

Lab 9.

Topic: Points of Inflection

You have learned in class how to find points of inflection, that is to say points at which the concavity of a curve changes from positive to negative or vice versa. To locate possible points of inflection we find points where the second derivative is zero. Frequently we can do this numerically. This is based on the premise that if a function is twice differentiable on an interval then:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h},$$

and

$$f''(x) \approx \frac{f'(x+h) - f'(x)}{h}.$$

If we stipulate that h is positive then these two forms of the difference quotient are called "right derivatives". We could easily have used "left derivatives":

$$f'(x) \approx \frac{f(x-h) - f(x)}{-h},$$

and

$$f''(x) \approx \frac{f'(x-h) - f'(x)}{-h}.$$

Assignment.

(a) By using the appropriate forms of these show that;

$$f''(x) \approx \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}.$$

(Note here that the denominator is h^2 and so it is positive. Thus the sign of the second derivative is the same as that of the numerator.)

(b) Consider now the following logistic function,

$$f(x) = \frac{1}{1+3e^{-x}}.$$

(i) Plot this function on $[0,6]$ and describe in your own words the features of the graph. Use the mouse to estimate the point of inflection.

(ii) Find the point of inflection the traditional way.

(iii) Now use the formula in (a) to find the point of inflection. You might proceed in this way:

The point is obviously between 0 and 3.

Divide the interval $[0,3]$ into say 90 equal subintervals.

Now use Maple to evaluate the **numerator** in (a) in successive pairs of subintervals until it changes sign.

(iv) How many subintervals are needed to find this point to 3 decimal places?

Incidentally Wal-Mart assumes that sales of goods are modeled by logistic curves. They use this procedure monthly for each item to see if the point of inflection has been reached. When it is they discontinue stocking the goods. It must work since Wal-Mart rarely has sales on discontinued items.