

Project 2

Topic: Elementary curve fitting.

Curve fitting is a very important topic in mathematics and in all areas which use mathematics, such as engineering and science. The situation frequently arises in which we are given a set of data which we would like to link through some given function. As with all topics some cases will be easy and some hard.

(a) Let's first look at a simple problem: given three points - find a parabola which passes through them. To see how we can do this let's suppose the points are (1,4), (2,6) and (4,2). Any parabola has the form $y = ax^2 + bx + c$. In rough terms then we think that having three points will determine the three constants a, b and c . Maple V does this as follows:

```
>f:=x->a*x^2+b*x+c;
```

$$f := x \rightarrow ax^2 + bx + c$$

```
>u:=solve({f(1)=4,f(2)=6,f(4)=2},{a,b,c});
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$$u := \left\{ a = \frac{-4}{3}, b = 6, c = \frac{-2}{3} \right\}$$

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>y=subs(u,f(x));
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$$y = -\frac{4}{3}x^2 + 6x - \frac{2}{3}$$

Suppose now that we asked to fit a straight line through three or more points which are not in a straight line. In particular suppose we want to find the "best fit" line through the points (1,1), (2,5/2), (3,3), (4,3), (5,6), (6,5), (7,9) and (8,8). One approach to this problem would be to take the line $y = m*x+b$, say, and to find m and b so as to minimize the sum of the vertical distances between the given points and the corresponding points on the line. The following Maple V commands do this. The first thing you should do is to enter these commands into a Maple V worksheet and execute them and then figure out the mathematical steps which have been used.

```
> x1:= [1,2,3,4,5,6,7,8];
> y1:= [1,5/2,3,3,6,5,9,8];
> dist:=sum((m*x1[r]+b-y1[r])^2,r=1..8);
> with(student):
> u:=expand(dist);
> completesquare(u,m);
> completesquare((28/17)*b^2+(9/17)*b+1379/204);
> line:=(107/102-(3/17)*(-9/56))*x-9/56;
> l:=x1[1],y1[1];
> for n from 2 to 8 do l:=l,x1[n],y1[n]: od:
> a:=plot([l],style=point):
> b:=plot(line,x=0..8):
> with(plots):
> display({a,b});
```

Assignment:

- (a) Follow the above procedure and find the best fit line through the points: $(1,2)$, $(2,2)$, $(3,3)$, $(4,3)$, $(5,4)$, $(5,5)$, $(6,7)$ and $(7,7)$.
- (b) By writing a similar procedure find the best fit parabola through the points in (a).