MA 114-002
Test 1-B

Spring 2006

Name Key

Row ______

Read each question carefully. You must SHOW ALL WORK for full credit. Put all work and answers on this paper. Good luck.

28 points

(1) Use the following matrices to answer the questions if possible, otherwise write "not possible."

$$A=\begin{vmatrix} 1 & 5 & 4 & 0 \\ 2 & 3 & 1 & 3 \end{vmatrix} \quad B=\begin{vmatrix} 1 \\ 2 \\ 0 \\ 1 \end{vmatrix} \quad C=\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} \quad D=\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}$$

Find:

a) The size of matrix A 2×4

b) C + 2D

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} + 2\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 7 \\ 2 & 2 \end{bmatrix}$$

c) BA

$B_{4\times 1}$ $A_{2\times 4}$ Not possible

d) a_{32} Not possible

e) c_{12} 3

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f) CAB Remember: CAB = C(AB) = (CA)B

$$\begin{bmatrix}1 & 3\\0 & 2\end{bmatrix}\begin{bmatrix}1 & 5 & 4 & 0\\2 & 3 & 1 & 3\end{bmatrix}\begin{bmatrix}1\\2\\0\\1\end{bmatrix} = \begin{bmatrix}1 & 3\\0 & 2\end{bmatrix}\begin{bmatrix}11\\2+6+0+3\end{bmatrix}$$

$$= \begin{bmatrix}1 & 3\\0 & 2\end{bmatrix}\begin{bmatrix}11\\11\end{bmatrix} = \begin{bmatrix}11+33\\22\end{bmatrix} = \begin{bmatrix}44\\22\end{bmatrix}$$

g) What is the ADDITIVE identity matrix for B (that is B + ___ = B)

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

(8 points) (2) Multiply

$$\begin{vmatrix}3\\5\end{vmatrix}\ [a \quad 2] = \begin{bmatrix}3a & 6\\5a & 10\end{bmatrix}$$

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(24 points) (3) Solve the following system of equations using the Gauss-Jordan method. SHOW ALL STEPS and label all row operations.

$$\begin{aligned} 2x + 4y &= 4 \\ 3x - 4y &= 16 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 4 & 4 \\ 3 & -4 & 16 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 3 & -4 & 16 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -10 & 10 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right] \qquad \begin{aligned} x &= 4 \\ y &= -1 \end{aligned}$$

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(14 points) (4) Graph the region determined by the following system of inequalities and find all intersection points.

$x + 2y \geq 2$ $\quad$ $y \leq 2$ $\quad$ $x \leq 2$

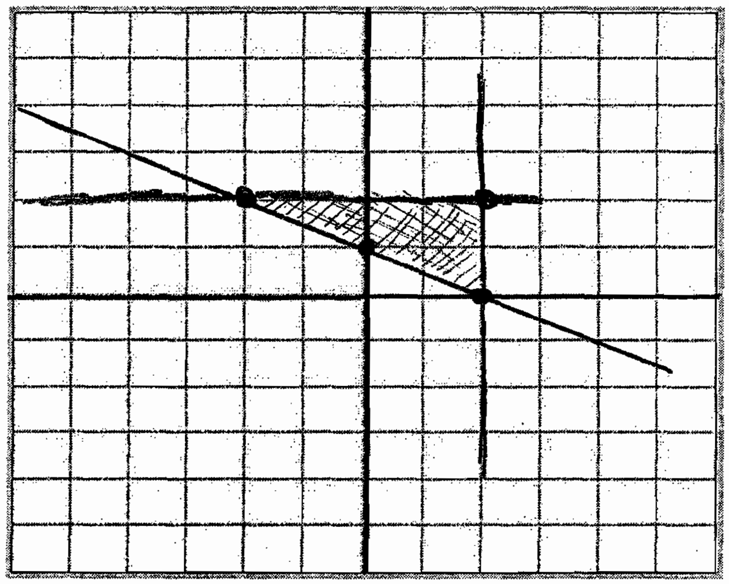
① $x + 2y = 2$ above line

$x = 0 \quad y = 1$

$x = 2 \quad y = 0$

② $y = 2$ below line

③ $x = 2$ left of line



① + ②

$x + 2y = 2$

hits $y = 2$

at $x + 4 = 2$

$x = -2$

$\boxed{(-2, 2)}$

① + ③

$x + 2y = 2$

hits $x = 2$

at $2 + 2y = 2$

$2y = 0$

$y = 0$

$\boxed{(2, 0)}$

② + ③

$x = 2$ hits $y = 2$

at $\boxed{(2, 2)}$

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(26 points) Name ____________________ Row ___________
(5)

$$A = \begin{vmatrix} 1 & 2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Calculate A^{-1} (that is the MULTIPLICATIVE inverse of matrix A) using an augmented matrix and the Gauss-Jordan method.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

so $$A^{-1} = \begin{bmatrix} 1/3 & -2/3 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

check

$$\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1/3 & -\frac{2}{3} & 0 \\ 1/3 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ works!

$$A^{-1} = \begin{bmatrix} 1/3 & -2/3 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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BONUS

(1) (2 points) In general for matrices, AB = BA is not a true statement. If

$$A = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}$$

Find ALL matrices B that make AB = BA true. let $B = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} 2c & a \\ 2d & b \end{bmatrix} = \begin{bmatrix} b & d \\ 2a & 2c \end{bmatrix}$$ so $b = 2c$, $a = 2d$

B is all matrices of the form $\begin{bmatrix} a & b \\ 2b & a \end{bmatrix}$ + 2

(2) (~~1~~ 2 point) Find x and y that satisfy

$$\underset{1\times 3}{[\,x \quad y \quad 2\,]} \underset{3\times 2}{\begin{vmatrix} 1 & 0 \\ 2 & 5 \\ 17 & 29 \end{vmatrix}} = [\,x \quad 5\,] \begin{vmatrix} 3 & 4 \\ y & 2y \end{vmatrix}$$

$$\begin{bmatrix} x + 2y + 34 & 5y + 58 \end{bmatrix} = \begin{bmatrix} 3x + 5y & 4x + 10y \end{bmatrix}$$

$x + 2y + 34 = 3x + 5y \Rightarrow 2x + 3y = 34$

$5y + 58 = 4x + 10y \Rightarrow 4x + 5y = 58$

$$\begin{array}{rrcr} & -4x & -6y & = -68 \\ + & 4x & +5y & = 58 \\ \hline & & -y & = -10 \\ & & y & = 10 \end{array}$$

$2x + 3(10) = 34$

$2x = 4$

$x = 2$

$\boxed{\begin{array}{l} x = 2 \\ y = 10 \end{array}}$ +1 +1