Total Delay Compensation in LAN Control Systems and Implications for Scheduling[1]

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Abstract

In the first part of this paper it is shown that long access delays are not necessarily detrimental to the stability of local area network embedded control systems. In the second part we show that (under some mild conditions on the control system) scheduling in the return path is not needed. This is a consequence of the fact that for local area networks the access delays can be exactly determined and completely eliminated from the system representation.

1 Introduction

Integrated communication-control systems have become increasingly popular over the last decade. Especially the case of control systems that are connected through local area network (LAN) have attracted significant attention. One of the obvious problems of these type of control systems are the effects of communication delays [1]-[3]. Since the major (or only) source of delays in LANs are access delays, this problem was approached through scheduling. In essence, the access to the network for members of a networked control system is scheduled in such a way, that each member (controller, plant, etc) gets a "turn" sufficiently often to avoid instability of the loop. This often resulted in conservative maximum delay bounds that need to be satisfied [4, 5].

In this paper we take a fresh look at the problem of delay effects in networked control systems by pursuing an alternate approach. We consider the case of digital controller-plant pairs being connected via a LAN, where delays are allowed to be time-variant, uncertain, and possibly large compared to the sampling time. We further assume synchronization of all controller-plant pairs. We will show, that under certain conditions, large time-variant access delays can have a positive effect on stability and performance. Furthermore, we will also show, that at least on one communication link, time-variant access delays can be totally compensated for, regardless of the size of the delay uncertainty. We will then show that, in general, only the forward communication path needs to be scheduled or in some cases, none of the paths need to be scheduled.

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2 The Nature of Time-Variant Access Delays in LANs

Even though a discussion of the general case of time-variant delays and possible interfaces with linear systems is a non-trivial subject and beyond the scope of this paper [7], we will briefly introduce the delay trajectories resulting from access delays in discrete time. In our model we make assumptions regarding the sender, the network link and the receiver. These assumptions arise naturally from the properties of typical LANs.

2.1 Assumptions

- Network link: depending on the data traffic, the communication bus is either busy or available. If available, communication between sender and receiver is instantaneous.
- Sender: after each clock cycle the sender attempts to send a packet containing continuously updated information (sensor measurement for example).
- Receiver: after each clock cycle the receiver examines its input queue for newly received information. If no new packet was received, the previous one is used.

For scalar outputs of linear systems, only one signal sample (the most recent one) is sent per packet. For convenience, discrete time n is expressed in terms of number of clock cycles from an arbitrary starting instant $n_0 = 0$.

2.2 Definitions

The transmission delay $\tau(n)$ of a received packet at time n is the difference between the receiving time n

and its sending time n_s: $\tau(n) = n - n_s$.

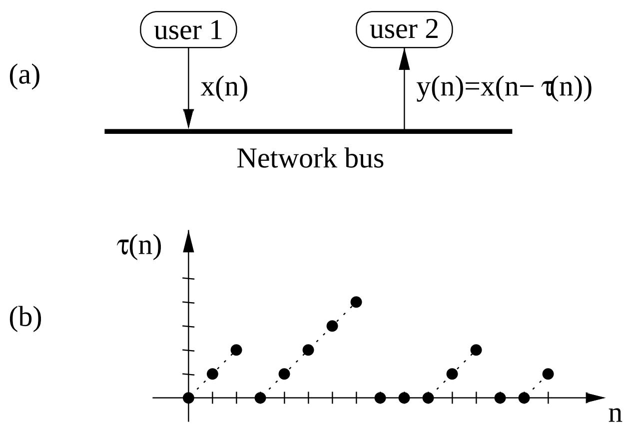

Figure 1: (a) User 1 – user 2 LAN connection. (b) Resulting access delays.

The following fact follows immediately from the definition, considering the case of a busy network link:

Fact: *Under the assumptions made in section 2.1 the communication delays between two synchronized linear systems in an LAN are given by:*

$$\tau(n+1) = \begin{cases} \tau(n) + 1 & \text{if the bus is busy at instant n} \\ 0 & \text{if the bus is idle at instant n} \end{cases}$$

Proof:

By our assumptions, if the bus is idle at time instant n the sender can immediately send and the receiver will receive the message instantaneously which corresponds to zero delay. If the bus is busy, the previous sample is held which results in:

$$\tau(n+1) = n + 1 - n_s = (n - n_s) + 1 = \tau(n) + 1.$$

$\square$

The resulting delay trajectories are consecutive ramps of various amplitude (see Figure 1(b)).

3 The Effect of Uncompensated Access Delays in one of the Communication Links

We will consider the case when $\tau(n) = n \bmod T_0$, where by $x \bmod y$ we denote the remainder of the integer division of x by y, i.e $x \bmod y = x - y \left\lfloor \frac{x}{y} \right\rfloor$. The integer T_0 denotes the period of the periodic ramp access delays.

By $\lfloor z \rfloor$ we denote the largest integer smaller or equal to z. The formulation $\tau(n) = n \bmod T_0$ corresponds to periodic delay ramps. The overall control system is displayed in Figure 2.

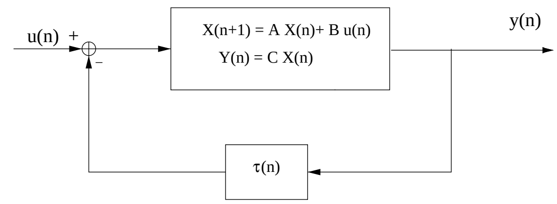

Figure 2: Linear discrete time system with access delays in the feedback path

The case of periodic LAN delay ramps is given by:

$$\tau(n) = n \bmod T_0. \qquad (1)$$

Consider a system with the discrete-time description

$$\begin{aligned} X(n+1) &= AX(n) + Bu(n) \\ y(n) &= CX(n) \end{aligned} \qquad (2)$$

and the time-variant linear feedback control law:

$$u(n) = -ky(n - \tau(n)). \qquad (3)$$

From equations (1),(2) and (3) we obtain the closed loop equation:

$$X(n+1) = AX(n) - kBCX\left(T_0 \left\lfloor \frac{n}{T_0} \right\rfloor\right) \qquad (4)$$

For $n \in [mT_0, \quad (m+1)T_0 - 1]$ the zero input system (4) has a constant feedback term, and therefore it can be described by:

$$\begin{aligned} X(n+1) &= A^{n-mT_0+1} X(mT_0) - \\ &- k \left(\sum_{i=0}^{n-mT_0} A^i \right) BCX(mT_0) \end{aligned} \qquad (5)$$

For $n = (m+1)T_0 - 1$ equation (5) becomes:

$$X((m+1)T) = \left(A^{T_0} - k(A^{T_0} - 1)(A - I)^{-1}BC\right) X(mT_0) \qquad (6)$$

and therefore:

$$X(mT) = \left(A^{T_0} - k(A^{T_0} - I)(A-I)^{-1}BC\right)^m X(0) \quad (7)$$

The system (7) is stable if and only if the ramp transition matrix Φ_{T_0} is Schur stable, where Φ_{T_0} is defined as follows:

$$\Phi_{T_0} = \left(A^{T_0} - k(A^{T_0} - I)(A-I)^{-1}BC\right) \quad (8)$$

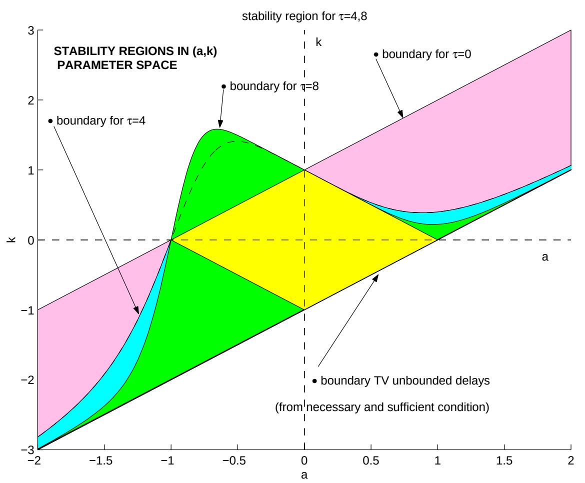

Figure 3: Stability region in the case of periodic ramp delays, for $\tau_{\max} = 4$ and $\tau_{\max} = 8$; $A = a$ scalar (first order system)

This approach can be extended to the case of non periodic delay ramps, as they naturally appear in LANs. The ramp state transition Φ_T transfers the state of the system from the beginning to the end (dropping flank) of a delay ramp of length T. Since the delay trajectory is completely defined by a succession of ramps of various length $(T_1, T_2, ..., T_n)$, the global state transition matrix can be explicitly determined for every instant. In fact the control system can be described for time instances of zero access delay as:

$$X(n + T_i) = \Phi_{T_i}(n)X(n)$$

where $\Phi_{T_i} \in \{\Phi_0, \Phi_1, \ldots, \Phi_{T_{\max}}\}$. This corresponds to a subsampled description of the state vector sequence $X(n)$.

Asymptotic stability of the origin of this system can be determined using a Theorem in [9] after some modifications:

Theorem 1

The system (4) is asymptotically stable if and only if there exists a positive integer k sufficiently large such that:

$$||\Phi_{T_1}\Phi_{T_2}...\Phi_{T_k}|| < 1$$

for any k-tuples $(\Phi_{T_1}, \Phi_{T_2}, ..., \Phi_{T_k})$, $0 \le T_i \le T_{\max} = \tau_{\max} + 1$

Since we are dealing with a linear time-variant system, global asymptotic stability of the origin implies global asymptotic stability of any equilibrium point of the system. Furthermore, the original sequence $X(n)$ is also global asymptotically stable, since as shown in equation (5), the gain over a finite time interval smaller than $T_{\max}$ is finite for $X(n)$.

Finally, $||\Phi_{T_i}|| < 1, 0 \le T_i \le (\tau_{\max} + 1)$, is obviously sufficient to guarantee stability, which is obtained from the Theorem for $k = 1$.

An example of a first order system is illustrated in Figure 3. The central diamond shaped region corresponds to the stability region for the time-variant system with unbounded delays [6].

Note that the system with time variant ramp delays as given by (1),(2) may be stable, although some of its instantaneous frozen time systems may be unstable. This shows that a stability analysis considering frozen time systems is conservative in this case, since it dismisses the compensation of instantaneous unstable systems by consecutively occurring stable systems. Furthermore, as illustrated in Figure 3, the stability region for $\tau = 0$ (smallest possible delay) does not contain the stability region produced by large delays. However, this positive effect of large access delays occurs mostly for oscillatory type of plants.

4 Smith Predictors for Time-Variant Access Delays in LAN Embedded Control Systems

In the last section, positive effects of uncompensated access delays were highlighted. In this section, we will concentrate on perfect delay cancellation in LAN embedded control systems.

4.1 Access Delay Identification at the Receiver

Let us briefly consider Figure 1 again. Assume user 1 sends a packet to user 2, and only access delays exist. Then user 2 can easily determine the access delay by counting the time-instances (cycles) for which it has not received a sample. When it receives a new sample, user 2 knows it is "fresh", i.e. the delay is zero. It then uses this sample as an input until it receives the next sample. If the next sample does not arrive for several time instances, the received sample "ages" with time, hence the ramp delay trajectories. Therefore the number of consecutive cycles for which no packet is received is the access delay. (Of course user 1 can use the same procedure to find the access delays encountered by user 2).

4.2 Access Delay Identification at the Sender

Considering once again the case of user 1 sending a packet to user 2, we claim that user 1 can also determine the delay of the data samples sent to user 2. Whenever user 1 succeeds in sending a data sample, user 2 will instantly have a "fresh" sample with zero delay. If after that user 1 cannot transmit for several time cycles, it will exactly know the age of the last successfully transmitted sample and thus the "age" of the sample used by user 2.

Combining this observation with the one made in section 4.1 we conclude that for a LAN it is possible for both users to effectively know the delays encountered on *both* the forward and the return path.

4.3 Controller Based Smith Predictor

Next we will show that LAN induced access delays can be totally eliminated from the control loop. There are two methods to achieve this. This section will address the first method of controller based Smith predictors, which is the more useful one of the two.

Consider Figure 4, where the internal compensation loop is closed around the controller. It is assumed that the plant is known.

Notice that in general a linear system and a time-variant delay are *not* commutative [7, 8]. Two notable exceptions are the cases when the linear system is a proportional gain or when the delay is constant. (We can for example force $\tau_1(n)$ to be constant using scheduling.)

If the plant and the time-variant delay $\tau_1(n)$ are commutative, then, as in the classic Smith predictor case, the forward delay is removed from the loop and appears as a gain block before the output. However, the time-variant uncertain return path delay is totally eliminated from the system! Even in the case where commutativity does not hold, the delays $\tau_2(n)$ in the feedback paths can be totally eliminated (Figure 4b).

This is possible since, as described in section 4.1, $\tau(n)$ can be determined in real time. Therefore, the traffic on the return path does not need to be scheduled and $y(n)$ can be transmitted back to the controller "whenever possible". This allows to *utilize the capacity of the communication channel more effectively than static or dynamic scheduling could.*

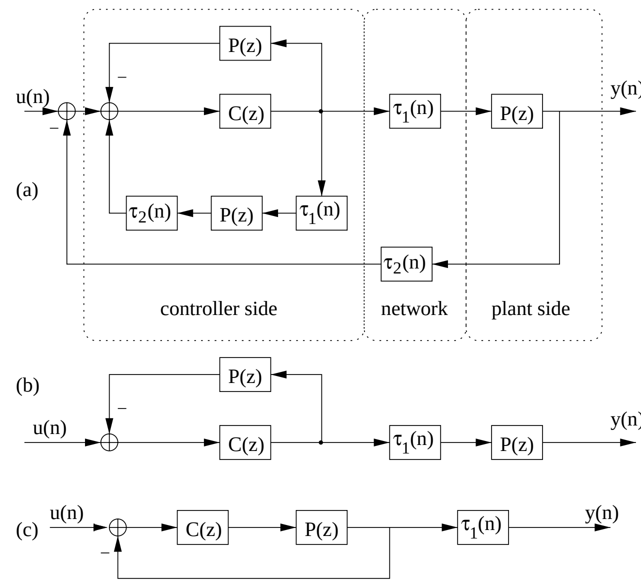

Figure 4: (a) Controller based Smith predictor to compensate time-variant access delays in the return path. (b) Equivalent control system. (c) Equivalent control system if the plant $P(z)$ and the time-variant delay $\tau_1(n)$ are commutative

4.4 Plant Based Smith Predictor

Similarly to Figure 4, the delay compensation loop can also be installed at the plant side of the network. The added advantage of such a scheme is that plant uncertainties and disturbances are also compensated (Figure 5).

If the time-variant delay in the forward path is commutative with the controller $C(z)$ we can totally eliminate the delay in the return path, while removing the delay in the forward path from the loop. In order to make the controller and the delay commutative we can schedule the delay in the forward path (to obtain a constant delay $\tau_1(n) = \tau_1$). Scheduling forward path traffic would also ensure a small delay thus reducing the dead time in the plant response to an input change. Alternatively, we can choose our controller to be a proportional gain if it will stabilize the plant $P(z)$ and satisfy the performance requirements. In this case the system can be stabilized regardless of the two delays, and loop cancellation does not require knowledge of the plant and disturbances. Of course, using the controller on the plant side might not be possible for a particular application or might not be desired (for example in the case of different control loops closed through a single processing unit), in which case the controller side implementation (presented in section 4.3) must be employed. Furthermore, if controller implementation at the plant is possible, there is no need for a feedback loop through the network and the control system in Figure 5 (c) could

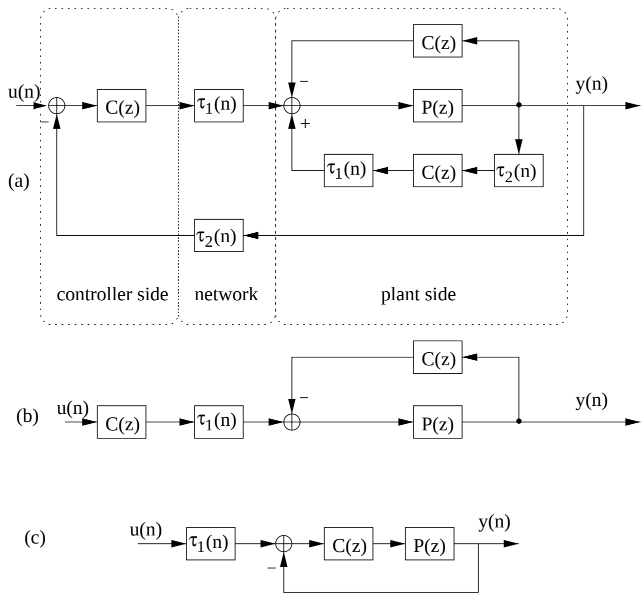

Figure 5: (a) Plant based Smith predictor for time-variant access delay in the return path. (b) Equivalent control system. (c) Equivalent control system if the controller $C(z)$ and the time-variant delay $\tau_1(n)$ are commutative.

be directly implemented.

5 The Effect of Plant Uncertainties

Consider the case of a controller based Smith predictor as shown in Figure 6 (a). Since for a synchronized controller-plant pair, there is no delay uncertainty, the remaining major uncertainty to be considered is given by the plant. If we denote the 'real' plant as $P(z)$ and the modeled plant in the Smith loop as $\tilde{P}(z)$, we have the following block diagram for the controlled system.

Applying the small gain theorem to the error system in Figure 6 (b), it is obvious that for sufficiently small plant uncertainties stability can be guaranteed since:

$$||\frac{C(z)}{1+C(z)\tilde{P}(z)}||.||\tau_1||.||\Delta P(z)||.||\tau_2|| < 1 \qquad (9)$$

can always be satisfied for $||\Delta P(z)||$ sufficiently small. For example using the fact that:

$$\sup_{X(n)} \frac{||\tau_{1,2}\{x(n)\}||_{l_\infty}}{||x(n)||_{l_\infty}} = 1 \qquad (10)$$

and using the l_∞ measure for all control and plant signals we obtain from (9):

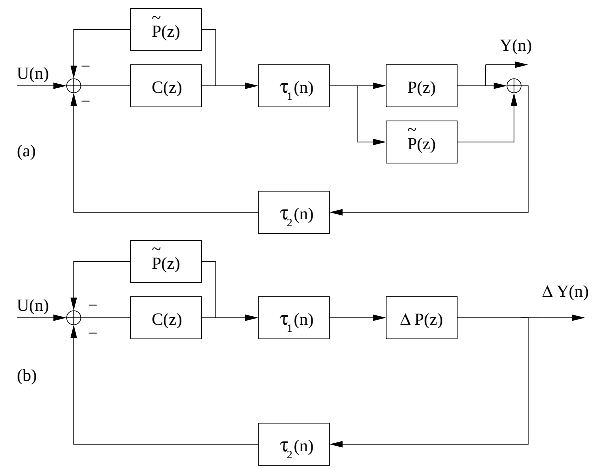

Figure 6: (a) Feedback system with Plant model uncertainties. (b) Error system resulting from plant uncertainties.

$$||Z^{-1}\{\frac{C(z)}{1+C(z)\tilde{P}(z)}\}||_{l_1}.||Z^{-1}\{\Delta P(z)\}||_{l_1} < 1 \qquad (11)$$

as a sufficient condition for stability of the error system in Figure 6(b). (Z^{-1} denotes the inverse Z-transform.)

For the special case where the delay $\tau_1(n)$ in the forward path is constant, achieved e.g. by scheduling, the analysis in section 3 can be applied directly with $k = 1$. The resulting system then takes the form as shown in Figure 7:

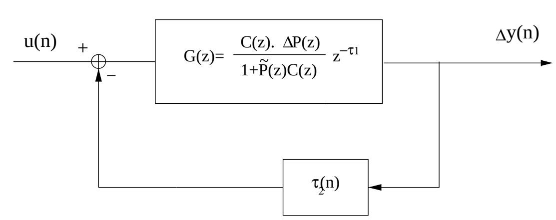

Figure 7: Resulting error system with a constant delay in the forward path

If ΔP is sufficiently small the small gain theorem can be invoked again resulting in the condition:

$$||Z^{-1}\{\frac{C(z).\Delta P(z)}{1+C(z)\tilde{P}(z)}\}||_{l_1} < 1. \qquad (12)$$

This often results in more conservative conditions than the one presented in Theorem 1 of section 3.

6 Conclusion

This paper has shown that, long delays are not always detrimental to the stability of a LAN embedded control system and in some cases are even beneficial. It was also shown that for LAN embedded control systems, scheduling in the return path is (under certain mild conditions on the control system) not needed. This result is based on the fact that time-variant access delays can be exactly determined in synchronized systems and totally eliminated from the system representation. As a consequence, return path information can be sent whenever possible, resulting in a much more efficient use of the network capacity.

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