The Effect of Uncertain Time Variant Delays in ATM Networks with Explicit Rate Feedback

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Abstract

A new, more realistic model for the ABR traffic class in ATM network congestion control is introduced and analyzed. The new discrete time model takes into account the effect of time-variant buffer occupancy levels of ATM switches, thus treating the case of time-variant delays between a single congested node and the connected sources. For highly dynamic situations, such a model is crucial for a valid analysis of the resulting feedback system. The new model also handles the effects of the mismatch between the RM cell rates and the variable bit rate controller sampling rate as well as buffer and rate nonlinearities. A stability study is presented, that shows an equilibrium in the buffer occupancy level is not possible if time-variant delays are present in the forward path. Stability conditions for the case of time-variant delays in the return path are derived. Finally, illustrating examples are provided.

1 Introduction

Previous work [1]-[4] on explicit rate feedback of the ABR class of traffic in ATM networks is dealing with the analysis of the real feedback system using a number of simplifying assumptions. These assumptions range from linear time-invariant systems with no delay [1] to linear time-invariant systems with uncertain delays [2]. Some results even take nonlinear effects such as the saturation of the buffer occupancy into account [3]. Even though most papers deal with the case of a single congested switch, there is some recent work where multiple congested switches were allowed [4].

In this paper, we develop a model for a rate based congestion control system, taking rapidly changing buffer levels into account. This not only allows to account for the real situation of time-variant delays between congested node and sources, but, as will be explained later, can also cope with the rate dependent RM-cell rate and the resulting mismatch with the fixed controller cycle time. Furthermore, we will also include the effects of the buffer and rate saturation nonlinearity. The resulting time-variant linear feedback system model (nonlinearities are modeled through time-variant sector gains) is then analyzed for its stability using stability theory for uncertain time-variant systems.

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In section 2 of this paper, we will introduce the new time-variant uncertain delay model for congestion control of ABR traffic in ATM networks. Section 3, addresses the problem of stability and existence of equilibria for the developed models and section 4 introduces two examples in order to illustrate the results. Section 5 provides the conclusion and an outlook for future work.

2 The Time-Variant Delay Model

For the analysis and model development that will follow, we make the following assumptions:

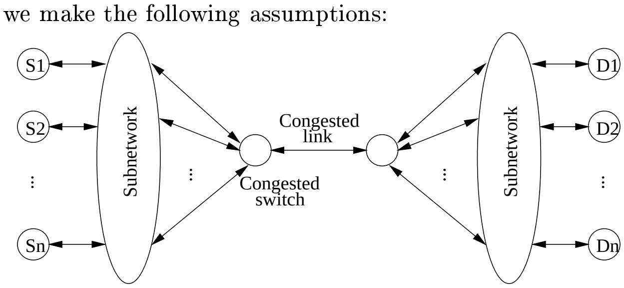

Figure 1: Single Congested Node ATM Network

- We consider a simple network with a single congested node (shown in Figure 1) and end to end RM cell routing.
- The number of sources trying to send cells through the same output link of the congested node is $M(n)$.
- All sources are greedy and hence will always send at the maximum allowable rate.
- Bandwidth for the ABR traffic on the congested link is b_0.
- The variable bit-rate controller is located at the congested switch and uses a fixed sampling time T.
- The congested switch uses the RM cells on the return path to inform the sources about the rate at which they should transmit. The delay these RM cells undergo from the congested node to the source will be time-variant in nature.
- The effect of a rate change at the source is "felt" at the congested switch only after a time-variant delay,which is due to the buffer or queue delays of all switches that the data has to pass before it arrives at the congested node.

Figure 2 depicts the case of a single source transmitting data through the congested switch. We will analyze the case of multiple sources as soon as we complete the model for the case of a single source.

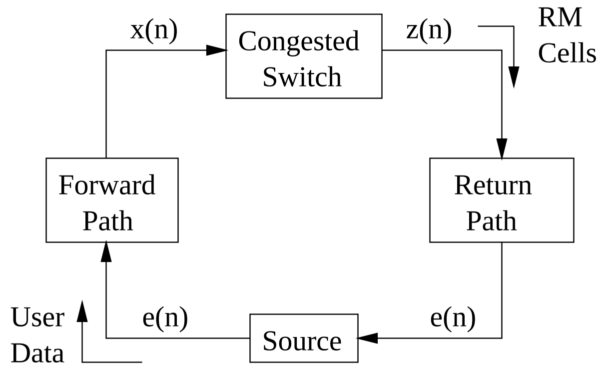

Figure 2: The single source case.

The two paths presented in Figure 2 are in reality one single communication link (or a chain of links and switches) but qualitatively they transport two different types of data. On the return path RM cells travel from the switch to the source. On the forward path the user data travels from the source through the congested switch. We need two different models corresponding to the two different types of data .

2.1 The "Hold Freshest Sample" Model

The RM cells sent on the return path may encounter different delays as the lengths of the queues of the intermediate switches vary. The source adjusts its transmission rate to the one specified in the most recent received RM cell and continues to transmit at that rate until another RM cell arrives. Since the source "holds" the same rate until it receives "fresh" information, we will call this the "Hold Freshest Sample" (HFS) [5] delay interface model.

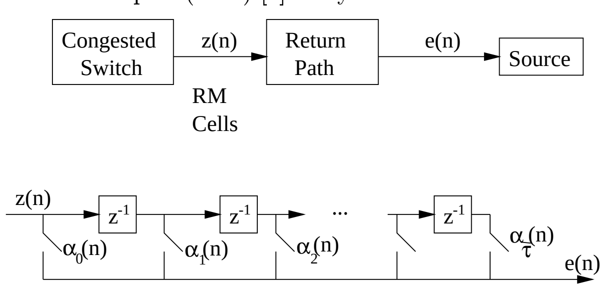

Figure 3: HFS model for the communication link: a tapped delay line with varying tap positions

Figure 3 depicts the HFS model for the return path. We denote with $z(n)$ the rate computed at the congested node at time instant n, with $e(n)$ the rate at which the source transmits at time instant n and with $\bar{\tau}$ (integer) the maximum delay encountered by an RM cell on the return path. The time-variant coefficients $\alpha_j(n)$ turn on and off exactly one switch at every time instant n. Which switch is turned on is determined by the "age" of the last received RM cell. Thus if we have $e(n) = z(n - \tau(n))$ then:

$$\alpha_j(n) = \begin{cases} 1 & \text{if } j = \tau(n) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Notice that by HFS definition the coefficients $\alpha_j(n)$ can not vary arbitrarily from one time instant to another [5]: the delay $\tau(n)$ is restricted by $\tau(n+1) \le \tau(n) + 1$, and hence we have:

$$\alpha_j(n) = 1 \Rightarrow \alpha_k(n+1) = 0 \;\; \forall k > j+1 \tag{2}$$

In other words the used sample from the time-variant delay output ages with time, but not faster.

2.2 The Variable Bit Rate Model

On the forward path we need to quantify the number of cells sent through the communication link at any given time n. We assume that there are no cell losses on the communication channel.

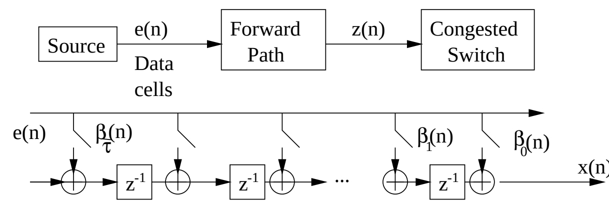

Figure 4: VBR model for the forward path

In the VBR model presented in Figure 4, $e(n)$ denotes the number of cells transmitted by the source between time instant $n-1$ and time instant n, $z(n)$ is the number of cells that arrive at the congested switch between time instant $n-1$ and time instant n. The time-variant coefficients $\beta_i(n)$ turn on exactly one "switch" (in Figure 4) at every time instant n. Which "switch" is turned on is determined by the "average" delay the cells transmitted between time instants $n-1$ and n will encounter. Thus if the delay at time n is $\tau(n)$ then:

$$\beta_j(n) = \begin{cases} 1 & \text{if } j = \tau(n) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

2.3 Total system model

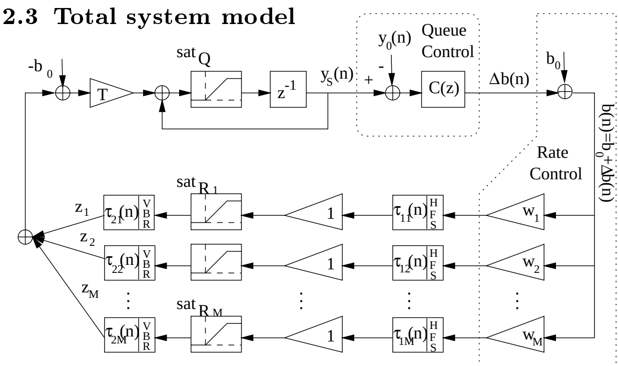

Figure 5: Total system model

Figure 5 depicts the total system model for the case of multiple sources. We denote with M the maximum number of sources that may connect to the congested switch at one time. T is the sampling period of the discrete time system; the controller uses this fixed period to compute the new rates which will be included in the RM cell that travel on the return path. The generation of RM cells in general does not follow a fixed period creating thus a rate mismatch between the controller rate and the RM cells rate. There are two cases for the mismatch:

(a) the controller rate is greater than the RM cell rate at the switch
(b) the controller rate is smaller than the RM cell rate at the switch

Typically the controller rate should be designed so that is somewhere in the middle between the maximum and minimum RM cell rate. In case (a) the controller output is subsampled by the RM cells and transported to the source. This effect can be modeled by skipping samples at the delay line in Figure 3 resulting in a sawtooth delay time function. Case (b) can simply be handled by holding the last controller sample and repeatedly inserting it into the RM cell stream until the next one becomes available.

The weights $w_i(n)$ represent the "fair" share of the bandwidth allocated to source i and can be computed using a max-min fairness algorithm [6]. The weights $w_i(n)$ vary with time as virtual circuits connect or disconnect from the congested switch; their sum is equal to one:

$$\sum_{i=1}^{M} w_i(n) = 1. \quad (4)$$

$b(n)$ is the rate computed at the controller of the congested switch and it represents the total desired incoming rate for the congested switch. b_0 is the output bandwidth available for ABR traffic (equal to the total bandwidth minus the bandwidth reserved for others classes of traffic). Matching the incoming rate to the output rate assures a stable steady state for the queue, but the queue length can be arbitrary in the absence of queue control. $\Delta b(n)$ is the queue control component which aims to stabilize the congested switch queue length to a fixed set point y_0. In addition, the queue control component can correct a number of non-ideal phenomena of the rate control system like saturation, quantization, cell-loss, etc.

$y(n)$ represents the length of the queue of the congested switch (i.e without the saturation effects taken into account). $y_s(n)$ is the queue length after the saturation is accounted for:

$$y_s(n) = sat_Q(y(n)) \quad (5)$$

where the saturation function is defined as follows:

$$sat_Q(y) = \begin{cases} 0 & \text{if} \quad y < 0 \\ y & \text{if} \quad 0 \le y \le y_{\max} \\ y_{\max} & \text{if} \quad y_{\max} < y \end{cases} \quad (6)$$

where $y_{\max}$ is the buffer capacity (in cells). We can model the saturation by a sector description around an equilibrium point y_0:

$$sat_Q(y) = Q(n)(y - y_0) + y_0 \quad \text{where} \quad Q(n) \in [Q_{\min},\ 1] \quad (7)$$

y_0 is the buffer set point. Ideally, at steady state, the buffer will have $y_s(n) = y_0$ which will ensure that the buffer will not overflow (loosing data cells) or underflow (missing service opportunities).

Similar to the queue saturation rate we model the saturation of the rate for the sources which corresponds to the sources transmitting with positive rates and not more than the available bandwidth. We define the rate saturation for the i^{th} source:

$$sat_{R,i}(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x & \text{if} \quad 0 \le x \le R_{i,\max} \\ R_{i,\max} & \text{if} \quad R_{i,\max} < x \end{cases} \quad (8)$$

We can also use a sector description for the rate saturation around an equilibrium point $x_{0,i}$:

$$sat_{R,i}(x) = R_i(n)(x - x_{0,i}) + x_{0,i} \quad \text{where} \quad R_i(n) \in [R_{i,\min},\ 1] \quad (9)$$

$R_{i,\min} > 0$ is an essential condition for the stability of the system: if $R_{i,\min} = 0 \quad \forall i = 1,\ldots,M$ we have an open loop system with an unstable plant.

The delays $\tau_{1,i}(n)$ and $\tau_{2,i}(n)$ represent the HFS/VBR models for the return paths and the forward path respectively of source i with $i = 1...M$. We assume that the delays are bounded:

$$0 \le \tau_{1,i}(n) \le \bar{\tau}_{1,i} \quad (10)$$
$$0 \le \tau_{2,i}(n) \le \bar{\tau}_{2,i} \quad (11)$$

Let us denote with $\alpha[j,i](n),\ j = 1,\ldots,\bar{\tau}_{1,i},\ i = 1,\ldots,M,\ n \ge 0$ the j^{th} time-variant coefficient α_j of the HFS model for the i^{th} return path at time instant n. Similarly denote with $\beta[j,i](n),\ j = 1,\ldots,\bar{\tau}_{1,i},\ i = 1,\ldots,M,\ n \ge 0$ the time-variant coefficient β_j of the VBR model for the i^{th} forward path at time instant n. The coefficients $\alpha[j,i](n)$ and $\beta[j,i](n)$ are computed using the equations (1) and (3).

For the sake of brevity in what follows we will not explicitly show the dependence of $\alpha[j,i]$,$\beta[j,i]$, w_i, Q and R_i on n.

We can use a state space representation for the weight w_i together with the two time-variant delays $\tau_{1i}(n)$ and $\tau_{2i}(n)$ and the source rate nonlinearity corresponding to source i:

$$x_i(n+1) = A_i(n)x_i(n) + B_i(n)b(n) \quad (12)$$
$$z_i(n) = C_i(n)x_i(n) + D_i(n)b(n) \quad (13)$$

$$B_i(n) = \begin{pmatrix} w_i \\ 0 \\ \vdots \\ 0 \\ R_i\beta[\bar{\tau}_{2,i}, i]\alpha[0,i] \\ R_i\beta[\bar{\tau}_{2,i}-1, i]\alpha[0,i] \\ \vdots \\ R_i\beta[1,i]\alpha[0,i] \end{pmatrix} \quad (15)$$

$$C_i(n) = (R_i\beta[0,i]\alpha[1,i];\ R_i\beta[0,i]\alpha[2,i];\ \ldots\ R_i\beta[0,i]\alpha[\bar{\tau}_{1,i},i];\ 0;\ \ldots;\ 0;\ 1) \quad (16)$$

$$D_i(n) = \begin{pmatrix} R_i w_i \beta[0,i]\alpha[0,i] \end{pmatrix} \quad (17)$$

$$A_i(n) = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ R_i\beta[\bar\tau_{2,i},i]\alpha[1,i] & R_i\beta[\bar\tau_{2,i},i]\alpha[2,i] & \dots & \dots & R_i\beta[\bar\tau_{2,i},i]\alpha[\bar\tau_{1,i},i] & 0 & \dots & 0 & 0 \\ R_i\beta[\bar\tau_{2,i}-1,i]\alpha[1,i] & R_i\beta[\bar\tau_{2,i}-1,i]\alpha[2,i] & \dots & \dots & R_i\beta[\bar\tau_{2,i}-1,i]\alpha[\bar\tau_{1,i},i] & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ R_i\beta[1,i]\alpha[1,i] & R_i\beta[1,i]\alpha[2,i] & \dots & \dots & R_i\beta[1,i]\alpha[\bar\tau_{1,i},i] & 0 & \dots & 1 & 0 \end{pmatrix} \quad (14)$$

$x_i(n)$ corresponds to the state of the i^{th} delay line corresponding the i^{th} source, including the RM cells path from the congested switch to the source and the data cells from the source to the congested switch. $x_i(n)$ has the dimension $\bar\tau_{1,i} + \bar\tau_{2,i}$. $z_i(n)$ corresponds to the data cells sent by the i^{th} source that reach the congested switch between time instant $n-1$ and time instant n.

$w_i(n)b(n)$ corresponds to the fair share of the total bandwidth $b(n)$ as it is computed by the controller of the congested switch at the time instant n. It includes both the queue control and the rate control components and it represents an imperative command for the source to start transmitting at (or below) that new rate (exactly at that rate with our assumption of greedy sources).

The LTI controller $C(z)$ can be chosen to be of the general form:

$$x_c(n+1) = A_c x_c(n) + B_c(y_s(n) - y_0(n)) \quad (18)$$

$$\Delta b(n) = C_c x_c(n) + D_c(y_s(n) - y_0(n)) \quad (19)$$

where by $x_c(n)$ we denote the state of the controller, $\Delta b(n)$ represents the queue control component of the bandwidth and $y_0(n)$ represents the set point for the queue length. $x_c(n)$ has the dimension N_c, input and output are scalars.

We can express the closed loop system in state space form as follows:

$$x(n+1) = A(n)x(n) + B(n)\begin{pmatrix} b_0(n) \\ y_0(n) \end{pmatrix} \quad (20)$$

$$y_s(n) = Cx(n) + D\begin{pmatrix} b_0(n) \\ y_0(n) \end{pmatrix} \quad (21)$$

where $A(n)$, $B(n)$, $C(n)$ and $D(n)$ are defined in the equations (22)-(25).

$$B(n) = \begin{pmatrix} QT\sum_{i=1}^M D_i - QT & -QT\left(\sum_{i=1}^M D_i\right)D_c \\ 0 & -B_c \\ B_1 & -B_1 D_c \\ B_2 & -B_2 D_c \\ \vdots & \vdots \\ B_M & -B_M D_c \end{pmatrix} \quad (23)$$

$$C = \begin{pmatrix} 1, & 0, & \dots, & 0 \end{pmatrix} \quad (24)$$

$$D = \begin{pmatrix} 0, & 0 \end{pmatrix} \quad (25)$$

The state vector $x(n)$ is composed by the queue length, the state of the controller and the state of the delay lines: $x(n) = (y_s(n) \quad x_c(n) \quad x_1(n) \quad x_2(n) \ \dots \ x_M(n))^T$. $x(n)$ has the dimension $1 + N_c + \sum_{i=1}^M(\bar\tau_{1,i} + \bar\tau_{2,i})$. The inputs are the available bandwidth $b_0(n)$ and the queue set point $y_0(n)$ while the output is the queue length $y_s(n)$ which is the best measure of the performance of the control system. Ideally the output should track the input $y_0(n)$ with zero steady state error, should not saturate and reject any disturbances due to changes in $w_i(n)$ as quick as possible.

3 Stability

There are a number of results in the literature [7],[8] concerning the control of a plant through a communication network while taking into account the variation of the delays. In [7] the plant to be controlled is a buffer which closely matches the problem discussed in this paper but the stability criterion employed in these papers required an FIR controller. Also the sufficient stability conditions were often shown to be conservative.

In this paper we will pursue another avenue: we will use a necessary and sufficient stability condition presented in [9].

The system model is time-variant, the matrices $A(n)$ and $B(n)$ depending on the particular combination of delays $\tau_{1,i}(n)$ $\tau_{2,i}(n)$ and on whether the queue and/or the source rates saturate.

In what follows, we describe the system around the equilibrium point $y_s(n) = y_0, \quad b(n) = b_0$. This allows to find the non-zero minimum sector gain for the nonlinearities and it allows for a zero input model after the subtraction of the equilibrium point.

At first we will introduce a Lemma, that essentially shows that the overall system cannot stay at the equilibrium point if the delays $\tau_{2,i}(n) \quad i = 1, \dots, M$ in the forward path are non-constant.

Lemma 1 *The system (20),(21) cannot have a stable, non-zero equilibrium if the delay trajectories on the forward path are time-variant i.e.* $\tau_{2,i}(n) \neq const$.

Proof: Consider the i^{th} feedback loop described by equations (12)-(17), with the VBR part shown in Figure 4. An equilibrium at $b(n) = b_0 \quad \forall n \geq 0$ implies that $z_i(n) = z_0, \quad x_i(n) = x_0 \quad \forall n \geq 0$. From (12),(14) and (15), it is clear that only $\tau_{2,i} = const$ results in a constant state vector $x_i(n)$ and also a constant output $z_i(n)$. Only for the degenerate (and not meaningful) case of $x_i(n) = 0$ will there be an equilibrium corresponding to $b(n) = z_i(n) = 0$.

$$A(n) = \begin{pmatrix} Q + QT\left(\sum_{i=1}^{M} D_i\right) D_c & QT\left(\sum_{i=1}^{M} D_i\right) C_c & QTC_1 & QTC_2 & \dots & QTC_M \\ B_c & A_c & 0 & 0 & \dots & 0 \\ B_1 D_c & B_1 C_c & A_1 & 0 & \dots & 0 \\ B_2 D_c & B_2 C_c & 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ B_M D_c & B_M C_c & 0 & 0 & \dots & A_M \end{pmatrix} \quad (22)$$

Since the i^{th} feedback path as described in (12),(13) does not have a non-zero equilibrium if the delays $\tau_{2,i}$ are non-constant, the overall system in (20),(21) will not have a non-zero equilibrium if the delays $\tau_{2,i}$ are non-constant.

Comments:

- It is interesting to note, that for $R_i(n) = 1$ (i.e. the saturation non-linearities for the rates are not activated) the output $z_i(n)$ for a constant input $b(n) = b_0$ can only take the values $0, w_i b_0, 2w_i b_0, \dots, (\bar{\tau}_{2,i} + 1)w_i b_0$.
- If the delays on the VBR side are constant i.e. $\tau_{2,i} = const$, an equilibrium in the i^{th} loop (12)-(17) is achieved regardless of $\tau_{1,i}(n)$, if the input rate $b(n) = b_0$ is constant. This assumes $w_i(n) = const$.
- A similar result (as in Lemma 1) can be formulated for the case when $w_i = w_i(n)$. Even though $\sum_{i=1}^{M} w_i(n) = 1 \quad \forall n \geq 0$ the time-variant delays $\tau_{1,i}(n)$ will result in the actual rate splitting between the sources to occur at different times if $\tau_{1,i}(n) \neq \tau_{1,j}(n), \quad i \neq j$. Hence the sum of all rates $\sum_{i=1}^{M} z_i(n) \neq b_0$ and there will be no equilibrium, i.e. $y_s(n) \neq y_0$.

Since the system with time-variant delays in the forward path does not have an equilibrium, and the delays in the return path are usually more critical (due to abrupt delay variations caused by the mismatch of varying RM cell spacing and the fixed controller sampling time), we will address the case of time-invariant uncertain delays in the forward path, time-variant uncertain delays in the feedback path and time-invariant weight distribution. Typically the weights are piecewise constant, which justifies such an analysis.

In order to address stability of the desired equilibrium point in the overall system (20),(21) we choose for the equilibrium:

$$\begin{aligned} b(n) &= b_0 && (26) \\ z_i(n) &= w_i b_0 \quad (\text{assuming } w_i b_0 \leq R_{i,\max}) && (27) \\ y_s(n) &= y_0(n) = y_0 && (28) \end{aligned}$$

The fact that equations (26)-(28) correspond to an equilibrium is easily verified using either Figure 5 or the equations (20),(21).

In this case

$$x_{i,0} = (w_i b_0,\ w_i b_0, \dots, w_i b_0)^T. \quad (29)$$

A new system description around the equilibrium point is obtained by letting $x_0 + \hat{x}(n) = x(n)$ in (20). This yields the zero input system description:

$$\hat{x}(n+1) = A(n)\hat{x}(n) \quad (30)$$

with $A(n)$ given in (22).

In order to reduce notation, we will formulate the stability test for the case of a known set of delays $\tau_{2,i},\ i = 1, \dots, M$, i.e. the delays in the forward path are assumed to be fixed for now. In this case the matrix $A(n)$ in (30) depends on the instantaneous values of the delays in the return path $\tau_{1,i},\ i = 1, \dots, M$ and the values of the sectors $Q(n)$ and $R_i(n),\ i = 1, \dots, M$. In order to show this dependency, we will express $A(n)$ in (30) as

$$\begin{aligned} A(n) &= A(\tau_{1,1}; \dots; \tau_{1,M}; Q; R_1; \dots; R_M) && (31) \\ 0 &\leq \tau_{1,i} \leq \bar{\tau}_{1,i} \quad i = 1, \dots, M && (32) \\ Q &\in [Q_{\min},\ 1] && (33) \\ R_i &\in [R_{i\,\min},\ 1], \quad i = 1, \dots, M && (34) \end{aligned}$$

Denote the set of matrices described by (31)-(34) by $\mathcal{S}$. Note that $\tau_{1,i}$ are integers whereas Q and R_i are real numbers.

Now for the set of matrices described by (31)-(34), fix $\tau_{1,1}, \dots, \tau_{1,M}$ at any delay combination satisfying (32). The resulting uncertainty set is due only to the sector intervals and can be shown to be of polytopic nature. We will denote this polytope as $\mathcal{P}_{\tau_{1,1},\dots,\tau_{1,M}}$ and it is obtained as the convex hull of the vertex matrices $A(\tau_{1,1}; \dots; \tau_{1,M}; Q; R_1; \dots; R_M)$ with $Q \in \{Q_{\min},\ 1\}$, $R_i \in \{R_{i\,\min},\ 1\}$, i.e.

$$\mathcal{P}_{\tau_{1,1},\dots,\tau_{1,M}} = \underset{\substack{Q \in \{Q_{\min},\ 1\} \\ R_i \in \{R_{i\,\min},\ 1\}}}{conv} \{A(\tau_{1,1}; \dots; \tau_{1,M}; Q; R_1; \dots; R_M)\} \quad (35)$$

where *conv* denotes the convex hull of the arguments. The uncertainty set given by (31)-(34) can then be described as the union of all possible polytopes $\mathcal{P}_{\tau_{1,1},\dots,\tau_{1,M}}$ i.e.

$$\mathcal{S} = \bigcup_{\substack{(\tau_{1,1}, \dots, \tau_{1,M}) \\ 0 \leq \tau_{1,i} \leq \bar{\tau}_{1,i}}} \mathcal{P}_{\tau_{1,1},\dots,\tau_{1,M}} \quad (36)$$

$\mathcal{S}$ is the uncertainty set that needs to be tested for stability.

In the next step we will overbound $\mathcal{S}$ by the convex hull formed by all vertices of all polytopes $\mathcal{P}_{\tau_{1,1},\dots,\tau_{1,M}}$. Hence

$$\mathcal{P} = \underset{\substack{Q \in \{Q_{\min},\ 1\} \\ R_i \in \{R_{i\,\min},\ 1\} \\ 0 \leq \tau_{1,i} \leq \bar{\tau}_{1,i}}}{conv} \{A(\tau_{1,1}; \dots; \tau_{1,M}; Q; R_1; \dots; R_M)\} \quad (37)$$

where $\mathcal{S} \subset \mathcal{P}$.

It is well known [9] that even though $\mathcal{P}$ includes points in its interior that cannot occur, asymptotic stability of (30) with respect to only the vertices of $\mathcal{P}$ or with respect to the entire polytope results in the same stability conditions. (This is why all tests involving time-variant systems with polytopic uncertainties only involve the vertex matrices. This of course is quite different in the time-invariant case). There exist a number of necessary and sufficient stability tests for this type of system. For reasons explained later we will use a modification of the test in [9]:

Theorem 2 *Denote the exposed vertices of the polytope $\mathcal{P}$ as $V_1, \ldots, V_N$. Then the system*

$$\hat{x}(n+1) = A(n)\hat{x}(n), \quad A(n) \in \mathcal{P} \tag{38}$$

is asymptotically stable, iff $\exists$ a finite k, such that:

$$\|V_{i_1} \cdot \ldots \cdot V_{i_k}\| \leq \gamma < 1, \quad \forall (i_1, \ldots, i_k) \in \{1, \ldots, N\}^k \tag{39}$$

where $\|\cdot\|$ is any induced matrix norm.

Remarks:

- Theorem 2 uses a matrix norm test where k highly depends on the chosen norm.
- The number of exposed vertex matrices N is bounded by the sum of all vertices in $\mathcal{P}_{\tau_{1,1},\ldots,\tau_{1,M}}$ i.e. $N \leq 2^{M+1} \prod_{i=1}^{M} (\bar{\tau}_{1,i} + 1)$. In other words not all vertices of $\mathcal{P}$ are necessarily exposed vertices.
- An additional reduction in the complexity is possible by limiting the k-tuples of matrices to be tested in the product norm to only those that correspond to admissible sequences of the HFS interface. This is the main reason why we favor the test in [9]. The other available tests do not easily allow to include information on permissible matrix sequences $A(n)$.
- A total number of $\prod_{i=1}^{M} (\bar{\tau}_{2,i} + 1)$ such tests would be necessary, if stability is to be determined for all possible forward path delay combinations.

4 Examples

4.1 The effect of time-variant delays on the forward path

We will present here a simple example to illustrate the effect of time-variant delays on the forward path. We will show that the equilibrium of the system is perturbed as soon as the delays on the forward path change. To isolate the phenomenon we will assume a very simple system with only one loop ($M = 1$), no non-linearities ($Q(n) = R_1(n) = 1$), no delays on the return path ($\tau_{1,1}(n) = 0$), the sampling period $T = 1ms$, the buffer length $y_{\max} = 1000 cells$, the buffer set point $y_0 = y_{\max}/2$ and the available bandwidth $b_0 = 100 cells/ms$. The delays on the forward path can vary $\tau_{2,1}(n) \in \{0, 1, 2, 3, 4, 5\}$, i.e. $\bar{\tau}_2 = 5$. We use a proportional controller having a gain such that all the frozen time systems are stable.

We start the simulation with the system at the equilibrium point ($y_S(n) = y_0$) and with $\tau_{2,1}(n) = 0$. After 10 sampling periods we randomly change the delays in the forward path. In order to illustrate the fact that even slow changes will perturb the equilibrium, we will only change the delays by one unit at a time. In Figure 6 (a) the time trace of the delay on the forward path is shown. In Figure 6 (b) the effect of those changes on the buffer occupancy level is shown. It is clear that, as predicted, the equilibrium has been perturbed. If the same delay trace is applied on the return path (HFS interface) while keeping the delays on the VBR side constant, the equilibrium point can be maintained (see Figure 6 (c) where the buffer is initially empty).

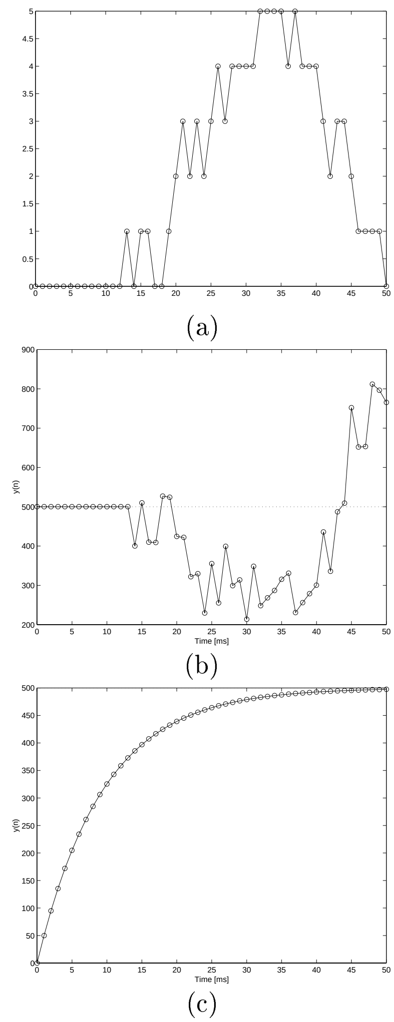

Figure 6: (a) The time-variant delay trace; (b) Buffer occupancy when the delay is on the VBR side; (c) Buffer occupancy when the delays is on the HFS side

4.2 Stability analysis

We will analyze in detail the case of a single source. We will use the following parameters for our system:

- The bandwidth available for ABR traffic $b_0 = 1500$ cells/sec.
- The maximum rate $R_{1,\max} = 2b_0 = 3000$ cells/sec.
- The buffer length $y_{\max} = 10000$ cells.
- The buffer set point $y_0 = \frac{1}{2} y_{\max} = 5000$ cells.
- The controller cycle time $T = 1$ msec.
- The maximum delay on the return path $\bar{\tau}_{1,1} = 10$ msec.
- The delay on the forward path is fixed: $\tau_{2,1}(n) = 1$ msec.
- We used a proportional controller with a gain $-G$.

Product length k	2	3	4	5	6	7	8	9
Controller gain $0 < G \leq$	3.92	21.67	31.23	34.42	36.66	37.58	38.31	38.68

Table 1: The case of a single source for the linear case: Stable proportional controller gains as a function of k

Product length k	5	6	7	8
Controller gain $0 < G \leq$	0.2421	0.3212	0.3613	0.4013

Table 2: The case of a single source with nonlinearities and constant delays

At first we assume that the buffer and the source work in the linear range ($Q(n) = R_1(n) = 1$). In this case the time invariant analysis results in the following stability condition:

$$G \in (0,\ 136.48) \tag{40}$$

In this case the sufficient stability condition for the time-variant case introduced in [7] yields the following stability condition:

$$G \in (0,\ 6.57) \tag{41}$$

In Table 1 the results of the stability test discussed in Theorem 2 are presented for the linear time-variant case.

If we consider the rate and queue nonlinearities, we first perform a constant delay analysis. Using Theorem 2 we analyze for stability each polytope $\mathcal{P}_{\tau_{1,1}}$ defined in equation (35) for all possible delays $\tau_{1,1} \in \{0, 1, \ldots, 10\}$. The results are presented in Table 2.

Finally we analyze the time-variant system with the queue and rate nonlinearities using Theorem 2. The results are presented in table 3.

Product length k	3	4	5
Controller gain $0 < G \leq$	0.1577	0.2359	0.3093

Table 3: The case of a single source with nonlinearities and time-variant delays

It can be seen from tables 1, 2 and 3 that the resulting stabilizing gains in the presence of nonlinearities are considerably smaller than in the linear case. This is expected and suggest that if one designs a fixed controller for the linear range of rates and buffers, it is imperative to stay away from the "nonlinear" operating region of the network.

5 Conclusion

This paper introduced a time-variant delay model for the ABR option of ATM networks. The introduced congestion control system is capable of modeling time-variant communication delays between a single congested node and several sources (in both directions), rate and buffer non-linearities, RM-cell losses as well as the mismatch between time-variant RM cell periods and the controller cycle time. Furthermore the model was analyzed for stability of a chosen equilibrium point (typically given as a nominal buffer occupancy level). It was shown that the forward path does not allow for an equilibrium, if the delays in this link are varying over time. In the case where the delays in the forward path can be modeled as time-invariant an equilibrium exists and its stability can be analyzed using Theorem 2. More work is needed to further reduce the complexity of the introduced stability test.

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