MATH 407, SAMPLETEST 1

(20 pts.) (1)(a) Show that in an abelian group G the set consisting of all elements of G of finite order is a subgroup of G.

(b) Is the same result true if G is not abelian? EXPLAIN!

(20 pts.) (2) The set $\{2, 4, 6, 8\}$ forms a group under multiplication modulo 10.
(a) What is the identity of this group?

(b) Is this group cyclic? If so, list all generators.

(30 pts.) (3) For subset S of a group G define the normalizer of S to be the set:

$$N_G(S) := \{g \in G \mid gxg^{-1} \in S \text{ for all } x \in S\}.$$

(a) Is $N_G(S)$ a subgroup of G? If yes, give a proof. If no, give a counter example.

(b) Assume $G = S_3$ and $S = \{(12), (23)\}$. Compute the centralizer $C(S)$ and normalizer $N_G(S)$ of S.

(30 pts.) (4) Let G be a group. State whether each of the following is true **always, sometimes or never.** If **sometimes**, then explain under what circumstances it is true.

(a) The intersection of 3 subgroups of G is a subgroup of G.

(b) If g is a group element and $g^n = e$, then $|g| = n$.

(c) If G is a cyclic group then it is abelian.

(d) If G is infinite, then an element a in G generates an infinite subgroup.

(e) If a and b are elements of an Abelian group, then $|ab|$ is the lcm $(|a|, |b|)$.

(f) Any group has only one element of order 1.

(g) If $a, b, c \in G$ and $b \in C(a)$ then $abc = bac$.