1. (10 pts)* Sketch the surfaces given by the following equations:
   (a) \( y = 9 - x^2 - z^2 \), (b) \( x^2 + y^2 = z^2 \)
   **SOLUTION:** The first is an elliptic paraboloid that has vertex at \((0, 9, 0)\) and opens along the negative y-axis. The second is a cone opening along the z axis. See your textbook for pictures.

2. (10 pts)* Write the complete Maple commands that you would enter on a Maple worksheet to carry out the following steps. Assume that you have already entered the command with(linalg):
   (a) Define the vectors \( \vec{A} = 3\vec{i} - 4\vec{j} + 7\vec{k} \) and \( \vec{B} = -2\vec{i} - 12\vec{j} + 16\vec{k} \).
       **SOLUTION:** \( A := [3, -4, 7] \); \( B := [-2, -12, 16] \); or replace : with ;
   (b) Compute the angle between vectors \( \vec{A} \) and \( \vec{B} \).
       **SOLUTION:** \( \text{angle}(A, B) \); or \( \arccos(\text{innerprod}(A, B)/(\text{norm}(A, 2)*\text{norm}(B, 2))) \)
   (c) Compute the area of the parallelogram spanned by vectors \( \vec{A} \) and \( \vec{B} \).
       **SOLUTION:** \( \text{norm(\text{crossprod}(A, B), 2)} \)

3. (30 pts)** Consider the parametric curve \( C \) given by
   \[ \vec{r}(t) = (2t)\vec{i} + (t^2)\vec{j} + (-\frac{1}{3}t^3)\vec{k} \]
   (a) Compute the **velocity vector** \( \vec{v}(t) \), the **acceleration vector** \( \vec{a}(t) \), and the **speed** \( v(t) \) as functions of the parameter \( t \).
       **SOLUTION:** \( \vec{v}(t) = <2, 2t, -t^2> \), \( \vec{a}(t) = <0, 2, -2t> \), \( v(t) = \sqrt{4 + 4t^2 + t^4} \)
   (b) Compute the **tangential and normal components** \( a_T(t) \) and \( a_N(t) \) of acceleration.
       **SOLUTION:** \( a_T = \frac{\vec{v} \cdot \vec{a}}{v(t)} = \frac{4t - 2t^3}{v(t)} \), \( a_N = |\vec{a} \times \vec{v}| / v(t) = \sqrt{16 + 16t^2 + 4t^4} / v(t) = 2 \)
   (c) Find **parametric equations of the tangent line** to the curve at the point \( (2, 1, -\frac{1}{3}) \) on the curve.
       **SOLUTION:** The point \( (2, 1, -\frac{1}{3}) \) corresponds to the parameter value \( t = 1 \). (You find this valued by solving \( x(t) = 2t \) with \( x = 2 \)). The tangent vector to the curve at \( t = 1 \) is \( v(1) = <2, 2, -1> \), which is the direction vector for the line. Hence the parametric equations are
       \[ x = 2 + 2t, \ y = 1 + 2t \text{ and } z = -\frac{1}{3} - t \]
4. (20 pts)** Let \( \vec{A} = 2\hat{i} + 4\hat{j} - 8\hat{k} \) and \( \vec{B} = \hat{i} + \hat{j} + \hat{k} \). Find the orthogonal decomposition of \( \vec{A} \) with respect to \( \vec{B} \). That is, find two vectors \( \vec{A}_\parallel \) and \( \vec{A}_\perp \) where \( \vec{A}_\parallel \) is parallel to \( \vec{B} \) and \( \vec{A}_\perp \) is perpendicular to \( \vec{B} \), and such that \( \vec{A} = \vec{A}_\parallel + \vec{A}_\perp \)

**SOLUTION:**

\[
\vec{A}_\parallel = (\vec{A} \cdot \vec{B}) \frac{\vec{B}}{|\vec{B}|^2} = \left(\frac{-\frac{2}{3}}{\frac{2}{3}}\right) \vec{B} = < -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3} >
\]

\[
\vec{A}_\perp = \vec{A} - \vec{A}_\parallel = < 2, 4, -8 > - < -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3} > = < \frac{2}{3}, 4\frac{2}{3}, -7\frac{1}{3} >
\]

Of course answers can also be given in terms of \( \hat{i}, \hat{j} \) and \( \hat{k} \).

5. (20 pts)** Find an equation for the plane that contains the three points \( P = (2, 1, 1) \), \( Q = (0, 4, 1) \) and \( R = (-2, 1, 4) \).

**SOLUTION:** \( \vec{PQ} = < -2, 3, 0 > \) and \( \vec{PR} = < -4, 0, 3 > \). Both vectors are parallel to the plane, so their cross product \( \vec{PQ} \times \vec{PR} = < 9, 6, 12 > \) will be a normal vector for the plane. Choosing \( P_0 = P_1 \) (You can choose any of the three points for \( P_0 \)) we obtain the equation (there are three possible answers)

\[
9(x - 2) + 6(y - 1) + 12(z - 1) = 0 , \text{ or any non-zero multiple.}
\]

\[
9(x - 0) + 6(y - 4) + 12(z - 1) = 0 , \text{ or any non-zero multiple.}
\]

\[
9(x + 2) + 6(y - 1) + 12(z - 4) = 0 , \text{ or any non-zero multiple.}
\]

6. (10 pts)*** Find parametric equations for the line through the point \( (0, 1, 2) \) that is parallel to the plane \( x + y + z = 2 \) and perpendicular to the line \( x = 1 + t, y = 1 - t \) and \( z = 2t \).

**SOLUTION:** The point on the line is given to us as \( P_0 = (0, 1, 2) \). Now we need a direction vector \( \vec{v} \). If the line we want is parallel to the plane \( x + y + z = 2 \), then the normal vector \( \vec{n} = < 1, 1, 1 > \) for the plane must be orthogonal to the direction vector \( \vec{v} \) that we seek. Also, if the line we seek is perpendicular to the line \( x = 1 + t, y = 1 - t \) and \( z = 2t \), then the direction vector \( \vec{v}_1 = < 1, -1, 2 > \) for this line must also be orthogonal to the direction vector \( \vec{n} \) that we seek. Hence \( \vec{v} \) must be orthogonal to both \( \vec{n} = < 1, 1, 1 > \) and \( \vec{v}_1 = < 1, -1, 2 > \), so we can take the cross product of \( \vec{n} = < 1, 1, 1 > \) and \( \vec{v}_1 = < 1, -1, 2 > \) as the direction vector \( \vec{v} \). So \( \vec{v} = < 1, 1, 1 > \times < 1, -1, 2 > = < 3, -1, -2 > \), or any non-zero multiple. Hence the equations for the line are

\[
x = 0 + 3t, \ y = 1 - t, \ z = 2 - 2t
\]