

## Lab 4

Topic: Piecewise defined functions.

In this lab we'll look at functions which have different definitions on different intervals. Let's call these function "piece wise defined" functions.

(a) A well-know such function is the Heaviside function, defined as follows:

$$\text{Heaviside}(x) = 1, \quad x \geq 0,$$

$$\text{Heaviside}(x) = 0, \quad x < 0.$$

By using this function we can give one line definitions of many piece wise defined functions. For example figure out the multiline definitions of each of the following:

$$(i) \ y = x * \text{Heaviside}(x), \quad (ii) \ y = x^2 * (1 - \text{Heaviside}(x)) + (\exp(x) - 1) * \text{Heaviside}(x),$$

$$(iii) \ y = x^2 * (\text{Heaviside}(x-1) - \text{Heaviside}(x-3)),$$

$$(iv) \ y = \text{abs}((\text{abs}(x)-1)) * (\text{Heaviside}(x+2) - \text{Heaviside}(x-2))).$$

By the way if you get tired of writing Heaviside you can use the alias command:

```
>alias(H=Heaviside);
```

which now allows you to write H(x).

(b) Another method which will allow you to enter such functions into MAPLE is to use "if" statements. For example function (a)(i) can be put into MAPLE as follows:

```
>f:=x->if x<0 then 0 else x;fi;
```

```
>plot('f(x)',x=-4..4);
```

(Note the single quotes in the plot command.)

(a)(ii) would be:

```
>f:=x->if x<0 then x^2 else exp(x)-1;fi;
```

while a(iv) would be:

```
>f:=x-> if x>-2 and x<2 then abs(abs(x)-1) else 0;fi;
```

Now do the following both ways, verifying your results by plotting both versions in MAPLE.

c(1)

$$\begin{aligned} f &= -1, & x < -1, \\ &= x, & -1 < x < 1, \\ &= 1, & 1 < x. \end{aligned}$$

c(2)

$$\begin{aligned} f &= -x, & x < -1, \\ &= x^2, & -1 < x < 1, \\ &= x, & 1 < x. \end{aligned}$$

c(3)

$$\begin{aligned} f &= \sin(x), & -\pi < x < \pi, \\ &= 0, & \text{elsewhere.} \end{aligned}$$

c(4)

Work problem 49 on page 91 of your textbook. (Hint: There are 12 hours of sunlight at the equator and  $S$  is continuous at  $x_0$ .)