Angle of Arrival Localization for Wireless Sensor Networks

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***Abstract*— Awareness of the physical location for each node is required by many wireless sensor network applications. The discovery of the position can be realized utilizing range measurements including received signal strength, time of arrival, time difference of arrival and angle of arrival. In this paper, we focus on localization techniques based on angle of arrival information between neighbor nodes. We propose a new localization and orientation scheme that considers beacon information multiple hops away. The scheme is derived under the assumption of noisy angle measurements. We show that the proposed method achieves very good accuracy and precision despite inaccurate angle measurements and a small number of beacons.**

I. Introduction

The emergence of Wireless Sensor Networks (WSNs) has facilitated our interaction with the physical environment. A WSN consists of a large number of distributed sensor nodes, which are generally inexpensive and resource constrained. The network is often configured such that the communication between the sensor nodes and the base stations requires multiple hops. Such a network topology can be traced back to the ancient defensive systems. Instead of using electronic sensors, in the past, beacon towers would send signals (e.g., beacon fires, flags, smoke and drums) upon the observation of enemy activity. The signals usually passed through several towers before reaching the command center. In contrast to this ancient system, modern WSNs require no or minimal human attendance.

In many WSN applications, including monitoring and tracking, the data collected is meaningless without the positions of the corresponding sensor nodes. The positions can be discovered either by equipping each sensor nodes with a global positioning system (GPS) or by hand-placing the sensors. However, both are impractical for many WSN applications due to the expense in terms of cost and human effort.

Another technique is to use a limited number of nodes that are aware of their positions (either from GPS or by being hand-placed). These nodes are referred to as *beacons*. The rest of the nodes are referred to as *unknowns* and utilize beacons' positions to localize themselves. Depending on the mechanisms used, localization schemes can be classified into two categories:

- Range-free or proximity-based.
- Range-based.

While proximity-based schemes infer constraints on the proximity to the beacon nodes, range-based schemes rely on the range measurements (received signal strength (RSS), time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA)) among the nodes. Most of the existing approaches fall into the second category. In [1]–[3], the RSS, which is the easiest to obtain for the current sensors, is utilized. The other types of measurements, although require specialized hardware, can provide much better accuracy. Localization methods using time difference are discussed in [4], [5], while angle measurements are exploited in [6]–[8].

In this paper, assuming that all beacon nodes have omnidirectional antennas (which cannot estimate AOA) and unknown nodes are capable of detecting the angles of incoming signals, we propose a new scheme that discovers both the position and the orientation by exploiting the angle measurements among neighboring nodes. We show that, even with inaccurate measurements and a small number of beacons, the proposed angle-based approach can achieve better accuracy and precision than [6]. We also show that, for the same network configurations (density, number of beacons, number of unknowns, etc), the proposed approach allows more unknowns to be localized (and oriented).

The rest of the paper is organized as follows: the next section discusses the basic angle-based localization techniques. Section III presents our proposed approach and the corresponding implementation issues. In section IV we present the simulation results and their analysis. Section V concludes the paper.

II. Overview of Localization Using Angle of Arrival

AOA is defined as the angle between the propagation direction of an incident wave and some reference direction, which is known as orientation. *Orientation*, defined as a fixed direction against which the AOAs are measured, is represented in degrees in a clockwise direction from the North. When the orientation is 0° or pointing to the North, the AOA is absolute, otherwise, relative. One common approach to obtain AOA measurements is to use an antenna array on each sensor node. Other techniques to detect the angles between nodes are discussed in [6] and [7]. We assume that the beacons have no information about their orientations and the unknowns can detect the AOA information between neighbor nodes by using one of the above methods.

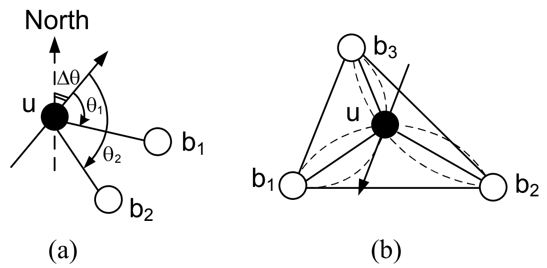

Fig. 1. Triangulation in AOA localization: (a) Localization with orientation information; (b) Localization without orientation information

The orientations of the unknowns may or may not be known at the time of deployment. Localizations under both scenarios can be solved using triangulation. We first consider the case when the orientations of the unknowns are known. In Fig. 1(a), angles θ_1 and θ_2, which are measured at unknown u, are the relative AOAs of the signals sent from beacons b_1 and b_2, respectively. Assuming the orientation of the unknown is $\Delta\theta$, the absolute AOAs from b_1 and b_2 can be calculated as $(\theta_i + \Delta\theta)(\text{mod } 2\pi), i = \{1,2\}$. Each absolute AOA measurement corresponding to a beacon restricts the location of the unknown along a ray starting at the beacon. The location of the unknown u is located at the intersection of all rays when two or more non-collinear beacons are available. When the orientations of the unknowns are not available, in other words, when the absolute AOAs cannot be obtained, the AOA differences can be used instead. In Fig. 1(b), angles $\angle b_1ub_2$, $\angle b_1ub_3$ and $\angle b_2ub_3$ can be computed using the knowledge of the relative AOAs. All angles subtended by the same chord are equal. Thus, given two points and the chord joining them, a third point from which the chord subtends a fixed angle is constrained to an arc of a circle. For example, the angle $\angle b_1ub_2$ and the chord b_1b_2 restrict u's position on the arc passing through b_1, u and b_2. Since each chord determines one arc, the location of an unknown is at the intersection of all arcs when three or more non-collinear beacons are available.

In summary, at least two non-collinear neighbor beacons are required to discover the location when the orientations are known, and at least three to discover both the location and the orientation.

Similar to localization using other measurements, AOA localization is also susceptible to measurement noise and additional problems if unknowns cannot hear directly from a sufficient number of beacons. In [6], the AOA measurements are exchanged between neighbor nodes, and the relative AOA with respect to each beacon (even multiple hops away) can be calculated based on geometry relations among the nodes. This enables the use of triangulation for localization. In [8], the problems were solved by using a semidefinite programming relaxation based method.

The proposed probabilistic localization scheme solves both challenges. In contrast to the existing schemes, the proposed scheme is derived under the assumption of AOA measurement inaccuracies. The measurement noise is modeled probabilistically and the algorithm utilizes the model to localize the unknowns. The position information of the beacons and the AOA information at each unknown are flooded within a limited number of hops. Therefore, even without a sufficient number of *neighbor* beacons, the position information of the beacons can be utilized indirectly by unknowns several hops away.

III. Probabilistic Localization

Without loss of generality, the following assumptions are made throughout the rest of the paper:

- All of the angle related variables are in the range of $[0, 2\pi)$.
- The transmission between nodes are bounded by a maximum transmission range d_{max}. Any packets received from a node outside of the transmission range is considered too weak to contribute.
- There are no major obstructions between the transmission of any two nodes. Therefore, the AOA measurements can be assumed to have certain distribution centered around the direction of the line-of-sight (LOS).

Each sensor node uses both the position of the beacons and the AOA measurements to estimate its position. Similar to the approach of probabilistic localization using RSS measurements [9], the position of each sensor node is estimated through a probability distribution function of the two-dimensional coordinate random variable (X,Y) in a collaborative and distributive manner.

A. Distribution of AOA measurements

AOA measurement inaccuracies can be caused by the wireless communication channel, the measuring device/method, or both. The spatial properties of the wireless channel have significant impact on the detection of AOA [10]. A considerable effort has been dedicated to finding good models to characterize these properties. Existing statistical models for the distribution of the AOA received at a wireless node include: Laplacian [11], von Mises [12] and Gaussian AOA distribution [13]. The measuring device and method also play important roles in the accuracy of the AOA measurements [14], [15].

Since the distribution of the AOA measurements highly depends on the communication environment and the AOA detection device/method, it is very difficult and outside the scope of this paper to find a single model that can be applied to all situations. Although the proposed localization approach is suitable for all AOA distributions mentioned above, for analytical convenience, we used the Gaussian distribution to characterize the AOA measurements combining the errors caused by both the channel and the device/method. Given that the direction of the LOS is θ_{LOS}, the AOA measurements can be described by a Gaussian distribution with mean of θ_{LOS} and standard deviation of σ_θ, which is used to describe the spread of the AOA measurements and varies with the environment.

Conversely, given a measured AOA of θ_{AOA}, the distribution of the LOS direction is:

$$\theta_{LOS} \sim N(\theta_{AOA}, \sigma_\theta). \tag{1}$$

A Gaussian noise model for the AOA measurement is also considered in [6], [8].

B. Probabilistic Localization with Orientation Information

We refer to our approach as *probabilistic* because the positions (and later the orientations) are estimated through probability distribution functions. In this section, we assume that the orientation of each sensor node is *a priori* known. The absolute AOA can be calculated using both the orientation information and the relative AOA as depicted in Fig. 1(a). Recall that for an unknown with known orientation, the minimum number of neighbor beacons required to estimate the position is two. However, since the nodes are usually assumed to be (very) sparse, most of the nodes can only hear directly from one or no beacon, thus making localizations for them impossible. In the proposed approach, the position information of beacons multiple hops away is used, such that the estimation can be performed at each node.

We define, a *pseudo-beacon*, as an unknown with an estimated position probability density function (pdf). To propagate the position information of the beacons, both beacons and pseudo-beacons send out beacon packets to their one-hop neighbors. Initially, each unknown initiates its position to be uniformly distributed over the entire network deployment area. An unknown node j receiving a beacon packet from a beacon/pseudo-beacon node i executes the following steps:

- Measures the relative AOAs of the received packets and calculates the absolute AOAs;
- Updates its position distribution and pdf using both the position information of beacons/pseudo-beacons and the computed absolute AOAs;
- All unknowns with updated pdfs become pseudo-beacons and send out their updated pdfs to their one-hop neighbors.

Assume the angle $\widetilde{\theta}_{ij}$ is j's measured absolute AOA of i's beacon packet, according to (1), the distribution of i's LOS direction 'seen' at j is:

$$f_{\Theta_{ij}}(\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{(\theta-\widetilde{\theta}_{ij})^2}{2\sigma_\theta^2}}. \tag{2}$$

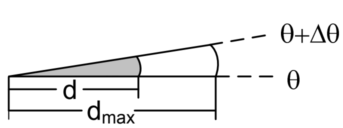

Fig. 2. An infinitesimal sector for a given angle of θ_j.

Variable Θ_{ij} in (2) can be considered as the angular coordinate in the polar coordinate system. In addition, we define distance random variable D_{ij}, which represents the distance from node i to j. Variables Θ_{ij} and D_{ij} are independent. In order to find the joint pdf $f_{\Theta_{ij},D_{ij}}(\theta, d)$, the pdf of D_{ij} has to be determined. As we use only angle measurement, the only information about the distance between nodes i and j is that $D_{ij} \leq d_{max}$. Recall that in calculus, a sector with an infinitesimal angle $\Delta\theta$ is used in integration to approximate an angle Θ_{ij} within $[\theta, \theta+\Delta\theta]$ as in Fig. 2. If the radius of this sector is d_{max}, since node j can be anywhere in the sector, the probability that $D_{ij} \leq d$ is the same as the probability that node j falls in the gray area:

$$P(D_{ij} \leq d) = F(d) = \frac{d^2}{d_{max}^2}, \tag{3}$$

where $F(d)$ is the cumulative distribution function (cdf) for variable D_{ij}. The pdf of the distance coordinate can be found simply by taking the first order derivative of (3):

$$f_{D_{ij}}(d) = \frac{2d}{d_{max}^2}. \tag{4}$$

Thus j's joint pdf for the polar coordinate random variables Θ_{ij} and D_{ij} is the product of (2) and (4). For convenience, this joint pdf is transformed into the joint pdf for the Cartesian coordinate random variables X_{ij} and Y_{ij} by using the *Jacobian* of the transformation

$$\begin{aligned} f_{X_j,Y_j}(x_j,y_j) &= \frac{1}{|d_j|} f_{D_j,\Theta_j}(d_j,\theta_j)\Bigg|_{\substack{d_j=\sqrt{x_j^2+y_j^2} \\ \theta_j=\vartheta(x_j,y_j)}} \\ &= \frac{\sqrt{2}}{\sqrt{\pi}\sigma_\theta d_{max}^2} e^{-\frac{(\vartheta(x,y)-\widetilde{\theta}_{ij})^2}{2\sigma_\theta^2}}. \end{aligned} \tag{5}$$

where $\vartheta(x,y)$ is the polar angular function of x and y:

$$\vartheta(x,y) = \begin{cases} \frac{\pi}{2} - \tan^{-1}\frac{y}{x}, & x \geq 0, \\ \frac{3\pi}{2} - \tan^{-1}\frac{y}{x} \pmod{2\pi}, & x < 0. \end{cases} \tag{6}$$

Assume the position pdf of the beacon/pseudo-beacon i is $f_{X_i,Y_i}(x,y)$, the position pdf for unknown j can be calculated by:

$$f_{X_j,Y_j}(x,y) = f_{X_i,Y_i}(x,y) ** f_{X_{ij},Y_{ij}}(x,y), \tag{7}$$

where $**$ is the two-dimensional convolution.

After computing $f_{X_j,Y_j}(x,y)$ using i's information, node j calculates its probability distribution of the position $P_j(x,y) = P_j(X_j = x, Y_j = y)$, which represents the probability of unknown j being placed at coordinates (x,y), by:

$$P_j(x,y) = \int_{y-\Delta y}^{y} \int_{x-\Delta x}^{x} f_{X_j,Y_j}(x_j,y_j)\,dx_j\,dy_j, \tag{8}$$

with $x_{min} \leq x \leq x_{max}$ and $y_{min} \leq y \leq y_{max}$. The constants $x_{min}, x_{max}, y_{min}$ and y_{max} are the bounding coordinates of the network. Both Δx and Δy are arbitrarily small numbers.

If unknown j's original probability distribution estimation is $P(x,y) = P(X = x, Y = y)$ (either the initial uniform distribution or an updated one), assuming that (X,Y) and (X_j,Y_j)

are independent, unknown j's probability distribution of the position can be updated by intersecting the newly computed and the original probability distribution:

$$P(x,y) = P(x,y)P_j(x,y). \tag{9}$$

Consequently, Node j locates itself at:

$$(x^*, y^*) = \arg\max_{x,y} P(x,y), \tag{10}$$

and updates its normalized position pdf as:

$$f_{X,Y}(x,y) = \frac{P(x,y)}{\sum_{y_{min}}^{y_{max}} \sum_{x_{min}}^{x_{max}} P(x,y)} \cdot \frac{1}{\Delta x \Delta y}, \tag{11}$$

which is broadcasted to j's neighbors. If there are other neighbors of unknown j (in addition to i) that send beacon packets to j, for each neighbor, one probability distribution constraint is computed and intersected with the current $P(x,y)$ as in (9). The intersection combining the original and new probability distribution information is possible only when all coordination random variables are mutually independent. To assure independence, an approach to eliminate the dependency among the above random variables is discussed in [9].

Example

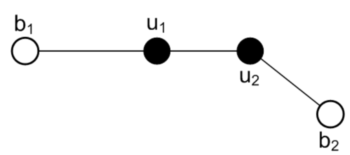

Fig. 3. Network topology with two beacons (b_1, b_2) and two unknowns (u_1, u_2).

Assume that the beacons (b_1 and b_2) and unknowns (u_1 and u_2) are deployed as depicted in Fig. 3. Upon the receipt of the beacon packets from the beacons, nodes u_1 and u_2 update their position probability distributions as shown in Figs. 4(a) and (d). Both unknowns become pseudo-beacons and broadcast their updated pdfs. After receiving from u_2, node u_1 computes a new distribution constraint as shown in Fig. 4(b) and intersects it with the one in Fig. 4(a) to obtain the final estimation in Fig. 4(c). Unknown u_2 uses u_1's pdf to calculate a new distribution as shown in Fig. 4(e) and obtain the final estimation in Fig. 4(f).

C. Log Format and Dependency Elimination

The position information of the beacons and the updated pdfs of the unknowns are transmitted in the network. Since the pdf estimation of each unknown is different from the others and too complicated to be expressed analytically, to record such pdfs, each unknown maintains a log, in which only a cascade of measured AOAs are stored as shown in Fig. 5.

Each entry in a log starts with the length of that entry. Since the only nodes providing accurate position information are beacons, and all unknowns estimate their positions based on

L_1	ID_{b1}	x_{b1}	y_{b1}	ID_{u1}	θ_{b1u1}	θ_{u2u1}	ID_{u2}	θ_{u1u2}	$\cdots$
L_2	ID_{b1}	x_{b1}	y_{b1}	ID_{u4}	θ_{b1u4}	θ_{u5u4}	ID_{u5}	θ_{u4u5}	$\cdots$
L_3	ID_{b2}	x_{b2}	y_{b2}	ID_{u4}	θ_{b2u4}	θ_{u5u4}	ID_{u5}	θ_{u4u5}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Fig. 5. Beacon packet format

this information, the first node appearing in an entry is a beacon. Beacon identifier (ID_{bi}) and the coordinates of the beacon (x_{bi}, y_{bi}) occupy the second to fourth columns. Each entry represents a transmission path from the beacon to the unknown node storing the log. ID_{ui}s are the IDs of the intermediate unknowns. A beacon can be in multiple entries, as the first two entries shown in the table, since all neighbor unknowns of the beacon relay its position information. Similarly, a single unknown can appear in multiple entries. Upon the receipt of a beacon packet from a neighbor node, an intermediate unknown appends its node ID to the packet entry and records two AOAs: one is the measured AOA of the incident packet from the ancestor node; the other is the measured AOA of the next node along the path. If the orientations are known, the recorded AOAs are absolute, otherwise, relative.

Beacon packets from an unknown contain one to several entries from that unknown's log. Packets from beacon nodes follow the same format (but having only the beacon information).

Each destination unknown appends its ID and the measured AOA from its ancestor to the packet, and stores the new packet in its log. The log is then processed using the method introduced in [9] to eliminate the dependencies among entries. The processed log has the following properties to ensure the information in the log is independent [9]:

- A node can appear only once in an entry.
- If an intermediate unknown appears in more than one entry and occupies column C_i in *ith* entry, then $(L_i - C_i)$ should be the same for all entries where the unknown appears. In other words, in all entries where the intermediate unknown appears, the number of hops between the intermediate unknown and the node storing the log (i.e., the node that estimates its position) must be the same.
- If there is more than one entry containing a path starting from an intermediate unknown i to another intermediate unknown j, the nodes between i and j must be same.

The processed log is then used at each unknown to estimate its position based on the approach introduced in section III-B when its orientation is known. When unknowns are not aware of their orientations, the processed logs are used to find both orientation and position.

D. Probabilistic Localization without orientation information

In some scenarios, orientation information is not available to the sensor nodes. For example, the sensor nodes may be dropped from a helicopter or an Unmanned Aerial Vehicle (UAV) without embedding a compass in each sensor. For such situation, only relative AOAs can be obtained and the

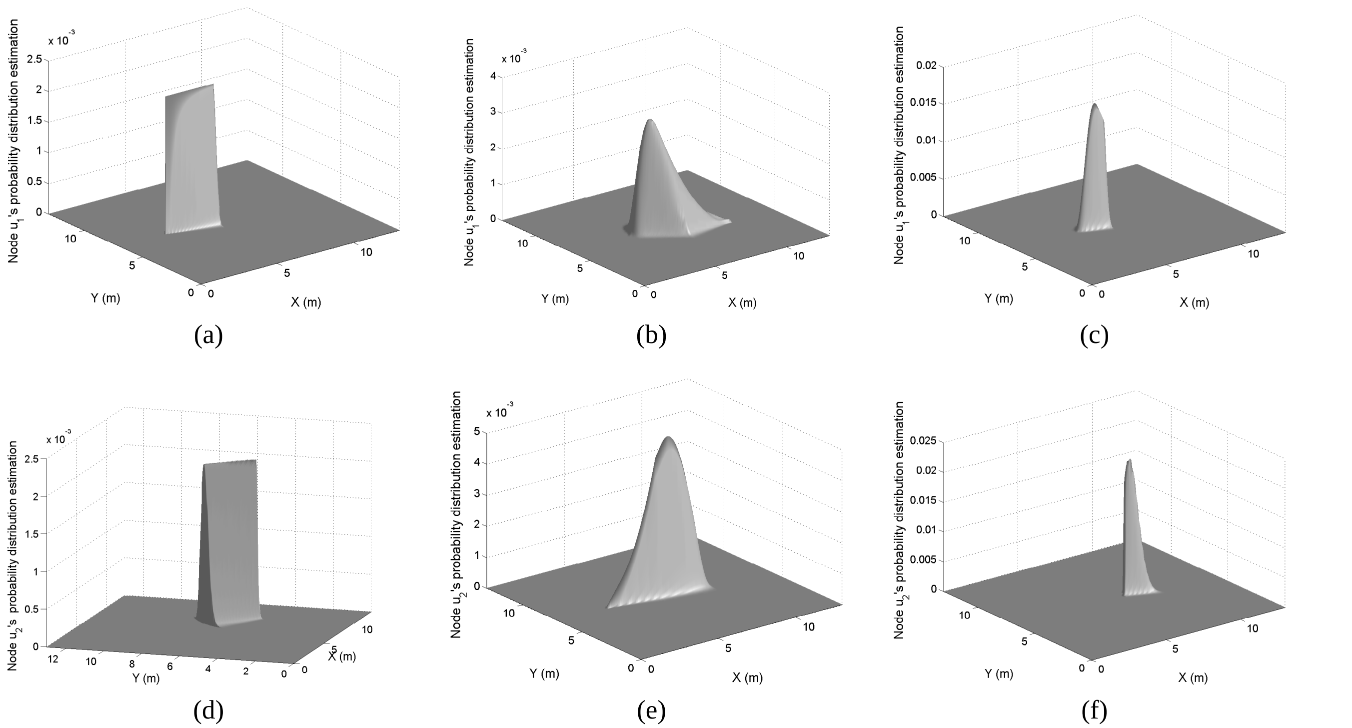

Fig. 4. Localization evolution for the network topology in Fig. 3: (a) position distribution estimation of u_1 using the beacon packet from b_1; (b) position distribution estimation of u_1 calculated by using u_2's beacon packet; (c) Final position distribution estimate of u_1 computed by intersecting the distributions in (a) and (b); (d) position distribution estimation of u_2 using the beacon packet from b_2; (e) position distribution estimation of u_2 calculated by using u_1's beacon packet; (f) Final position distribution estimate of u_2 computed by intersecting the distributions in (d) and (e).

localization scheme discussed above does not apply. In section II, we showed that at least three neighbor beacons are required to determine both location and orientation as in Fig 1(b). However, when this requirement is not satisfied, one possible scheme to solve the localization problem is to utilize the angle difference between AOAs from two different neighbors (either beacons or pseudo-beacons). Assume node j receives packets from two neighbors i and k with AOAs $\Theta_{ij} \sim N(\widetilde{\theta}_{ij}, \sigma_\theta)$ and $\Theta_{kj} \sim N(\widetilde{\theta}_{kj}, \sigma_\theta)$, respectively. The angle difference $\angle ijk$, which is defined by variable $\Theta_{ijk} = \Theta_{ij} - \Theta_{kj}$, has a pdf of:

$$f_{\Theta_{ijk}}(\theta) = \frac{1}{2\sqrt{\pi}\sigma_\theta} e^{-\frac{(\theta-(\widetilde{\theta}_{ij}-\widetilde{\theta}_{kj}))^2}{4\sigma_\theta^2}} \tag{12}$$

Variable Θ_{ijk} is a function of both (X_{ij}, Y_{ij}) and (X_{kj}, Y_{kj}), which are the random variables for the distribution constraints calculated using i's and k's information, respectively. Recall that in (5), when Θ_{ij} is a function of (X_{ij}, Y_{ij}), the two-dimensional convolution in (7) requires double integral on both X_i and Y_i to obtain the position pdf for node j. Similarly, using the pdf of Θ_{ijk} to calculate j's position pdf, a 4th-order integral on X_j, Y_j, X_k and Y_k is required. Furthermore, if node j has n neighbors, it leads to a computation of C_n^2 position pdfs. When n is large, the computational cost can be very high.

To avoid computing C_n^2 4th-order integrals, we propose a new scheme to estimate the orientations of the unknowns without three neighbor beacons.

First, we define a new term *AOA pair*: an AOA pair consists of two angle measurements between two neighbor nodes i and j: one is j's relative AOA measured at i and the other is i's relative AOA measured at j.

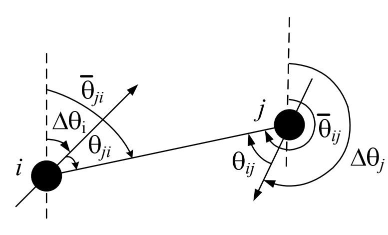

Fig. 6. θ_1 and θ_2 consist of an AOA pair

As shown in Fig. 6, θ_{ij} and θ_{ji} consist of an AOA pair. In the figure, angles $\Delta\theta_i$ and $\Delta\theta_j$ represent the orientations for i and j. Angles $\overline{\theta}_{ij}$ and $\overline{\theta}_{ji}$ are the absolute AOAs corresponding to the relative ones (θ_{ij} and θ_{ji}). When the AOA estimation is accurate, we have

$$|\overline{\theta}_{ij} - \overline{\theta}_{ji}| = \pi. \tag{13}$$

All six angles ($\Delta\theta_i$, $\Delta\theta_j$, $\overline{\theta}_{ji}$, $\overline{\theta}_{ij}$, θ_{ji}, θ_{ij}) in the figure are in $[0, 2\pi)$. Since the values of θ_{ji} and θ_{ij} can be measured, given any one of the remaining four angles, the other three can be uniquely determined. For example, if the value of $\Delta\theta_i$

is known, then

$$\overline{\theta}_{ji} = (\theta_{ji} + \Delta\theta_i) \pmod{2\pi}, \tag{14}$$

$$\overline{\theta}_{ij} = \begin{cases} \overline{\theta}_{ji} + \pi, & \overline{\theta}_{ji} \leq \pi, \\ \overline{\theta}_{ji} - \pi, & \overline{\theta}_{ji} > \pi, \end{cases} \tag{15}$$

$$\Delta\theta_j = (\overline{\theta}_{ij} - \theta_{ij}) \pmod{2\pi}. \tag{16}$$

Recall that when the unknowns are unaware of their orientations, the AOAs recorded in the log (as shown in Fig. 5) are relative. Assuming that the estimation node's orientation is $\Delta\theta_0$, using the AOA pair relation discussed above, the orientations of the unknowns that are one hop before the estimation node can be determined. Using these unknowns' newly determined orientations, the unknowns that are two hops before the estimation node can also be determined. Therefore, if we apply the AOA pair relation to all neighbor unknowns in the log, we can obtain a set of orientations for all unknowns in the log $\{\Delta\theta_k|\Delta\theta_0\}, ID_k \in \log$, in which each orientation is related to $\Delta\theta_0$. Consequently, the absolute AOAs corresponding to each relative AOA in the log can be calculated. If we rotate the estimation node's orientation by an step of $\Delta\theta'$, we will obtain another set of orientations $\{\Delta\theta'_k|\Delta\theta_0 + \Delta\theta'\}, ID_k \in \log$ that are related to $\Delta\theta_0 + \Delta\theta'$.

The localization scheme is based on calculating a position estimate of an unknown for each set of the orientations given an assumed orientation of that unknown. For different assumed orientations, there are different position estimations.

Example

In Fig. 7, b_1, b_2 and b_3 are beacons and u_1 and u_2 are unknowns. None of the unknowns can hear directly from all three beacons. The true orientations of both unknowns are 0. Unknown u_1 receives beacon packets from b_1, b_2 and u_2; and u_2 receives beacon packets from b_3 and u_1. First, consider the localization procedure for u_1. Assume the AOA measurements are accurate, if we let u_1's orientation $\Delta\theta_1 = 0$ (same as the true orientation), as shown in Fig. 7(a), the absolute AOA $\overline{\theta}_{b1}$ and $\overline{\theta}_{b2}$ are same as the relative ones θ_{b1} and θ_{b2}. Two absolute AOAs constrain u_1 to the intersection of two lines $\overrightarrow{b_1u_1}$ and $\overrightarrow{b_2u_1}$ (we use $\overrightarrow{AB}$ to represent a line segment that starts from A , passes through B, and has a length of d_{max}). Using u_1's orientation and the AOA pair relation, u_2's orientation can be calculated as 0. Thus, the AOA from b_3 constrains u_2 to $\overrightarrow{b_3u_2}$. Consequently, using AOA coming from u_2, u_1 locates itself in an area defined by an infinite number of lines with length of d_{max} and parallel to $\overrightarrow{u_2u_1}$. Each of these lines starts from a point on $\overrightarrow{b_3u_2}$. The intersection of these lines and the original intersection of $\overrightarrow{b_1u_1}$ and $\overrightarrow{b_2u_1}$ results in a unique point, which is the exact position of u_1. Similarly, at u_2, if we let u_2's orientation $\Delta\theta_2 = 0$, u_1's orientation can be computed as 0 and u_1's position can be obtained. The constraints from b_3 and u_1 intersect at u_2's exact position.

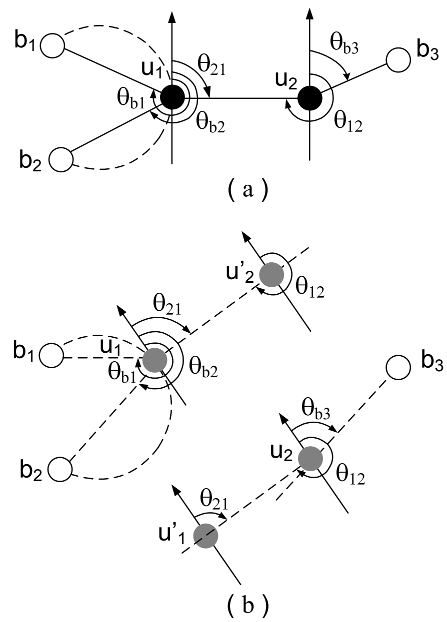

Fig. 7. Localization when the orientations are unaware

If unknown u_1 rotates its orientation to the direction shown in Fig. 7(b), $\Delta\theta'$, then constraints from b_1 and b_2 intersect at the point of u_1 as shown in 7(b) (a different point on the same arc as in the first figure). By AOA pair relation, u_1 can compute that u_2's orientation is also $\Delta\theta'$. By utilizing this newly computed orientation, u_2 is constrained to be on the dashed line $\overrightarrow{b_3u_2}$. Unknown u_2's constraint constrains u_1 to be in an area consisted of infinite lines parallel to $\overrightarrow{u_2u'_1}$ and started from a point on $\overrightarrow{b_3u_2}$. The intersection of all constraints is empty for u_1. Similarly, for u_2, the intersection also results to empty when its orientation is set to $\Delta\theta'$.

As stated in section III-B, when the AOA measurements are not accurate and when the unknowns are aware of their orientations, for each unknown node, a probability distribution as in (9) can be generated by intersecting all probabilistic constraints from its neighbors. The location of this node is estimated at (x^*, y^*) as in (10). When unknowns are not aware of their orientations, a location estimate $(x^*_{\Delta\theta}, y^*_{\Delta\theta})$ of the estimation node is obtained for each assumed value of orientation $\Delta\theta$. The closer the assumed orientation to the true one, the higher probability of the intersection. Therefore, for each assumed orientation of the unknown node, we record the probability of the estimated location $(x^*_{\Delta\theta}, y^*_{\Delta\theta})$:

$$\xi_{\Delta\theta} = P(x^*_{\Delta\theta}, y^*_{\Delta\theta}), \tag{17}$$

where $P(x^*_{\Delta\theta}, y^*_{\Delta\theta})$ is 0 when the intersection is empty.

The estimation of an unknown's orientation is

$$\Delta\theta^* = \arg\max_{\Delta\theta}\{\xi_{\Delta\theta}\}. \tag{18}$$

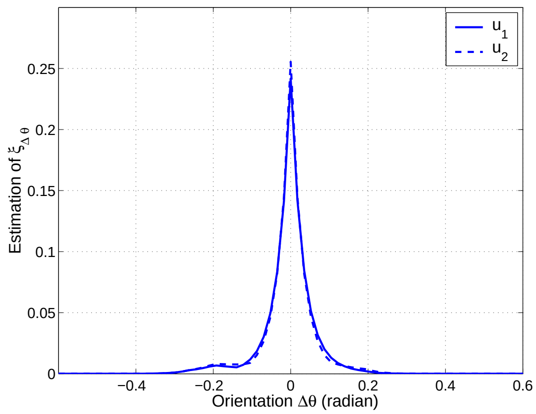

Fig. 8. Orientation estimation for u_1 and u_2 in Fig. 7

In Fig. 8, we rotate the orientations of u_1 and u_2 in Fig. 7 from $-\pi$ to π with a step size of 0.0028π (1°). For each orientation, the estimation of $\xi_{\Delta\theta}$ is obtained. The normalized $\xi_{\Delta\theta}$ is plotted against the orientations in radian (only a small range of the orientations are shown, the rest are empty or zero). Orientations within $[-\pi, 0)$ and $[\pi, 2\pi)$ have the same estimations, therefore we used orientations within $[-\pi, \pi)$ instead of $[0, 2\pi)$ to have the curve centering at zero.

After the orientation is determined, the position probability distribution and the position estimate can be determined. The rotation step size of the orientation affects the performance of the localization. The smaller the step, the more accurate the orientation estimation. There is a clear tradeoff between the accuracy and the computational cost.

IV. Simulation Results

To evaluate the performance of the proposed approach, we used a custom simulator implemented in Matlab. The results considered both the accuracy (i.e., the difference between the real position of the nodes and the position determined by the localization algorithm) and the precision (i.e., the uncertainty in the position estimate) of the proposed approach. We compare the results with the APS+AOA-based localization approach proposed in [6].

For the simulation we considered 5 beacons and 50 unknowns randomly placed in a square area (of variable size depending on the desired density). Unknowns are not aware of their orientations before the estimation. We evaluated the effects of the variations of several parameters (number of hops the beacon packet is allowed to travel, number of beacons, number of unknowns and measurement inaccuracy) on the estimation of both position and orientation. Gaussian noise was added to the AOA measurements. Unless otherwise specified, we used a network density of 0.024 nodes/m^2 (average degree$\approx$10) and a standard deviation for the AOA noise of $\frac{\pi}{18}$ (10°). To discover the orientation, we used a rotation step of 4°. For every graph we present the average of 10 simulations with different network topologies. All results are normalized with respect to the maximum transmission range.

For the compared algorithm APS+AOA, we used the following techniques proposed in [6] to control the estimation error:

- A threshold of 0.35 ($\approx 20^\circ$) is used to avoid the interference caused by small angles or degenerate triangles;
- outliers in the position estimations are eliminated to reduce the clustered errors.

A. The effect of TTL for beacon packets

The maximum number of hops (time to live (TTL) in IP) of the beacon packets has a considerable impact on the performance of the proposed algorithm. To evaluate this impact, we varied the TTL from 1 to 10. The position estimation accuracy is plotted in Fig. 9 against the TTL. Fig. 10 shows the percentage of the nodes that can discover their orientations and positions. Different lines represent different network densities including 0.016, 0.02, 0.024 and 0.028, which correspond to average degrees of 6, 8, 10 and 12, respectively.

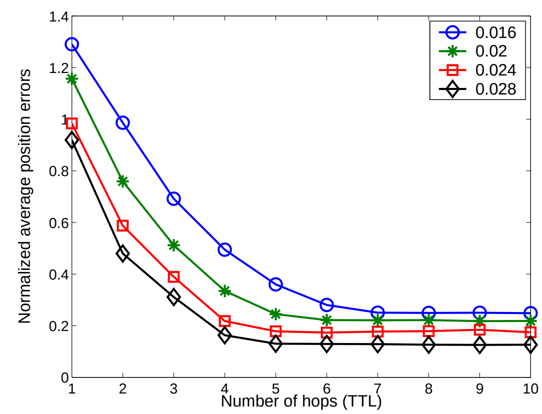

Fig. 9. Average accuracy as a function of the TTL for beacon packets for different node density

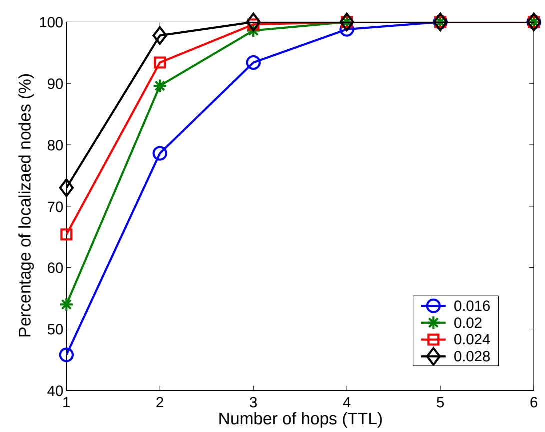

Fig. 10. Percentage of nodes that can be localized for different node density

Most of the nodes in the network cannot hear directly from enough beacons to perform triangulation. Thus, they rely on the beacon packets that are generated from beacons several hops away and relayed by other unknowns. Figures also show that, regardless of the network density, the accuracy of the localization improves along with the TTL for the first several hops. However, the improvement is not significant after $5-6$ hops.

The beacon packets become larger as they travel through the network. Larger packets require more computation resource for processing. Furthermore, larger TTLs result in more network traffic. In what follows, we used TTL of 4 (which results in 100% localization coverage for density of 0.024 nodes/m^2) for all considered schemes (i.e., both for the probabilistic and APS+AOA approaches).

B. The effect of number of beacons

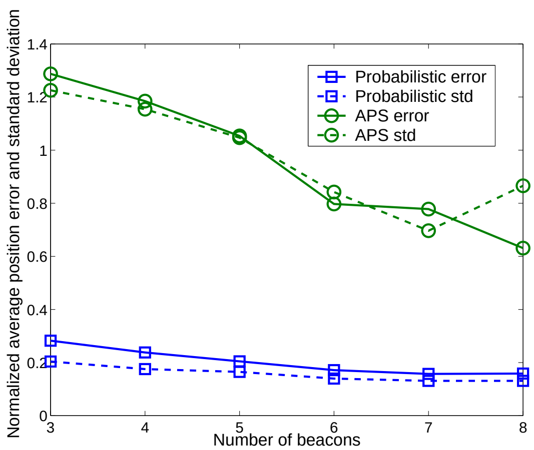

Fig. 11. Average accuracy and precision as a function of the number of beacons

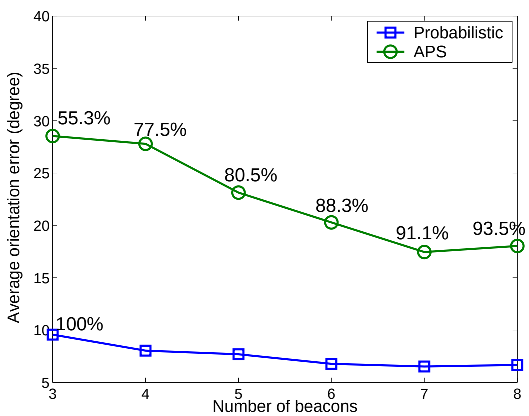

Fig. 12. Average orientation error (and the number of located node) as a function of the number of beacons

We varied the number of beacons in the network from 3 to 8. The estimations of position and orientation are shown in Figs. 11 and 12, respectively. Fig. 11 shows that both the accuracy (average error in solid lines) and the precision (standard deviation in dashed lines) of the position estimate. In Fig. 12, the orientation estimate is given. The percentages of the nodes that can be localized are also marked for each number of beacons. Our probabilistic approach achieves 100% coverage

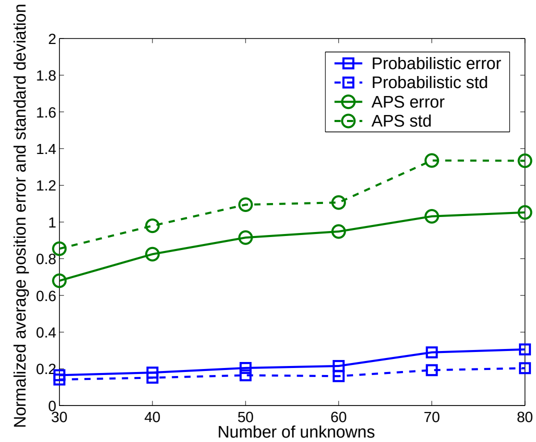

Fig. 13. Average accuracy and precision as a function of the number of unknowns

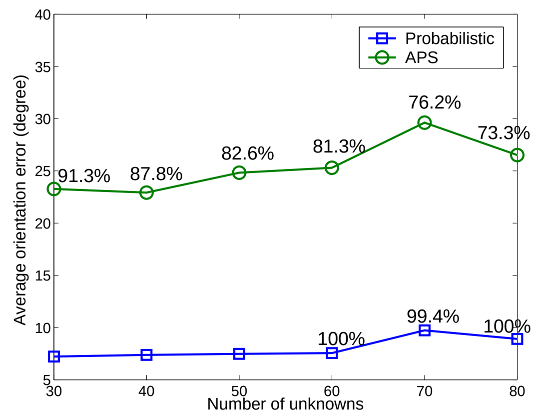

Fig. 14. Average orientation error (and the number of located node) as a function of the number of unknowns

for all numbers of beacons. The performance improves along with the number of beacons. However, unlike the APS+AOA approach, the improvement is not significant (especially when number of beacons is greater than 5). This suggests that even a low number of beacons can achieve good accuracy, precision, and coverage.

C. The effect of number of unknowns

In this scenario, we changed the number of unknowns from 30 to 80 while keeping the density constant. The simulation results are shown in Figs. 13 and 14. The estimation errors for both position and orientation increase slightly with the increase of the network size. Intuitively, since the network density is constant, the larger the network size, the fewer unknowns can hear directly from the beacons. Thus, localization for most nodes rely on relayed beacon packets with larger TTL. Since we limited the TTL to 4, the average number of beacons that an unknown can hear indirectly decreases for larger networks, which causes the slight increase of the estimation error.

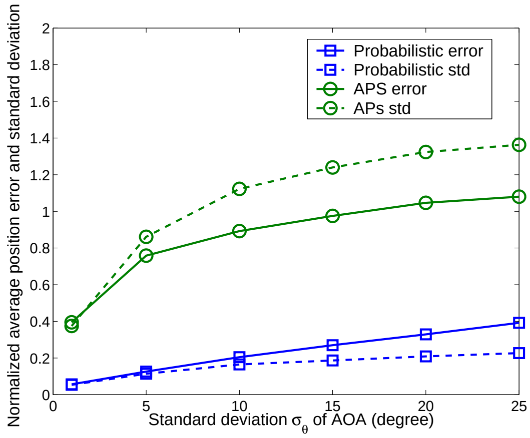

Fig. 15. Average accuracy and precision as a function of the number of AOA noise

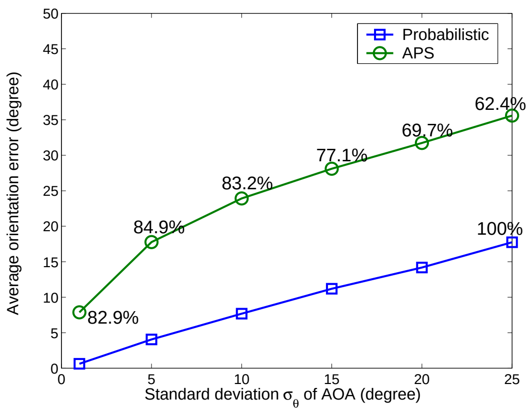

Fig. 16. Average orientation error as a function of the AOA estimation noise.

D. *The effect of measurement noise*

A good localization scheme must be robust to measurement errors. We varied the AOA measurement error from 1° to 25°. The localization and orientation results are shown in Figs. 15 and 16. The presence of AOA measurement noise has a significant impact on localization and orientation. However, the probabilistic approach behaves far more robustly than the APS+AOA scheme in both accuracy and precision; it also achieves much better coverage especially when the noise is large.

V. Conclusion

In this paper, we presented a distributed AOA-based localization and orientation approach for wireless sensor networks under the assumption that all unknown sensors are capable of detecting angles of the incident signal from the neighboring nodes. The simulation results show that even with inaccurate AOA measurements and a small number of beacons, the proposed approach exhibits very good accuracy and precision for the estimation, and achieves much better localization coverage than the existing approach in [6].

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