Asymptotic Stability of Congestion Control Systems with Multiple Sources

Mihail L. Sichitiu
Dept. of Electrical and Computer Eng.
NC State University
Raleigh, NC 27695
mlsichit@ncsu.edu

Peter H. Bauer
Dept. of Electrical Eng.
University of Notre Dame
Notre Dame, IN 46556, USA
pbauer@mars.ee.nd.edu

***Abstract*— Congestion in computer networks is the main reason for reduced performance and poor quality of service; therefore, a good congestion control system is essential. The basic property of any control system is stability. We consider the problem of stability in computer network congestion control systems with multiple sources, which is the most common case in general purpose computer networks, e.g., the Internet. The main result of the paper is the proof that for most congestion control systems (including TCP and ATM-ABR), the stability of the congestion control system with a single source is equivalent to the stability of the one with multiple sources. The proof is based on a well known necessary and sufficient stability test.**

I. Introduction

Congestion control in data networks usually refers to the problem of controlling the sending rate of the source hosts such that the throughput of the network is maximized under some additional constraints (e.g., fairness). The importance of congestion control schemes cannot be overemphasized, as congestion can bring even a correctly operating network to its knees [1]. When congested, the routers in the network receive more packets than they can forward, their internal buffers overflow, and packets are dropped. If the congestion control scheme is not well designed, in response to packets drops the sources will try to push even more packets through the network, thus worsening the congestion.

Over the years, many congestion control protocols have been proposed, among them is also the one that forms the basis of the current TCP congestion control in the Internet [2]. In time, several improvements have been proposed and implemented [3]–[5]. In the last decade, a large number of papers took a rigorous, control theoretic approach to congestion control [6]–[29]. In these papers, either one or multiple data sources are throttled in such a way that congestion in the network is eliminated after it occurs or, even better, avoided altogether.

Motivated by the importance of the congestion control system for practically any computer network (even more important if the network has to provide quality of service) and by the recent activity in the control theoretical approaches to congestion control, we studied the effect of having multiple sources on the asymptotic stability of the congestion control system. The main result of this work is that, for most congestion control systems, the stability of the system with one source is equivalent to the stability of the system with multiple sources.

If the reader is familiar with the stability of time-variant control systems, the proof is not very difficult to understand; however, the applicability of the result is potentially very broad. The direct result of this work can be utilized practically in any congestion control system that considers the stability of the system with multiple sources. If stability can be shown for one source, the stability of the system with multiple sources follows, automatically.

II. Model

Fig. 1 depicts a typical congestion control system in a data network with M sources transmitting data through a congested switch. The sources send data through a congested switch to their corresponding destinations (in reality, the circuits need not be distinct, e.g., a destination can get data from multiple sources, the separation is only conceptual). The congested switch sends congestion feedback information to the sources, and the sources may reduce or increase their data rate as a function of this feedback. The way in which the feedback is provided may be different from network to network. For example, in ATM networks [30], for available bit-rate flows, either a congested bit or an explicit rate request is sent back to the sources with the resource management (RM) cells on the return paths. In IP networks (e.g., the Internet), the transmission control protocol (TCP) uses an implicit feedback: if the packets are not acknowledged, it is assumed that they were dropped at the congested switch and the sending rate is reduced.

Some of the flows may be congested at different switches. Depending on the particular feedback mechanism, the feedback of all congested switches can be cumulative (e.g., in IP networks), or only the most congested switch on the path of the flow can impose the transmission rate of the source of the flow (e.g., in ATM networks). The flows that are not controlled by the congested switch (e.g., ATM ABR flows congested elsewhere), will not be considered in what follows, as they will not influence the stability congestion control system in any way (they will simply shift the equilibrium point by a constant value equal to their throughput).

Regardless of the feedback mechanism and the exact reaction of the sources to this feedback, the general feedback mechanism for multiple sources is similar to the one presented in Fig. 2.

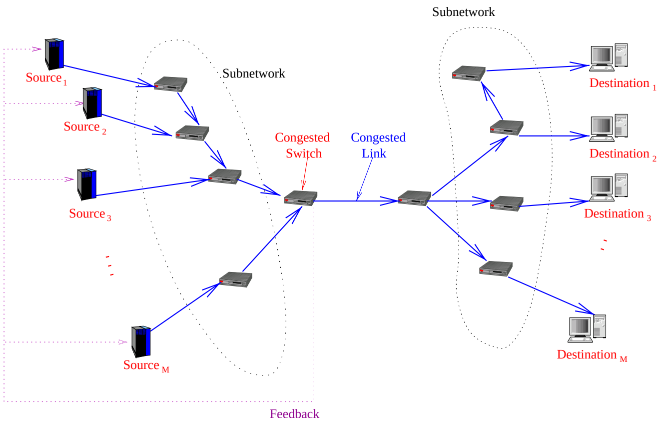

Fig. 1. A network with a congested node and multiple sources

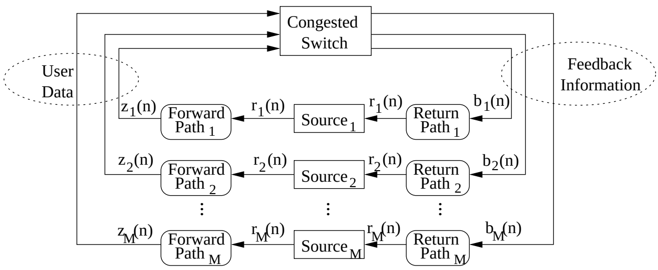

Fig. 2. The feedback loops of the congestion system

In reality, the forward and return paths presented in Fig. 2 can be a single communication link (or a chain of links and switches) or not (it may be a separate channel altogether as in the case of TCP, which has implicit feedback), but qualitatively, they transport two different types of data. On the return paths, feedback information travels from the switch to the sources. On the forward paths, user data travels from the sources to the congested switch. Two different discrete time models were formulated [8], [9], [31], [32] for the two quantities of interest: a "hold freshest sample"(HFS) model for the feedback information and a "variable bit-rate"(VBR) model for the user data volume.

Fig. 3 depicts a detailed model of an ATM-ABR congestion control system that fits most congestion control systems for the case of multiple sources. We denote with M the maximum number of sources that may connect to the congested switch at any one time (we shall see that this is not a restriction, as M can be arbitrarily large). The sampling period T of the discrete time model can be chosen to be as small as necessary to capture the dynamics of the system. The focus of this paper is not on the system model. A detailed description of the model is available in [8], [9], [31], [32].

The weights $w_i(n)$ represent the "fair" share of the bandwidth allocated to source i. In ATM ABR networks, the

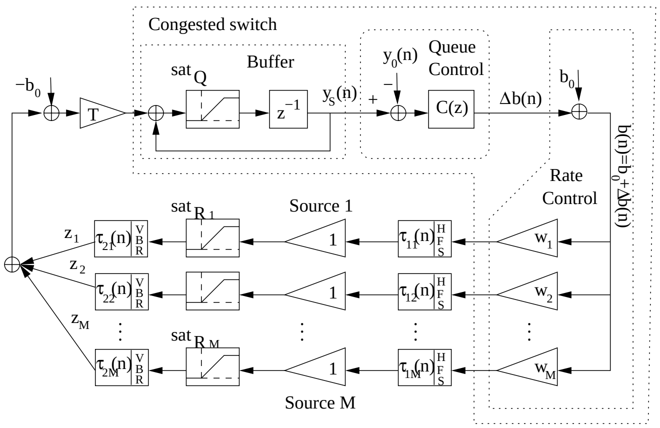

Fig. 3. System model for an ATM-ABR congestion control system [8], [9], [31], [32]

weights are computed using a max-min fairness algorithm [33], [34], and are sent to the sources using the explicit rate (ER) field in the resource management (RM) cells. In TCP/IP networks, if a plain forwarding mechanism is used, all flows will receive an approximately equal share of the bandwidth (although the flows with shorter round-trip times will be favored). If a diffserv mechanism is implemented, different flows will receive different shares of the available bandwidth depending on their diffserv class. The weights $w_i(n)$ vary with time as sources connect or disconnect from the congested switch; however, they are piecewise constant, and their sum is equal to one at any one time instant:

$$\sum_{i=1}^{M} w_i(n) = 1. \tag{1}$$

The weights $w_i(n)$ change as new flows join or depart the congested server. It was shown [7]–[9], [32] that when the weights change, the system does not have an equilibrium point, and hence, there can be no question on the stability of the system. Thus, an important assumption is that the weights are constant for relatively long periods of times (more than the time constant of the congestion control system).

The delays $\tau_{1,i}(n)$ and $\tau_{2,i}(n)$ correspond to the HFS and VBR models for the return paths and the forward path, respectively, where the index i denotes the source with $i = 1, ..., M$. We assume that the delays are bounded:

$$0 \leq \tau_{1,i}(n) \leq \bar{\tau}_{1,i}, \tag{2}$$
$$0 \leq \tau_{2,i}(n) \leq \bar{\tau}_{2,i}. \tag{3}$$

The constant rate b_0 is capacity of the switch (i.e., the departure rate of the switch). For systems with flows congested elsewhere, b_0 is the capacity of the switch minus the throughput of those flows. For the rate saturation nonlinearities, we can use a sector description for the rate saturation around a rate equilibrium point $r_{0,i}$:

$$sat_{R,i}(r_i) = R_i(n)(r_i - r_{0,i}) + r_{0,i}, \tag{4}$$

where $R_i(n) \in [R_{i,\min},\ 1]$. (This is not a linearization and models the full dynamic range of the nonlinearity [35]).

Let us denote

$$\bar{\tau} = \max\left\{ \max_{i=1..M} \bar{\tau}_{1,i},\ \max_{i=1..M} \bar{\tau}_{2,i} \right\}. \tag{5}$$

The model in Fig. 3 is fairly general and accurate considering the significant effects that can occur in congestion control systems. We will simplify it by representing the congested switch and the sources separately as in Fig. 4.

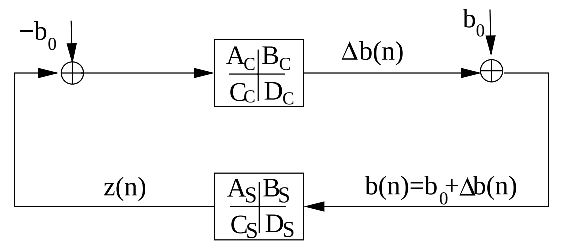

Fig. 4. Simplified model of the congestion control system in Fig. 3

Let us denote

$$z(n) = \sum_{i=1}^{M} z_i(n). \tag{6}$$

The sources can be represented in state space as:

$$x_s(n+1) = A_s(n)x_s(n) + B_s(n)b(n) \tag{7}$$

$$z(n) = C_s(n)x_s(n) + D_s(n)b(n) \tag{8}$$

We can represent the switch and and rate controller in state space form:

$$x_c(n+1) = A_c(n)x_c(n) + B_c(n)\left(z(n) - b_0(n)\right) \tag{9}$$

$$\Delta b(n) = C_c(n)x_c(n) + D_c(n)\left(z(n) - b_0(n)\right), \tag{10}$$

where the state $x_c(n)$ includes both the queue of the switch as well as the state of the queue controller $C(z)$. The matrix $D_c(n) = 0$ regardless of the chosen controller (due to the presence of the queue in the switch).

For the sake of brevity in what follows, we will not explicitly show the dependence of A_c, B_c, C_c, D_c, A_s, B_s, C_s, D_s, R_i and w_i on n. We can express the entire closed loop system in Fig. 4 in state space form, as follows:

$$x(n+1) = A(n)x(n) + B(n)b_0(n), \tag{11}$$

$$\Delta b(n) = C(n)x(n) + D(n)b_0(n), \tag{12}$$

where:

$$A(n) = \begin{pmatrix} A_c + B_cD_sC_c & B_cC_s \\ B_sC_c & A_s \end{pmatrix}, \tag{13}$$

$$B(n) = \begin{pmatrix} B_cD_s - B_c \\ B_s \end{pmatrix}, \tag{14}$$

$$C(n) = \begin{pmatrix} C_c \\ 0 \end{pmatrix}, \tag{15}$$

$$D(n) = (0). \tag{16}$$

The detailed model of the source for the single source congestion control system is presented in Fig. 5.

For the single source case, the matrices A_s, B_s, C_s and D_s are shown in (17-20).

For the case of multiple sources we can represent the system in Fig. 3 as depicted in Fig. 6. Notice that the assumption of piecewise constant weights w_i allows us to interchange the constant weights and the unit delay blocks.

In this case, the matrices A_s, B_s, C_s and D_s will have a similar structure as the ones in (17-20) except that each term $R_1\beta_k\alpha_j$ will be replaced by $\sum_{i=1}^{M} w_iR_i\beta_k\alpha_j$ for all $k = 1, \ldots, \bar{\tau}$ and $j = 1, \ldots, \bar{\tau}$.

III. Main Result

The main result of the paper is based on the following theorem [36] providing a necessary and sufficient condition for time-variant systems with polytopic uncertainties:

Theorem 1 [36]

The system

$$x(n+1) = A(n)x(n),\ \ A(n) \in conv(A_1, \ldots, A_N) \tag{21}$$

is asymptotically stable iff the system

$$\tilde{x}(n+1) = \tilde{A}(n)\tilde{x}(n),\ \ \tilde{A}(n) \in \{A_1, \ldots, A_N\} \tag{22}$$

is asymptotically stable, where by $conv(A_1, \ldots, A_N)$, we denote the convex hull (convex matrix polyhedron) of the set of constant matrices $\{A_1, \ldots, A_N\}$

In other words, the system (21) with $A(n)$ in the convex hull defined by the matrices $A_1, \ldots, A_N$ is stable iff the system with $\tilde{A}(n)$ that takes values only in the vertex matrices $A_1, \ldots, A_N$ is stable as well. The result is well known and the proof is available in [36].

Denote

$$Q_{j,k}(n) = A(n) \tag{23}$$

with $\alpha_j(n) = 1, \beta_k(n) = 1, A_s$ as in (17) and $A(n)$ as in (13). The matrix $Q_{k,l}(n)$ is the closed loop matrix of the single source system for a specific combination of delays in the return and forward path.

We are now in position to prove the main result of the paper.

Theorem 2

The system (11-12) for the multiple source case is asymptotically stable iff the system corresponding to the single source case is asymptotically stable.

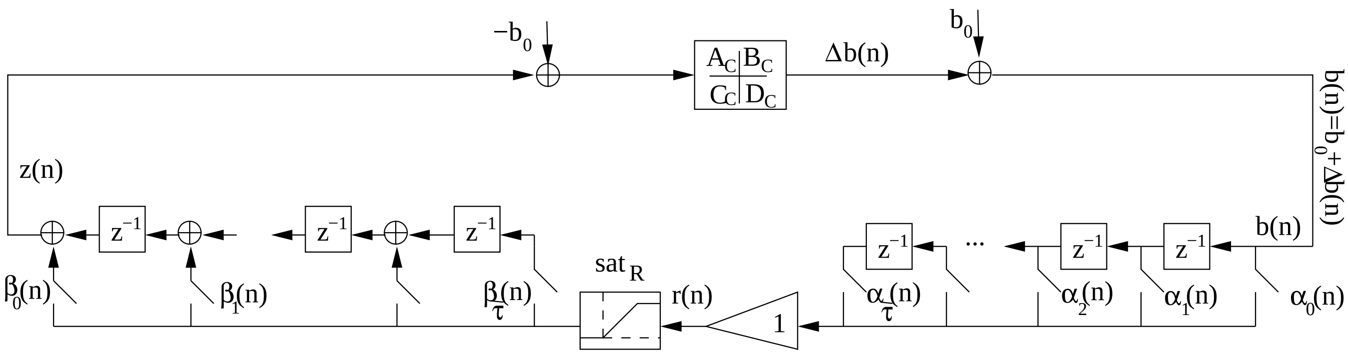

Fig. 5. Single source congestion control system

$$A_s = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \dots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ R_1\beta_{\bar\tau}\alpha_1 & R_1\beta_{\bar\tau}\alpha_2 & R_1\beta_{\bar\tau}\alpha_3 & \dots & R_1\beta_{\bar\tau}\alpha_{\bar\tau-1} & R_1\beta_{\bar\tau}\alpha_{\bar\tau} & 0 & 0 & \dots & 0 & 0 \\ R_1\beta_{\bar\tau-1}\alpha_1 & R_1\beta_{\bar\tau-1}\alpha_2 & R_1\beta_{\bar\tau-1}\alpha_3 & \dots & R_1\beta_{\bar\tau-1}\alpha_{\bar\tau-1} & R_1\beta_{\bar\tau-1}\alpha_{\bar\tau} & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ R_1\beta_1\alpha_1 & R_1\beta_1\alpha_2 & R_1\beta_1\alpha_3 & \dots & R_1\beta_1\alpha_{\bar\tau-1} & R_1\beta_1\alpha_{\bar\tau} & 0 & 0 & \dots & 1 & 0 \end{pmatrix}, \quad (17)$$

$$B_s = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ R_1\beta_{\bar\tau}\alpha_0 \\ \vdots \\ R_1\beta_1\alpha_0 \end{pmatrix}, \quad (18)$$

$$C_s = \begin{pmatrix} R_1\beta_0\alpha_1 & R_1\beta_0\alpha_2 & \dots & R_1\beta_0\alpha_{\bar\tau} & 0 & \dots & 0 & 1 \end{pmatrix}, \quad (19)$$

$$D_s = (\beta_0\alpha_0). \quad (20)$$

Proof

The proof is based on Theorem 1. We will show that the matrices $A(n)$ corresponding to the multiple sources case are in the convex hull defined by the matrices of the single source case $Q_{j,k}(n)$.

Indeed, $A_s(n), B_s(n), C_s(n)$ and $D_s(n)$ for the multiple source case are linear combinations of the corresponding matrices for the single source case. Moreover, the combination is of the form:

$$A_{s-multiple\ source}(n) = \sum_{i=1}^{M} w_i A_{s-single\ source}(n,i), \quad (24)$$

where the weights w_i satisfy (1).

Thus, the matrices $A_s(n), B_s(n), C_s(n)$ and $D_s(n)$ for the multiple source case are in the convex hull defined by the corresponding matrices for the single source case.

Notice the form of $A(n)$ in (13). The matrices $A_s(n), B_s(n), C_s(n)$ and $D_s(n)$ enter in each element of $A(n)$ exactly once, and thus the matrix $A(n)$ for multiple sources can be written as a function of the matrices of the single source $Q_{k,l}(n)$ like in (24).

In other words, the matrices $A(n)$ corresponding to the multiple sources are in the convex hull defined by the matrices of the single source. Therefore, by Theorem 1, the system (11-12) is asymptotically stable iff the system corresponding to the single source case is asymptotically stable.

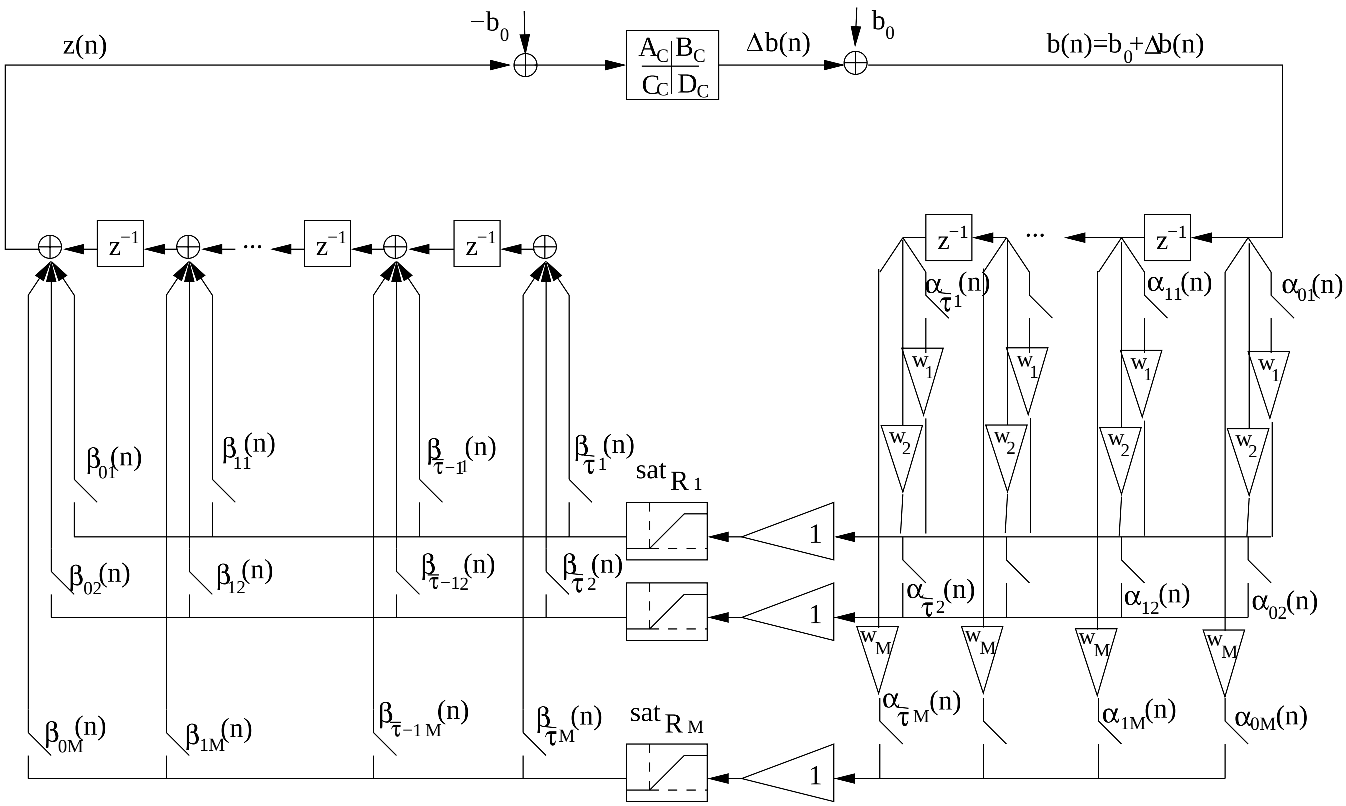

Fig. 6. More detailed model of the system in Fig. 3

Comments:

- The result is *independent* of the actual number of sources as long as (1) holds because the upper bound on the number of sources M can be made arbitrarily large.
- It was shown [8], [9], [32] that if the delays in the forward path are time-varying, the congestion control system does not have an equilibrium point.

IV. Conclusion

A detailed model of a class of congestion control systems is considered. For the considered systems, we prove a theorem that, simply put, states that for most computer congestion control systems, the stability of the system with a single source is equivalent to the stability of the system with multiple sources. The proof is based on a well-known result on the stability of control systems with time-variant delays.

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