

Study Guide for Test # 2
MA 242.601 and MA 242.651
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You should be able to work problems of the following type:

1. Determine the largest set on which a given function of 2 or more variables is continuous. In doing so you will need to use the following facts: (Section 11.2)
 - (a) Polynomial functions are continuous everywhere.
 - (b) Rational functions are continuous at all points where the denominator polynomial is non-zero.
 - (c) The composition of a continuous function with a continuous function is continuous. More precisely, if $f(x, y)$ is continuous at (x_0, y_0) and $g(x)$ is continuous at $f(x_0, y_0)$, then the composite function $g \circ f$ is continuous at (x_0, y_0) .
 - (d) The sum or difference of continuous functions is a continuous function.
 - (e) The product of continuous functions is a continuous function.
2. Use the following theorem plus the theorems listed above to determine if a function $f(x, y)$ is differentiable or not at a point. (Section 11.4)

A function $f(x, y)$ is differentiable at a point $P_0 = (x_0, y_0)$ if the x and y partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist near P_0 and $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous at P_0 .

3. Be able to compute first and second partial derivatives of multivariable functions (Section 11.3)
4. Be able to write down the chain rule formulas for the following situations:
 - (a) Given $f(x, y, z)$ where $x = g(t)$ and $y = h(t)$ and $z = l(t)$ are given functions of the independent variable t , compute

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

- (b) Given $f(x, y, z)$ where $x = g(r, t)$ and $y = h(r, t)$ and $z = l(r, t)$ are given functions of the independent variables r and t , compute

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial r}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}$$

5. Know how to compute the directional derivative $D_{\vec{u}}f(P_0)$ of a multivariable function f at a point P_0 in a specified direction.

6. Know how to compute the gradient $\nabla f(x, y, z)$ of a function $f(x, y, z)$, and know the geometrical significance of (1) the magnitude of $\nabla f(x, y, z)$ and (2) the direction of the gradient vector. (See theorem 15, page 804 in your textbook).
7. Be able to find: (1) the tangent plane to a level surface of a function of 3 variables, and (2) the tangent plane to the graph of a function of 2 variables.
8. Be able to find all critical points of a function $f(x, y)$ of two variables, and then be able to use the second derivative test to determine if the critical points correspond to local maxima, local minima or saddle points of the function. (Section 11.7)