Algebra Projects

All projects will be presented in the last week of classes: Nov. 29 and Dec. 1. Each project presentation should take about 15–20 minutes. Your group should explain what your project is about in enough detail so that all students get a good sense of the material. You will probably choose to present some of the exercises you were assigned, but you need not present them all. A careful selection is a good idea.

Your group will also be responsible for handing in a short write up of your work. Be sure to give a brief summary of your topic as well as to write up all the problems you have done (as a group). This write up is due December 1st, by 2pm.

Projects from Gallian

** **Conjugacy and Sylow Theorems.**
Note $C(a)$ is the centralizer of a which you studied last time and $Z(G) = C(G)$ is the center of the whole group. Problems #4, 5, 44, 45 in Gallian Chapter 24.

* **Generators and Relations.**
Chapter 26 in Gallian: problems #4,6,8, 10.

* **Data security and cryptography.**
Chapter 8 in Gallian: problems #68, 69, + extra problem on handout.

* **Symmetry groups.**
Chapter 27 in Gallian: problems #4, 6, 8, 10, 14.

* **Soccer Balls and Isometries in 3 dimensions.**
Chapter 7 and 27 in Gallian: problems #6, 8 in Chapter 27 and #42, 44, 46 in Chapter 7.

* **Frieze Groups and Wall paper patterns.**
Chapter 28 in Gallian. Problems # 1,2,11,12.

* **Cayley Digraphs.**
Chapter 30 in Gallian. Problems # 10, 14, 15, 17.

* **Coding Theory.**
Chapter 31 in Gallian. Problems # 3,10,13,17.

Projects from “A Concrete Introduction to Higher Algebra” by Lindsay N. Childs

* **Factoring large numbers**. Chapter 7E (needs 7C and 7D) in Childs: problems # E3, E5, E7.

* **Knapsack Cyptosystems**. Chapter 7F in Childs: problems # 1, 2.

* **Fractions in base *a***. Chapter 10A in Childs: problems # 10, 17, 21, 22.

* **RSA codes**. Chapter 10B in Childs: problems # 1, 2, 5.

* **Pseudoprimes**. Chapter 10C, D in Childs: problems # C5, C11, D2.

* **Error-Correcting Codes and Hill Codes**. Chapter 13E and 13F in Childs: problems # E3, E6 and F5, F8.

Other Algebra Projects

Project: Automata

Automata are one of the many reasons that computer scientists need to understand some abstract algebra.

The reading for your project is page 42-43 and 100-101 of Fraleigh. Do at least the following problems: Page 43 #34, 38. page 101 # 50, 52, 54, 56.

Notes: (i) Monoids and semigroups are sets with binary operations and properties that are weaker that a group. They are defined on page 52 of Fraleigh.
(ii) Ignore the sentences on Cayley digraphs - another group of students might talk about them.

J.B. Fraleigh: *A first course in Abstract Algebra*, Sixth Edition, 1999, Addison-Wesley.

PROJECT: CENTRALIZERS AND THE CENTER OF GROUPS

Not every group is commutative (abelian). In groups that are not, we can look at for which elements in the group the group operation is in fact commutative. For a specific element $a \in G$ (where G is a group), we define the *centralizer* of a to be the set

$$H_a = \{x \in G \mid xa = ax\}.$$

In other words, the centralizer of an element is the set of all elements in G which commute with that element.

(i) Let $G = \mathbf{GL}(2, \mathbf{R})$, the group consisting of all invertible 2×2 matrices with entries from $\mathbf{R}$. Find the centralizer of the element $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(ii) Now find the centralizer of

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(iii) Prove that the centralizer H_a is a subgroup of G.

(iv) We can generalize the notion of a centralizer to any set $S \subset G$.

$$H_S = \{x \in G \mid xs = sx, \text{for all } s \in S\}.$$

Returning to the example. Let $S = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\}$.
Find H_S for this set.

(v) If $a \in S$ what can be said about H_a and H_S? Do they always intersect? Must one contain the other?

(vi) The *center* of a group G is just H_G. Show that H_G is an abelian group.

(vii) Determine the center of $G = \mathbf{GL}(2, \mathbf{R})$. You might want to first find the centralizer of $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(vii) Determine the center of $G = \mathbf{GL}(3, \mathbf{R})$.