MIXING PROBLEMS

\[
\frac{dy}{dt} = R_i C_i - R_o C_o
\]

P553

18. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?

\[
\int \frac{dy}{1 - \frac{y}{10}} = \int dt
\]

\[
\int \frac{du}{u} = \int dt
\]

\[
-10 \int \frac{dy}{u} = +C
\]

\[
-10 \ln |1 - \frac{y}{10}| = +C
\]

\[
\ln |1 - \frac{y}{10}| = -\frac{1}{10} + +C_2
\]

\[
1 - \frac{y}{10} = e^{-\frac{t}{10}} + C_2
\]

\[
1 - \frac{y}{10} = Ke^{-\frac{t}{10}} \quad (k = \pm C_2)
\]
\[ 1 - \frac{y}{10} = k e^{-\frac{t}{10}} + \]
\[ 10 \left( 1 - k e^{-\frac{t}{10}} \right) = y \]
\[ y(0) = 0 \quad \text{(Pure water has no salt)} \]
\[ \rightarrow 10 \left( 1 - k e^{0} \right) = 0 \]
\[ \text{or } k = 1 \]
\[ y = 10 \left( 1 - e^{-\frac{t}{10}} \right) \]
\[ y(6) = 10 \left( 1 - e^{-\frac{6}{10}} \right) \approx 4.52 \]

P520

37. A vat with 500 gallons of beer contains 4% alcohol by volume. Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after 1 hour?

\[ \frac{dy}{dt} = r_i c_i - r_o c_o \quad \text{where } y(t) = \text{amount of alcohol} \]
\[ = \left( \frac{5 \text{gal}}{\text{min}} \right)(0.06) - \left( \frac{5 \text{gal}}{\text{min}} \right) \left( \frac{y(t)}{500 \text{gal}} \right) \]
\[ = 0.3 - \frac{y}{100} \]
\[
\frac{dy}{dt} = 0.3 - \frac{y}{100}
\]

\[
\int \frac{dy}{0.3 - \frac{y}{100}} = \int dt
\]

\[
U = 0.3 - \frac{y}{100} \quad du = -\frac{1}{100} dy
\]

\[
-100 du = dy
\]

\[
\Rightarrow -100 \int \frac{1}{u} du = +C
\]

\[
-100 \ln \left| 0.3 - \frac{y}{100} \right| = +C
\]

\[
\ln \left| 0.3 - \frac{y}{100} \right| = -\frac{1}{100} + C_2
\]

\[
0.3 - \frac{y}{100} = e^{-\frac{1}{100} + C_2} = e^{-\frac{1}{100}} e^{C_2}
\]

\[
0.3 - \frac{y}{100} = Ke^{\frac{1}{100} t}
\]

\[
100 \left( 0.3 - Ke^{\frac{1}{100} t} \right) = y
\]

\[
y(0) = 0.04(500) \quad (4\% \text{ alcohol to start})
\]

\[
= 20 \quad (500 \text{ gallons of beer})
\]

\[
100 \left( 0.3 - K \right) = 20
\]

\[
\Rightarrow K = 0.1
\]

\[
y = 30 - 10e^{-\frac{1}{100} t}
\]

Amount of alcohol after 1 hr (60 min)

\[
y(1) = 30 - 10e^{-\frac{60}{100}} = 24.5
\]

\[
\frac{24.5}{500} = 0.049 \rightarrow 5\% \text{ alcohol}
\]
If $r_1 \neq r_0$ then frequently the mixing problem will need to be solved using another method, which you will learn in Differential Equations.