Lab 6

Topic: Differentiability/tangent lines.

In this lab we want you to investigate the differentiability of a function by looking at the tangent lines to its graph.
As review let us determine the tangent line of f(x) = x^2 + 2*x +3, say at (1,6).To do this we would enter the following into Maple V:

```
>f:=x->x^2 + 2*x + 3;
```

Since we want the derivative to be a function we will use the D operator, and so

```
>derf:=D(f);
```

```
>tgtline:= (x-1)*derf(1)+f(1);
```

The output for this last line will be tgtline:= 4*x+2.
We now want to stretch this idea further. We want to find the tangent line at any point (t,f(t)) on the graph. All we do is write:

```
>tgtline:=(x-t)*derf(t)+f(t);
```

The output for this is will be tgtline:=(x - t) * (2*t + 2) + t^2 + 2*t + 3.
Let's now use Maple V to show us how the tangent line moves on the graph of f(x) as we vary t. We will a use command called "animate". Warning: be careful how you use this feature since it uses a lot of memory. After working each problem, save your work, close down your Maple V worksheet and then bring up a new one. In this way you should not run out of memory.
We first need:

```
>with(plots):
```

Now enter the command:

```
>animate(tgtline,x=-3..1,t=-3..1,color=red);
```

When the graphics window appears it will have some controls like a tape recorder or videocassette player. Put the mouse on the "play" button and click. The line should now move. You should experiment with the other controls so as to become familiar with them.
Now go back and modify the command to:

```
>a:=animate(tgtline,x=-3..1,t=-3..1,color=red):
```

(Note the colon, not a semicolon at the end of this command.)

```
>b:=animate(f(x),x=-3..1,t=-3..1,color=black):
```

```
>display({a,b},view=[-3..1,-8..10]);
```

Now click the "play" button. What you should see is the tangent line rolling smoothly along the curve, indicating (not proving) that the function is differentiable at all x's in (-3,1).

Now examine the following functions using the same general procedure as above. If you encounter any points at which the tangent line does not move smoothly examine them using the definition of the derivative and clearly state your conclusions. Are there any points where the tangent line appears to move smoothly but where the function is not differentiable?

(a) $f(x) = \sin(x)$, on $-2 <= x <= 2$.

(b) $f(x) = \text{abs}(\sin(x))$, on $-2 <= x <= 2$.

(c) $f(x) = x^2$ if $x < 0$ and $= x^3$ if $x >=0$, on $-3 <= x <= 3$.

(d) $f(x) = x^2$ if $x < 0$ and $= x - x^2$ if $x <= 0$, on $-3 <= x <= 3$.

(e) $f(x) = \cos(x)$ if $x < 0$ and $= \exp(x/50)$ if $x > 0$, on $-3 <= x <= 3$.