Lines & Planes

Lines: To find an equation of a line we need a point on that line and a vector in the direction (parallel) to the line.

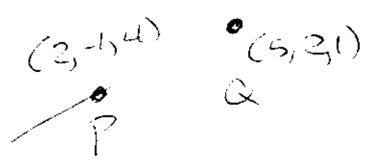
$\vec{v} = \langle a, b, c \rangle \quad P(x_0, y_0, z_0)$

Vector equation: $\vec{r} = \langle x_0, y_0, z_0 \rangle + t\vec{v}$

Parametric equations:
$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

① Find parametric equations of the line through the points $(2,-1,4)$ and $(5,2,1)$.

* We have a point on the line but we still need a vector in the direction of the line.

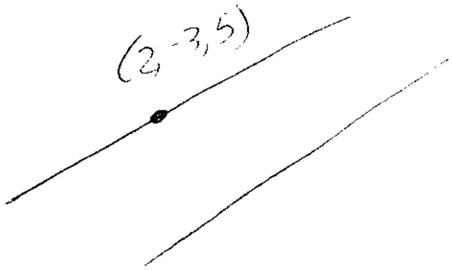


$\vec{PQ}$ is a vector on the line (so clearly // to it)

$\vec{PQ} = \langle 5-2, 2-(-1), 1-4 \rangle = \langle 3, 3, -3 \rangle$

$$x = 2 + 3t$$
$$y = -1 + 3t$$
$$z = 4 - 3t$$

(2) Find parametric equations of the line through the point $(2,-3,5)$ and parallel to the line $(x,y,z)=(t-1,\ 2t+3,\ -t)$



* We have a point on the line but we still need our vector parallel to the line. But since we have a line parallel to ours, its vector will do the job.

So $x = t-1,\ y = 2t+3,\ z = -t$

$\vec{v} = \langle 1, 2, -1 \rangle$

which gives us: $x = 2 + t$

$y = -3 + 2t$

$z = 5 + (-t)$

Planes

To find an equation of a plane we need a point in the plane and a vector perpendicular to the plane.

$\vec{n} = \langle a, b, c \rangle$ and $P(x_0, y_0, z_0)$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

③ Find an equation of the plane containing (2, 3, 3) and with normal vector $\langle 3, -2, 1 \rangle$

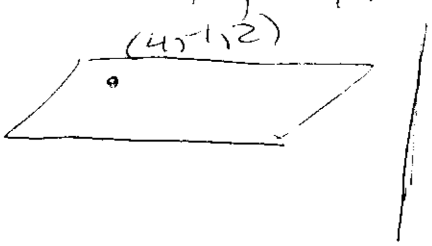
* They gave us all the information; don't get your hopes up for a problem like this on the next test

$$3(x-2) + (-2)(y-3) + 1(z-3) = 0$$

$$3x - 6 - 2y + 6 + z - 3 = 0$$

$$3x - 2y + z = 3$$

④ Find the equation of the plane containing (4, -1, 2) and orthogonal to the line $x = 3 - 2t$, $y = 4t + 5$, $z = t - 3$

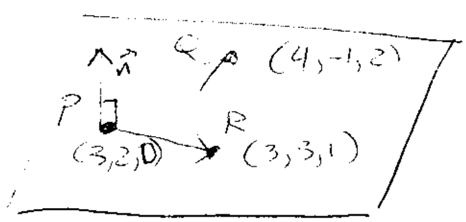


* We need a vector normal to the plane

but we have an equation of a line $\perp$ to the plane. We know from the equations we have of the line that $\langle -2, 4, 1 \rangle$ is parallel to the line. $\rightarrow \perp$ to the plane.

$$-2(x-4)+4(y+1)+1(z-2)=0$$
$$-2x+8+4y+4+z-2=0$$
$$\boxed{-2x+4y+z=-10}$$

⑤ find the equation of the plane containing $(4,-1,2)$, $(3,3,1)$ and $(3,2,0)$



* we have 3 points in the plane and again are missing a vector orthogonal to the plane.

First we'll find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, then we can find their cross product. The vector that results from the cross product is $\perp$ to both original vectors. If it is $\perp$ to vectors that lie in the plane, then it is $\perp$ to the plane.

$$\overrightarrow{PQ} = \langle 4-3, -1-2, 2-0 \rangle = \langle 1, -3, 2 \rangle$$

$$\overrightarrow{PR} = \langle 3-3, 3-2, 1-0 \rangle = \langle 0, 1, 1 \rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 0 & 1 & 1 \end{vmatrix}$$

$$(-3(1) - 2(1))\hat{i} - (1 - 2(0)) \quad + (1 - (-3)(0))\hat{k}$$

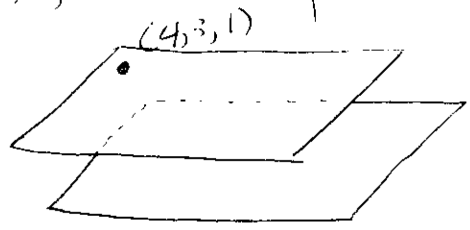
$$= -5\hat{i} - \hat{j} + \hat{k} = \langle -5, -1, 1 \rangle$$

* Now that I have my normal vector, I just have to pick any point in the plane. When we simplify all our results will match.

$P(4,-1,2)$

$$-5(x-4) - 1(y-(-1)) + 2(z-1) = 0$$
$$-5x + 20 - y - 1 + 2z - 2 = 0$$
$$-5x + 17 - y + 2z = 0$$

⑥ Find the equation of the plane containing $(4,3,1)$ and parallel to $2x - y + 3z = 5$



* We have 2 parallel planes which implies they have the same normal vector $\langle 2,-1,3 \rangle$

$$2(x-4) - (y-3) + 3(z-1) = 0$$
$$2x - 8 - y + 3 + 3z - 3 = 0$$
$$2x - y + 3z = 8$$