Project 4

Topic: Tangent line approximations.

This project looks at approximating functions using tangent lines, or as the book says, local linearization. Let's look at a specific example.
Suppose we are given the horrible function:

$$f := x \to \sqrt{(1+x)^{3/2} + (1-x)^{5/2}}$$

Note that the domain of the this function is [-1,1]. Now clearly we cannot use just one tangent line to approximate this function on its entire domain. However if we subdivide the interval into smaller subintervals and find a tangent line to the curve on each subinterval then this sequence of tangent lines may well give us a good approximation to f(x). A way of doing this on Maple V and seeing the outcome would be as follows:

```
> f:=x->((1+x)^(3/2)+(1-x)^(5/2))^(1/2);
> b:=plot(f(x),x=-1..1):
> dev:=D(f);
> for i from 0 to 8 do x.i:=i/4-1;  y.i:=evalf(f(x.i));od:
> tangent:=i->(x-x.i)*dev(x.i)+y.i;
> a:=j->plot(tangent(j),x=x.j..x.(j+1),color=red):
> with(plots):
> display({seq(a(j),j=0..7),b});
> c:=k->plot(f(x)-tangent(k),x=x.k..x.(k+1)):
> display({seq(c(k),k=0..7)});
> i:='i'; j:='j'; k:='k';
```

Study these commands making sure you can follow the mathematics and then enter them into a Maple V window and execute them.

Assignment.

a) For each line of input above write one comment line or a line of text explaining the input.

(b) By using the mouse on the last graph estimate the largest error between the tangent lines and the function. Now evaluate this error to 10 digits.

(c) In the above example the lefthand endpoint of each subinterval was chosen to to locate the tangent line. Repeat the question using the midpoint of the subinterval.