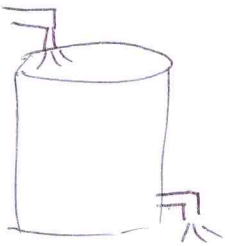
Mixing Problems

$$\frac{dy}{dt} = r_i c_i - r_o c_o$$

P553

18. A tank contains 100 L of pure water. Brine that contains .1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?



$$\frac{dy}{dt} = r_i c_i - r_o c_o$$

$$= \left(\frac{10L}{min}\right)\left(\frac{.1kg}{L}\right) - \left(\frac{10L}{min}\right)\left(\frac{y(t)}{100L}\right)$$

$$= 1 - \frac{y}{10}$$

$$\int \frac{dy}{1-\frac{y}{10}} = \int dt \qquad u = 1 - \frac{y}{10}$$

$$du = -\frac{1}{10}\,dt$$

$$-10\,du = dt$$

$$-10\int \frac{du}{u} = t + C$$

$$-10 \ln\left|1 - \frac{y}{10}\right| = t + C$$

$$\ln\left|1 - \frac{y}{10}\right| = -\frac{1}{10}t + C_2$$

$$\left|1 - \frac{y}{10}\right| = e^{-\frac{1}{10}t + C_2} = e^{-\frac{1}{10}t} e^{C_2}$$

$$1 - \frac{y}{10} = K e^{-\frac{1}{10}t} \quad (K = \pm e^{C_2})$$

$$1-\frac{y}{10}=ke^{-\frac{1}{10}t}$$

$$10\left(1-ke^{-\frac{1}{10}t}\right)=y$$

$y(0)=0$ (Pure water has no salt)

$$\rightarrow 10(1-ke^{0})=0$$

or $k=1$

$$y=10\left(1-e^{-\frac{1}{10}t}\right)$$

$$\boxed{y(6)=10\left(1-e^{-\frac{6}{10}}\right)} \approx 4.52$$

p520

37. A vat with 500 gallons of beer contains 4% alcohol by volume. Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after 1 hour?

$\frac{dy}{dt} = r_i c_i - r_o c_o$ where $y(t)$ = amount of alcohol

$$=\left(\frac{5\text{gal}}{\text{min}}\right)(.06)-\left(\frac{5\text{gal}}{\text{min}}\right)\left(\frac{y(t)}{500\text{gal}}\right)$$

$$=.3-\frac{y}{100}$$

$$\frac{dy}{dt} = .3 - \frac{y}{100}$$

$$\int \frac{dy}{.3 - \frac{y}{100}} = \int dt \qquad u = .3 - \frac{y}{100} \quad du = \frac{-1}{100} dy$$

$$-100\, du = dy$$

$$\Rightarrow -100 \int \frac{1}{u} du = t + C$$

$$-100 \ln\left|.3 - \frac{y}{100}\right| = t + C$$

$$\ln\left|.3 - \frac{y}{100}\right| = \frac{-1}{100} t + C_2$$

$$\left|.3 - \frac{y}{100}\right| = e^{\frac{-1}{100}t + C_2} = e^{\frac{-1}{100}t} e^{C_2}$$

$$.3 - \frac{y}{100} = K e^{\frac{-1}{100}t}$$

$$100\left(.3 - K e^{\frac{-1}{100}t}\right) = y$$

$y(0) = .04(500) = 20$ (4% alcohol to start, 500 gallons of bees)

$$100(.3 - K) = 20$$

$$\rightarrow K = .1$$

$$y = 30 - 10e^{-\frac{1}{100}t}$$

Amount of alcohol after 1 hr (60 min)

$$= y(1) = 30 - 10e^{-\frac{60}{100}} = 24.5$$

$$\frac{24.5}{500} = .049 \rightarrow \boxed{\approx 5\% \text{ alcohol}}$$

(4) Note:

If $r_i \neq r_o$ then frequently the mixing problems will need to be solved using another method, which you will learn in Differential Equations.