that has the minimum throughput per node can fully use the resources during the access zone λ_0, while the rest of subsections are constrained by the link capacities C_{BR_1}, C_{BR_2}, ..., C_{BR_R}. Thus, the following equation should be satisfied for absolute fairness:

$$\frac{\lambda_0 H_x}{n_x} = \frac{\lambda_1 C_{BR_1}}{n_1} = \frac{\lambda_2 C_{BR_2}}{n_2} = \; ... \; = \frac{\lambda_R C_{BR_R}}{n_R}, \tag{9}$$

where H_x and n_x are the average data rate and the number of active SSs of subsection S_x that has minimum throughput per node, the second term represents the throughput for nodes in S_1, the third term represents the throughput for nodes in S_2, etc. If we use (9) to substitute for λ_i in (4), λ_0 can be expressed as:

$$\lambda_0 = \frac{1}{\frac{n_1}{n_x}\frac{H_x}{C_{BR_1}} + \frac{n_2}{n_x}\frac{H_x}{C_{BR_2}} + \cdots + \frac{n_R}{n_x}\frac{H_x}{C_{BR_R}} + 1}. \tag{10}$$

By using λ_0 determined from (10), the cell throughput is:

$$\textit{Cell Throughput}_{AF} = \frac{\lambda_0 H_x}{n_x} \times N, \tag{11}$$

where N is the total number of active SSs in a cell.

C. Max-min Fairness (MF)

The principle of max-min fairness is to allocate network resources in such a way that the bit rate of a flow cannot be increased without decreasing the bit rate of a flow having a smaller bit rate. In 802.16j systems, the active SSs in S_0 can fully use resources during λ_0 without decreasing the throughput of the other SSs in a cell since they are not constrained by the relay zone transmissions. Therefore, using (6)-(10) the cell throughput under max-min fairness scheme can be expressed as:

$$\textit{Cell Throughput}_{MF} = \frac{\lambda_0 H_x}{n_x} \times (N - n_0) + \lambda_0 H_0. \tag{12}$$

D. Proportional Fairness (PF)

Another well known fairness scheme is proportional fairness [5] that represents a compromise between max-flow and absolute fairness. The achieved throughput of each node will be proportional to their associated link capacities. To be more precise, both BS to RS and RS to SS link capacities are considered to determine which nodes should achieve more throughput.

We denote with A_j^i the achievable maximum data rate of the j^{th} active SS ($j = 1, ..., n_i$) in subsection S_i ($i = 0, ..., R$) respectively. The achievable maximum data rate A_j^i is computed with the assumption that the whole DL frame is used for the j^{th} subscriber in subsection S_i. Therefore, the A_j^0 in subsection S_0 is equal to C_j^0 as the SSs are served by the BS directly, and the A_j^i in subsection S_i ($i = 1, ..., R$) is:

$$A_j^i = C_j^i \times \lambda_0 = C_{BR_i} \times (1 - \lambda_0), \tag{13}$$

as the received data rate of RS from the BS is equal to the forwarded data rate of RS to the SS. From (13), the λ_0 can be expressed as $\lambda_0 = \frac{C_{BR_i}}{C_j^i + C_{BR_i}}$, thus A_j^i can be rewritten as:

$$A_j^i = \frac{C_j^i \times C_{BR_i}}{C_j^i + C_{BR_i}}. \tag{14}$$

The throughput achieved by each node should be proportional to its A_j^i value:

$$C_1^0 t_1^0 : C_2^0 t_2^0 : \ ... \ : C_{n_R}^R t_{n_R}^R = A_1^0 : A_2^0 : \ ... \ : A_{n_R}^R. \tag{15}$$

In order to find out which subsection will fully use the access zone λ_0, we denote with T_i the sum of all time fractions t_j^i in subsection S_i:

$$T_i = \sum_{j=1}^{n_i} t_j^i, \quad i \in \{0, 1, ..., R\}. \tag{16}$$

For the optimal boundary between access zone and relay zone, the received throughput of RS_i from the BS should be equal to the sum of throughput of SSs in subsection S_i. Let

$$T_{max} := \max\{T_0, T_1, ..., T_R\}. \tag{17}$$

Then, T_{max} is equal to λ_0, and λ_1, ..., λ_R can be expressed as:

$$\lambda_i = \frac{C_1^i t_1^i + C_2^i t_2^i + ... + C_{n_i}^i t_{n_i}^i}{C_{BR_i}}, \quad i \in \{1, 2, ..., R\}. \tag{18}$$

Thus, the following equation is satisfied:

$$T_{max} + \lambda_1 + \ ... \ + \lambda_R = 1. \tag{19}$$

Using (13)-(19), t_j^i of each node is computed, and consequently the cell throughput can be computed by:

$$\textit{Cell Throughput}_{PF} = \sum_{i=0}^{R} \sum_{j=1}^{n_i} C_j^i t_j^i. \tag{20}$$

E. Subsection Fairness

We propose another fairness scheme, subsection fairness, which is also a compromise between max-flow and absolute fairness. As the name of scheme implies, node fairness is achieved only within a subsection, but each subsections' achievable throughput is not constrained by the time fractions λ_1, ..., λ_R for global fairness. The time fractions of the relay zone are allocated proportionally according to each subsections' average data rate H_i, i.e., the higher H_i, the larger the time fraction allocated in the relay zone. With this fairness scheme, the bandwidth utilization is always maximized since there are no wasted resources. This subsection fairness scheme can be viewed as the combination of two fairness schemes because max-min fairness is achieved within each subsection and proportional fairness is achieved in the relay zone period. Alternatively, it is possible to apply proportional fairness within each subsection instead of max-min fairness by defining an alternate average data rate:

$$H_i^* = \frac{C_1^i + C_2^i + ... + C_{n_i}^i}{n_i}, \quad i \in \{0, 1, ..., R\}. \tag{21}$$

The data rate H_i^* is the arithmetic mean value of active node's link capacities in S_i. We refer to these two subsection fairness schemes by "subsection max-min fairness (SMF)" and "subsection proportional fairness (SPF)". In the following we provide closed-form expressions for those two schemes.

The average data rate H_i of subsection S_i is computed using (6)-(8). The achievable throughput of S_i ($i = 0, ..., R$) is $H_i \times \lambda_0$. To ensure that the whole bandwidth is fully utilized, the data transferred from BS to RS_i is equal to the data transferred from RS_i to the subscribers in S_i:

$$H_i \lambda_0 = C_{BR_i} \lambda_i, \quad i \in \{1, 2, ..., R\}. \tag{22}$$

Using (22) and (4), the time duration of access zone λ_0 can be expressed as:

$$\lambda_0 = \frac{1}{\frac{H_1}{C_{BR_1}} + \frac{H_2}{C_{BR_2}} + \cdots + \frac{H_R}{C_{BR_R}} + 1}. \tag{23}$$

Once λ_0 is computed from (23), the cell throughput under subsection max-min scheme can be expressed as:

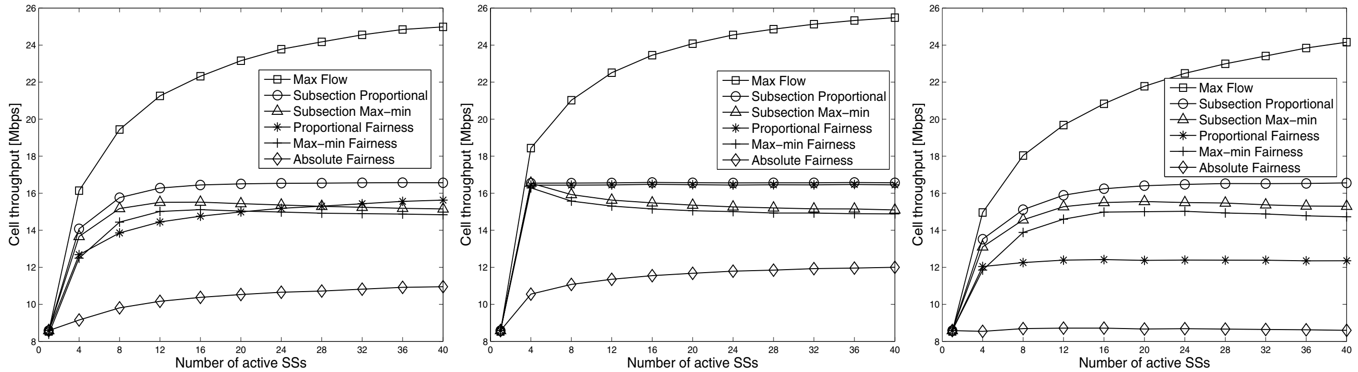
$$\textit{Cell Throughput}_{SMF} = \lambda_0 H_0 + \lambda_0 H_1 + \cdots + \lambda_0 H_R. \tag{24}$$

By substituting H_i^* for H_i in (24), the cell throughput under subsection proportional scheme can also be expressed as:

$$\textit{Cell Throughput}_{SPF} = \lambda_0 H_0^* + \lambda_0 H_1^* + \cdots + \lambda_0 H_R^*. \tag{25}$$

V. NUMERICAL RESULTS AND ANALYSIS

In this section we analyze the cell throughput with respect to the number of active SSs in a cell for several scheduling schemes. From (2), given the distance between the transmitter and receiver, the average achievable data rate can be computed. We consider this value as a link capacity. Therefore, whenever N active SSs are randomly placed within the cell, the cell



(a) Random distribution of SSs

(b) Uniform distribution of SSs

(c) Biased distribution of SSs

Fig. 2. Cell throughput as a function of the number of active SSs within a cell for different scheduling schemes.

throughput can be computed by using the equations of the scheduling schemes presented in Section IV.

Figure 2 shows the cell throughput as a function of the number of active SSs within a cell for three different distribution scenarios (random, uniform, and biased distributions). In each graph, the results from five different scheduling schemes are plotted. To obtain average cell throughput value, the simulation is repeated 10,000 times for each scenario with N active SSs randomly placed within a cell. We computed the 95% confidence intervals, but do not show them as they are very small and would clutter the graphs.

Overall, the cell throughput tends to increase with the number of active SSs because, as the number of active SSs in each different subsection increases, the access zone frequency reuse improves. The highest cell throughput is achieved by the max flow scheme, whereas absolute fairness scheme corresponds to the lowest cell throughput; the absolute fairness scheme is an extreme fairness and leads to bandwidth underutilization; especially, when the link capacities vary considerably, the cell throughput could be reduced significantly since most resources are consumed by those SSs having a low link capacity.

When the active SSs are randomly distributed in a cell as shown in Figure 2(a), the cell throughput achieved by subsection proportional scheme is always higher than that of proportional fairness, however the subsection max-min scheme can be slightly worse than that of proportional fairness for more than 28 active SSs because, as the number of SSs increases, it is more likely that each subsection can have SS connected with small link capacity. Figure 2(b) shows that the cell throughput from proportional fairness and max-min fairness are improved when the active SSs are equally distributed in each subsection, hence, the results from subsection fairness and well known fairness schemes are almost the same. On the other hand, as shown in Figure 2(c), when only one subsection accommodates half of the active SSs, the cell throughput from subsection proportional is much higher than proportional fairness. Therefore, it is clear that both subsection max-min and subsection proportional schemes achieve a higher throughput especially when the SSs are non-uniformly distributed.

VI. CONCLUSION

In this paper we study the scheduling problem in 802.16j based networks enhanced with non-transparent relays for the purpose of evaluating fairness scheduling schemes. We show how well known fairness schemes such as max-min and proportional fairness can be implemented in 802.16j systems and also present max flow and absolute fairness as extreme cases. We also proposed subsection max-min and subsection proportional schemes in order to improve the throughput in comparison with conventional fairness schemes. Numerical results show that subsection max-min and subsection proportional schemes have better throughput when the active SSs are non-uniformly distributed in a cell.

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