MA 114-002 Fall 2006
Test 1-A

Name Key Row____________________

Read each question carefully. You must SHOW ALL WORK for full credit. Put all work and answers on this paper. Good luck.

(1) Use the following matrices to answer the questions if possible, otherwise write "not possible."

$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 5 & 0 & 2 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ x \\ 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 0 & y & 2 \end{bmatrix}$

Find:

a) The size of matrix A $3 \times 4$

b) EB $\quad (1 \;\; 0 \;\; y \;\; 2)\begin{pmatrix} 2 \\ x \\ 1 \\ 0 \end{pmatrix} = (2\cdot 1 + 0\cdot x + y\cdot 1 + 2\cdot 0) = (2+y)$

c) BE $\quad \begin{pmatrix} 2 \\ x \\ 1 \\ 0 \end{pmatrix}(1 \;\; 0 \;\; y \;\; 2) = \begin{bmatrix} 2 & 0 & 2y & 4 \\ x & 0 & xy & 2x \\ 1 & 0 & y & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $a_{21}$ 5

e) DA $\quad \underset{2\times 3}{\begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}} \underset{3\times 4}{\begin{bmatrix} 1 & 0 & 2 & 0 \\ 5 & 0 & 2 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}} = \begin{bmatrix} 3+5+0 & 0+0+2 & 6+2+1 & 0+1+3 \\ 0+10+0 & 0+0+0 & 0+4+0 & 0+2+0 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 2 & 9 & 4 \\ 10 & 0 & 4 & 2 \end{bmatrix}$$

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f) $AB + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 5 & 0 & 2 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ x \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 2+0+2+0 \\ 10+0+2+0 \\ 0+2x+1+0 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 2x+1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ 2x+3 \end{bmatrix}$

g) C + 2D

Not possible

h) What is the ADDITIVE identity matrix for AB (that is AB + ___ = AB )

$A_{3\times4}\ B_{4\times1}$ so AB is $3\times1$ hence add. identity $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

i) DC

$D_{2\times3}\ C_{2\times2}$ Not possible

j) What is the MULTIPLICATIVE identity matrix for BE (that is BE x ____ = BE )

$B_{4\times1}\ E_{1\times4} = 4\times4$ matrix

$\therefore$ Mult. identity of BE is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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(2) The following are in Gauss-Jordan row reduced form. (i.e. After using Gauss-Jordan, the following augmented matrices result.) State the solution(s) if they exist.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 9 \end{array}\right]$ (d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

(a) $x=0$, $y=0$, $z=3$

(b) $\left.\begin{array}{l} x+z=5 \\ y=7 \end{array}\right\}$ either is ok

$\boxed{\text{OR}}$ $\left.\begin{array}{l} x=5-z \\ y=7 \\ z=\text{free} \end{array}\right\}$

(c) No solution $0 \neq 9$

(d) $x=2$, $y=1$, $z=3$

(3) Solve the following system of equations using the Gauss-Jordan method. SHOW ALL STEPS and label all row operations. HINT: you may want to start with a row swap to make the arithmetic easier.

$3x - 2y = 7$
$2x + 4y = 26$

$$\left[\begin{array}{cc|c} 3 & -2 & 7 \\ 2 & 4 & 26 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 2 & 4 & 26 \\ 3 & -2 & 7 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & 13 \\ 3 & -2 & 7 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 2 & 13 \\ 0 & -8 & -32 \end{array}\right] \xrightarrow{-\frac{1}{8}R_2} \left[\begin{array}{cc|c} 1 & 2 & 13 \\ 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array}\right]$$

$x=5$
$y=4$

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(4) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 5 \\ 1 & 3 & 0 \end{bmatrix}$

Calculate $A^{-1}$ (that is the multiplicative inverse of A) using an augmented matrix and the Gauss-Jordan method. Show all work and all row operations.
NO WORK = NO CREDIT

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 5 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 5 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & \frac{1}{5} & 0 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 3 & 0 & -2 \\ -1 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix}$$

check $AA^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 5 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & -2 \\ -1 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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(5) A = $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

Calculate $A^{-1}$ if it exists. If it does not exist state why. HINT: Try to find $A^{-1}$. If you can find the inverse state it, otherwise show where or explain why it fails.

$A^{-1}$ does not exist since $1 \cdot 6 - 2 \cdot 3 = 0$

OR

$R_2$ is a multiple of $R_1$ so $A^{-1}$ does not exist

OR

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 6 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{array}\right]$$

contradiction! $0 \neq -3$ $0 \neq 1$

can't get identity OR this is solving a system of equations, but there's no solution

BONUS: (1) Find the sum 1 + 2 + 3 + 4 + … + 998 + 999 + 1000. Show ALL work!
(2) Use Gauss-Jordan to derive the formula

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Then $A^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

You may assume that $a \neq 0$ and that $ad - bc \neq 0$

① $1 + 1000 = 1001$ $2 + 999 = 1001$ $3 + 998 = 1001$ …
there will be 500 such pairs $\therefore \sum_{i=1}^{1000} i = 500(1001) = 500{,}500$

② $$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{a}R_1} \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{R_2 - cR_1} \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & d - \frac{bc}{a} & -\frac{c}{a} & 1 \end{array}\right]$$

(common denominator; $d - \frac{bc}{a} = \frac{ad-bc}{a}$

$$= \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{array}\right] \xrightarrow{\frac{a}{ad-bc}R_2} \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$-\frac{b}{\cancel{a}} \cdot \frac{\cancel{a}}{ad-bc}$ since $\frac{-c}{\cancel{a}}\left(\frac{\cancel{a}}{ad-bc}\right) = \frac{-c}{ad-bc}$

$$\xrightarrow{R_1 - \frac{b}{a}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right] \text{ so } A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

in row 1 col 3 $\frac{1}{a} - \frac{b}{a}\left(\frac{-c}{ad-bc}\right) = \frac{1}{a} + \frac{bc}{a(ad-bc)} = \frac{ad-\cancel{bc}}{a(ad-bc)} + \frac{\cancel{bc}}{a(ad-bc)} = \frac{\cancel{a}d}{\cancel{a}(ad-bc)}$