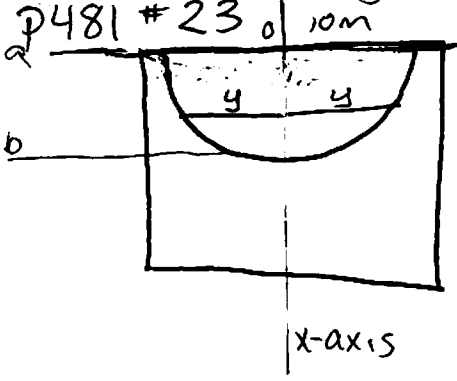
Hydrostatic Force Problems

Ex 1

Find the hydrostatic force on the submerged plate; do not evaluate.

P481 #23



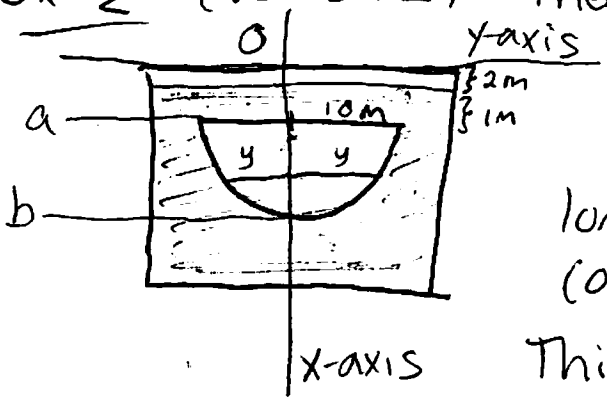
$x^2+y^2=100$

$y=\sqrt{100-x^2}$ $\quad L(h)=2y$

$$F=\int_0^{10}(1000\ kg/m^3)(9.8\ m/s^2)\,2\sqrt{100-x^2}\;x\,dx$$

(x: depth)

Ex 2 (version 1) There are **3** good ways to do this, (at least ☺)



1st notice: this is no longer the circle centered at $(0,0)$ with radius 10.

This circle is centered at $(3,0)$ with radius 10, so we have $(x-3)^2+(y-0)^2=100$

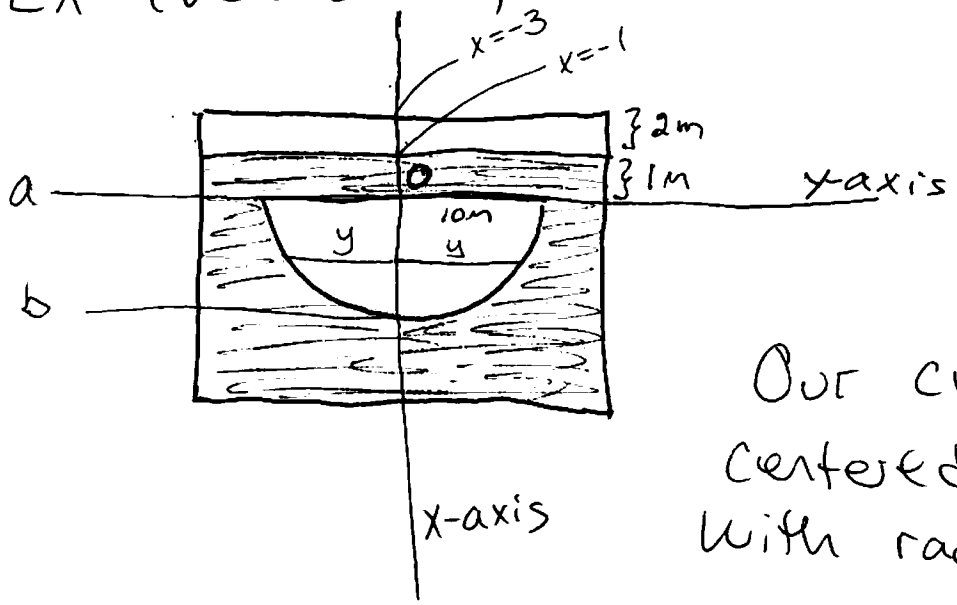
$y=\sqrt{100-(x-3)^2}$ $\quad L(h)=2y$

$$F=\int_3^{13}(1000\ kg/m^3)(9.8\ m/s^2)\,2\sqrt{100-(x-3)^2}\,(x-2)\,dx\ N$$

($x-2$: depth)

Note: If we just had x for the depth, we would have been including the 2m of empty space above the H_2O as H_2O!

Ex (Version 2)



Our circle is now centered at (0,0) with radius 10, so

$x^2+y^2=100$ $\quad y=\sqrt{100-x^2}$ $\quad L(h)=2\sqrt{100-x^2}$

$$F=\int_0^{10}(1000\,kg/m^3)(9.8\,m/s^2)\,2\sqrt{100-x^2}\,(x+1)\,dx \text{ N}$$

(depth.) Note: If we just had x, we wouldn't be including the 1m of water above the plate

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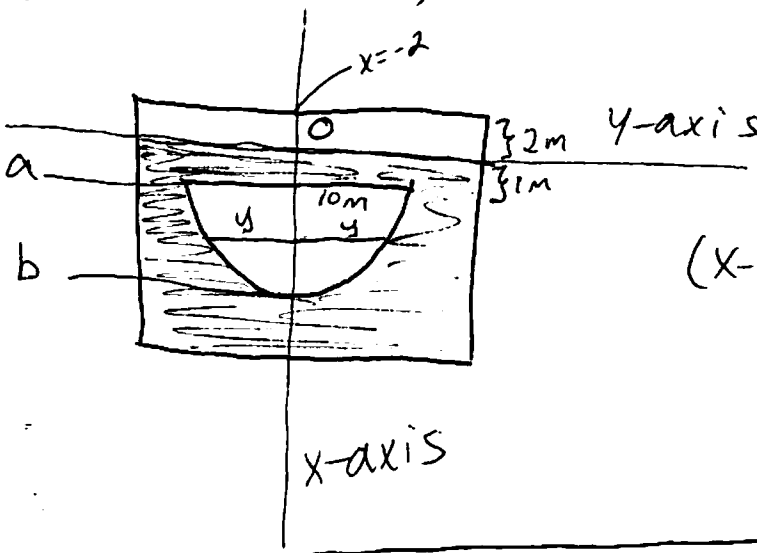
Ex (Version 3)



Our circle is centered at (1,0) with radius 10, so

$(x-1)^2+(y-0)^2=100$

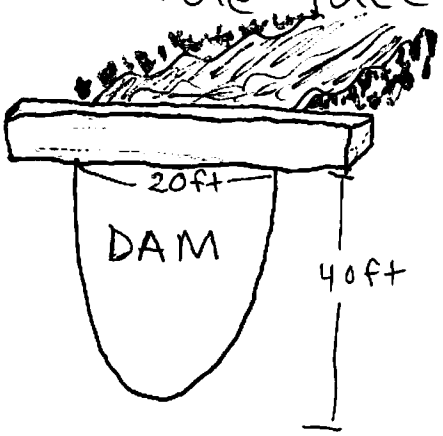
$y=\sqrt{100-(x-1)^2}$ $\quad L(h)=2y$

$$F=\int_1^{11}(1000\,kg/m^3)(9.8\,m/s^2)\,2\sqrt{100-(x-1)^2}\,x\,dx \text{ Newtons}$$

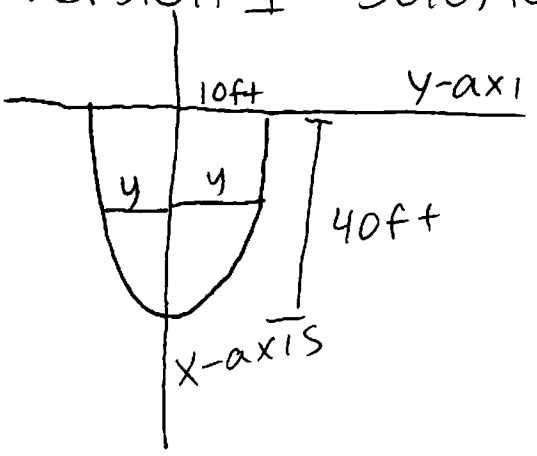
More Hydrostatic Force Problems

Ex Dam Problem

A reservoir is filled with water to the top of a dam. If the dam is in the shape of a parabola 40ft high and 20ft wide at the top as shown below, what is the total fluid force on the face of the dam?



Version 1 Solution:



We have a parabola that's been shifted, which can get a bit tricky (p38-39 give a nice refresher).

$x = ay^2 + b$, we know when $x=0$, $y=\pm 10$

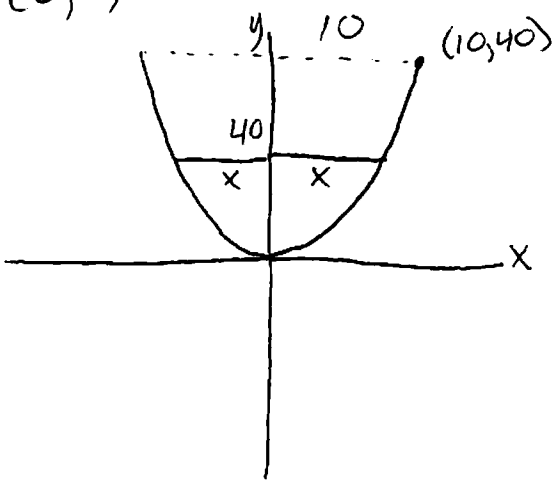
and when $y=0$ $x=40$, this gives us that $b=40$

$0 = a(10)^2 + 40$ $\quad a = \frac{-40}{100} = -\frac{2}{5}$ $\quad x = -\frac{2}{5}y^2 + 40$

or $y = \sqrt{-\frac{5}{2}(x-40)}$ $\quad L(h) = 2y$ so

$$F = \int_0^{40} 62.5\left(2\sqrt{-\tfrac{5}{2}x+100}\right) x\,dx \text{ lbs}$$

Version 2: This problem is easier (a bit) if we don't switch the x & y axis & put the bottom of the parabola at (0,0)



$y = ax^2$ $\quad x=10\ y=40$

$40 = a100 \quad a = 2/5$

$y = \frac{2}{5}x^2 \quad x = \sqrt{\frac{5}{2}y}$

$L(h) = 2x$

$$F = \int_0^{40} 62.5\left(2\sqrt{\tfrac{5}{2}y}\right)(40-y)\,dy \text{ lbs}$$

This is a little tricky we need to know our depth = distance below the surface of the water. The surface of the water is at $y=40$. If we put y for the depth we will have the distance above the river floor which is NOT what we want.

★ For all of these versions, try solving ★ them on your own and use your calculator to check your work & prove that the different versions are equivalent ☺