

## Study Guide for Test #3 MA 242.601 and MA 242.651

The test will cover the following sections of Chapter 12:1, 2, 3, 4, 5(see below), 7 and 8. In addition it will contain material from Chapter 9, section 7 on cylindrical and spherical coordinates.

1. Chapter 12, section 1: Double integrals over rectangles
  - (a) You should know the general definition (5) of the double integral of a function  $f(x,y)$  over a rectangular region  $R$ .
  - (b) You should know the definition in the box at the bottom of page 832 for the **volume below the graph of a function  $f(x,y)$**  and above a region  $R$  in the  $xy$ -plane.
  - (c) You should know the definition of the **average value of a function** on a rectangle given on page 835.
  - (d) You should know the properties of double integrals given on pages 836-837 of your text book.
  - (e) You should realize that the above items will be generalized in section 12.3 where we will no longer require the region of integration to be a rectangle.
  
2. Chapter 12, section 2: Iterated integrals and Fubini's Theorem.
  - (a) Be able to compute iterated integrals (double and triple) such as those given in problems 3 - 14 on page 843, and problems 3 - 8 on page 900. At most you will be required to use "substitution" to evaluate such integrals.
  - (b) You should know Fubini's theorem ((4) - page 840) and be able to apply it to problems like those worked in the text and the examples I worked for you in class.
  
3. Chapter 12, section3: Double integrals over general regions.
  - (a) You should know how to use the two basic theorems (3) and (5) for evaluating double integrals over type I and type II regions.
  - (b) You should be able to decompose a general region into a set of subregions, each of type I or type II. See problems 53 and 54 at the end of the section.
  - (c) You should be able to compute volumes below graphs of functions  $f(x,y)$  and above a general region in the  $xy$ -plane.
  - (d) You should be able to find the volume between the graphs of two functions  $f(x,y)$  and  $g(x,y)$  by reducing this problem to one you have already solved. For example, the volume of the region between the paraboloids  $z = x^2 + y^2$  and  $z = 18 - x^2 - y^2$  would be found as follows. These two paraboloids intersect in the circle of radius 3 centered on the origin in the  $xy$ -plane. You find this "curve of intersection" by solving the two equations simultaneously, eliminating  $z$ . Let the region inside this circle be denoted  $D$ . Then the volume between the two paraboloids would be given by the double integral of  $18 - x^2 - y^2$  over  $D$  MINUS the double integral of  $x^2 - y^2$  over the region  $D$ .

- (e) You should be able to "reverse the order of integration" on a given iterated integral. To do this you:
- Use the limits on the given iterated integral to write down the description of the region in set notation.
  - Use the set notation to sketch the region.
  - If the set notation indicates that the region is type I, then use the sketch to rewrite it as type II, and conversely.
  - Set up the iterated integral in the opposite order.
- (f) Chapter 12, section 4: Double integrals in polar coordinates
- You should know and be able to use theorems (2) and (3) to set up and evaluate double integrals in polar coordinates. This involves using the transformation equation  $x = r \cos(\theta)$  and  $y = r \sin(\theta)$ . See the examples worked in the textbook and the examples I worked for you in class.
- (g) Chapter 12, section 5: Applications of double integrals. Below are the applications you are responsible for.
- Volume below the graph of a function and above a general region in the xy-plane.
  - Average value of a function over a general region in the xy-plane
  - Area of a general region in the xy-plane: In this case the integrand of the double integral will be 1.
  - Densities: If  $\rho(x, y)$  is the mass (or charge) density of a region D in the xy-plane, then the total Mass (or charge) of the region is the double integral of  $\rho(x, y)$  over the region D.
- (h) Chapter 12, section 7: Triple integrals in Cartesian coordinates.
- - You should know and be able to apply the three versions (6), (7) and (8) of Fubini's theorem for triple integrals. See the examples worked in the book and the many examples I worked for you in class.
  - Know the formula (12) for the volume of the 3-dimensional region using triple integration.
4. Chapter 12, section 8: Triple integrals in cylindrical coordinates.
- See Chapter 9 section 7 for the definition of cylindrical and spherical coordinates.
  - Know the transformation equations: Cylindrical coords:  $x = r \cos(\theta)$ ,  $y = r \sin(\theta)$  and  $z = z$  and Spherical:  $x = \rho \sin(\phi) \cos(\theta)$ ,  $y = \rho \sin(\phi) \sin(\theta)$  and  $z = \rho \cos(\phi)$  in order to be able to transform an integrand given in terms of x,y and z to either of these coordinate systems.
  - Be able to use the above to set up a triple integral as a triple iterated integral in either cylindrical or spherical coordinates. See the examples worked in the text book and the many examples I worked for you in class.