MA 341 Test 2 NO WORK=NO CREDIT !

1. (15 points) Solve the IVP $y'' - y' - 2y = 8e^{3t} + 12t \quad y(0) = 2 \quad y'(0) = 3$

2. (10 points) Use the differential equation $y'' + 2y' + y = f(t)$ along with the given value of f(t) listed below to find the form of a particular solution. Do NOT solve for the coefficients.

a) $f(t) = e^{-t}$

b) $f(t) = \cos(4t)$

c) $f(t) = e^{3t}\sin t$

3. (15 points) Use the method of variation of parameters to find the general solution to $y'' + y = 2\csc t$

4. (7 points) A 18 - kg mass is attached a spring with stiffness 200N/m. The damping constant for the system is 140N - sec/m. If the mass is pulled 25 cm to the right of equilibrium and given an initial leftward velocity of 1m/sec and y(t) is the position of the spring at time t, formulate the IVP that describes this system, but do NOT solve for y.

5. (10 points) Use the definition of the Laplace transform to compute the Laplace transform of 4. State the domain of the transform. Show all of your work.

Use the Table provided below to answer the following:

$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0)$ $\quad \mathcal{L}\{\cos bt\} = \frac{s}{s^2+b^2}$ $\quad \mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2}$

$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$ $\quad \mathcal{L}\{\sin bt\} = \frac{b}{s^2+b^2}$ $\quad \mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2+b^2}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ $\quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ $\quad \mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$

$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$ $\quad \mathcal{L}\{1\} = \frac{1}{s}$

6. (9 points) Find the Laplace transform of $f(t) = \begin{cases} -3 & \text{if } 1 \le t < 2 \\ 4 & \text{if } 2 \le t < 4 \\ 5 & \text{if } 4 \le t \end{cases}$

7. (9 points) Find the Laplace transform of $f(t) = (t+1)^2 + e^{-6t}\sin 2t + 10\cos 5t$

8. (10 points) Find the inverse Laplace transform of $\dfrac{7s^2 - 41s + 84}{(s-1)(s^2 - 4s + 13)}$

9. (15 points) Use the method of Laplace transforms to solve the following initial value problem.
$y'' - 4y' + 4y = 4; \quad y(0) = 6, \quad y'(0) = 1$

MA 341 Test 2 Solutions

1. $r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$y_c = c_1 e^{2t} + c_2 e^{-t}$

$y_p = Ae^{3t} + Bt + C$

$y_p' = 3Ae^{3t} + B$

$y_p'' = 9Ae^{3t}$

$$\underline{9Ae^{3t}} - \underline{3Ae^{3t}} - B - \underline{2Ae^{3t}} - 2Bt - 2C = 8e^{3t} + 12t$$

$4A = 8 \quad A = 2$

$-B - 2C = 0 \qquad C = 3$

$-2B = 12 \quad B = -6$

$$y = c_1 e^{2t} + c_2 e^{-t} + 2e^{3t} - 6t + 3$$

$y(0) = 2 = c_1 + c_2 + 2 + 3$

$c_1 + c_2 = -3$

$y' = 2c_1 e^{2t} - c_2 e^{-t} + 6e^{3t} - 6$

$y'(0) = 3 = 2c_1 - c_2 + \cancel{6} - \cancel{6}$

$3c_1 = 0 \quad c_1 = 0 \quad c_2 = -3$

$$\boxed{y = -3e^{-t} + 2e^{3t} - 6t + 3}$$

2. $r^2+2r+1=0$

$(r+1)^2=0$

$y_c=c_1e^{-t}+c_2te^{-t}$

a) At^2e^{-t}

b) $A\cos 4t + B\sin 4t$

c) $e^{3t}(A\cos t + B\sin t)$

3. $r^2+1=0$

$r^2=-1$

$r=\pm i$

$y_c=C_1\cos t + C_2\sin t$

$y_p=V_1\cos t + V_2\sin t$

$V_1'\cos t + V_2'\sin t = 0 \longrightarrow V_1' = -V_2'\frac{\sin t}{\cos t}$

$V_1'(-\sin t) + V_2'\cos t = 2\csc t$

$V_2'\frac{\sin^2 t}{\cos t} + V_2'\cos t = \frac{2}{\sin t}$

$V_2'\sin^2 t + V_2'\cos^2 t = 2\frac{\cos t}{\sin t}$

$V_2 = \int 2\frac{\cos t}{\sin t}dt$ $\quad u=\sin t$, $du=\cos t$

$V_2 = 2\ln|\sin t|$

$V_1' = -2\frac{\cancel{\cos t}}{\cancel{\sin t}}\frac{\cancel{\sin t}}{\cancel{\cos t}}$

$V_1 = -2t$

$\boxed{y = C_1\cos t + C_2\sin t - 2t\cos t + 2\ln|\sin t|\sin t}$

4. $my'' + by' + ky = 0$

$$\boxed{18y'' + 140y' + 200y = 0 \qquad y(0) = .25\,m \quad y'(0) = -1\,m/s}$$

5. $\mathcal{L}\{4\} = \int_0^\infty 4e^{-st}\,dt = \lim_{n\to\infty} \int_0^n 4e^{-st}\,dt$

$$= \lim_{n\to\infty} -\frac{4}{s} e^{-st} \Big|_0^n$$

$$= \lim_{n\to\infty} -\frac{4}{s} e^{-sn} + \frac{4}{s} e^0$$

If $s = 0$ $\frac{1}{s}$ DNE

$s < 0$ $\lim_{n\to\infty} e^{-sn} \to \infty$

$$= \boxed{\frac{4}{s}} \quad \text{If } \boxed{s > 0}$$

6. $f(t) = -3u(t-1) + 7u(t-2) + u(t-4)$

$$\mathcal{L}\{f\} = -\frac{3e^{-s}}{s} + \frac{7e^{-2s}}{s} + \frac{e^{-4s}}{s}$$

7. $\mathcal{L}\{f\} = \mathcal{L}\{(t+1)^2\} + \mathcal{L}\{e^{-6t}\sin 2t\} + 10\mathcal{L}\{\cos 5t\}$

$$= \mathcal{L}\{t^2 + 2t + 1\} + \frac{2}{(s+6)^2 + 4} + \frac{10s}{s^2 + 25}$$

$$= \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s} + \frac{2}{(s+6)^2 + 4} + \frac{10s}{s^2 + 25}$$

8. $\dfrac{7s^2-41s+84}{(s-1)(s^2-4s+13)} = \dfrac{A}{s-1} + \dfrac{B(s-\alpha)+C\beta}{(s-\alpha)^2+\beta^2}$

$(s-\alpha)^2+\beta^2 = s^2-2\alpha s + \alpha^2+\beta^2 = s^2-4s+13$

$\alpha = 2 \quad \alpha^2+\beta^2 = 13$

$4+\beta^2 = 13 \quad \beta = 3$

$\dfrac{A}{s-1} + \dfrac{B(s-2)+C3}{(s-2)^2+9}$

$A(s^2-4s+13) + [B(s-2)+3C](s-1) = 7s^2-41s+84$

$As^2 - 4As + 13A + Bs^2 - 2Bs - Bs + 2B + 3Cs - 3C =$

$A+B=7$ | $A+B=7$ | $10A=50$
$-4A-3B+3C=-41$ | $9A-B=43$ | $-20-6+3C=-41$
$13A+2B-3C=84$ | | $A=5 \quad C=-15/3$
$B=2 \quad =-5$

$= \dfrac{5}{s-1} + \dfrac{2(s-2)+3(-5)}{(s-2)^2+9} = \dfrac{5}{s-1} + \dfrac{2(s-2)}{(s-2)^2+9} + \dfrac{(-5)3}{(s-2)^2+9}$

$$\boxed{\mathcal{L}^{-1} = 5e^t + 2e^{2t}\cos 3t - 5e^{2t}\sin 3t}$$

9. $s^2\mathcal{L}\{y\} - sy(0) - y'(0) - 4s\mathcal{L}\{y\} + 4y(0) + 4\mathcal{L}\{y\} = \frac{4}{s}$

$(s^2-4s+4)\mathcal{L}\{y\} = \frac{4}{s} + 6s + 1 - 24 = \frac{4}{s} + 6s - 23$

$\mathcal{L}\{y\} = \dfrac{4+6s^2-23s}{s(s-2)^2} = \dfrac{A}{s} + \dfrac{B}{s-2} + \dfrac{C}{(s-2)^2}$

$A(s^2-4s+4) + Bs(s-2) + Cs = 4 + 6s^2 - 23s$

$As^2 - 4As + 4A + Bs^2 - 2Bs + Cs =$

$A=1$

$B=5 \quad -4-10+C=-23$

$C=-7$

$$\boxed{y = 1 + 5e^{2t} - 7e^{2t}t}$$