Lab 11.

Topic: Newton's Method.

This lab is about testing a rule of thumb for finding square roots of numbers between 1 and 100. First of all recall Newton's method for finding zeros of a function $f(x)$. Let your initial guess or estimate for the root be x_0. Then a better estimate should be,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)},$$

provided $f'(x_0) \neq 0$.
We then keeping iterating with the scheme,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

provided $f'(x_n) \neq 0$.

Suppose we wish to find the roots of $f(x) = x^3 - 3x^2 - x + 2$. A Maple V program which will do the iteration could be the following:

```
>f:=x->x^3-3*x^2-x+2;
```

To get an idea of where the roots are we plot the function,

```
>plot(f(x),x=-2..5);
```

```
>newt:=proc(x0,n)
>local k,u;
>u:=x0;
>for k to n do u:=evalf(u-f(u)/D(f)(u));od;
end;
```

If you've typed this in correctly you will get the response

```
newt:=
        proc(x0,n)
        local k,n;
        u=x0; for k to n do u:=evalf(u-f(u)?D(f)(u)) od
        end
```

From the graph you will note that the largest root is near x=3, and so we ask for

```
>newt(3,100);
```

This says our initial guess is 3 and we want 100 iterations.
You should get the answer 3.114907542.
Find similar iterations for the other 2 roots.

Now do the following:

(a) Show that if we apply Newton's method to finding square roots, in particular $\sqrt{a}$, in other words find the zeros of the function $f(x) = x^2 - a$, the above formula becomes,

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

(b) A student has proposed the following rule of thumb concerning the initial guess x_0:

$$\textit{if} \quad 1 \le a \le 10 \quad \textit{then} \quad x_0 = \frac{a}{2},$$

$$\textit{if} \quad 10 \le a \le 20 \quad \textit{then} \quad x_0 = \frac{a}{3},$$

$$\textit{if} \quad 20 \le a \le 30 \quad \textit{then} \quad x_0 = \frac{a}{4},$$

and so on up to $a = 50$. The student claims that with just one iterate (x_1) this rule gives an accuracy to one decimal place. Is she correct?

(c) What accuracy does the above rule give if we go to the second iterate (x_2)?