MA 242 Test 3 Version 1

1. (15 points) Integrate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3}\, dy\,dx$ by first changing the order of integration.

Your answer should include a sketch of D.

2. (15 points) Evaluate $\iiint_E 2z\, dV$ where E is the solid bounded by the plane $z = 12$ and the cone $z = \sqrt{4x^2 + 4y^2}$ Your answer should include a sketch of E.

3. (23 points) E is the solid bounded by the hemisphere $z = \sqrt{9 - x^2 - y^2}$ and the xy - plane. The density of E at any point (x,y,z) is equal to $\sqrt{x^2 + y^2 + z^2}$

   a) Find the mass of the solid using spherical coordinates
   b) List what x, y, and z are each equal to in spherical coordinates.
   c) Briefly define the term centroid

4. (20 points) A lamina occupies a region D, where $D = \{(x,y) \mid 1 \le x^2 + y^2 \le 4,\ 0 \le y \le x\}$

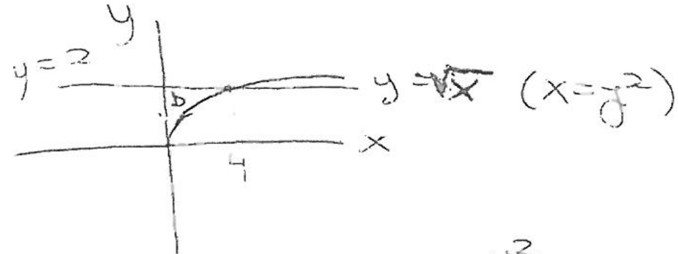
   a) Sketch and shade the region D
   b) Evaluate $\iint_D x\,dA$
   c) If $\rho(x,y) = x$, set up (**do NOT evaluate**) the integrals needed to find the lamina's center of mass

5. (15 points) Use a triple integral to find the volume of the solid E bounded by the planes $z = 3, z = -2$, $y = 4$, and the parabolic cylinder $y = x^2$. Your answer should include a sketch of E.

6. (12 points) Set up (**Do NOT evaluate**) a double integral to find the volume under $x^3 - 2y - z = 0$ and above the triangular region in the xy - plane with vertices (0,0), (1,1), and (0,3).

C3 T3 V1 Solutions

1. (15 pts)



$$\int_0^2 \int_0^{y^2} \frac{1}{1+y^3}\,dx\,dy = \int_0^2 \frac{x}{1+y^3}\Big|_0^{y^2} dy$$

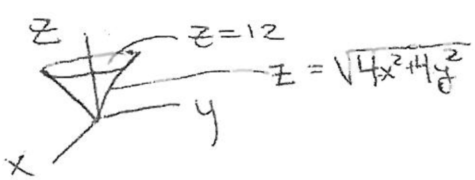
$$= \int_0^2 \frac{y^2}{1+y^3}\,dy \qquad \begin{aligned} u &= 1+y^3 \\ du &= 3y^2\,dy \\ \tfrac{1}{3}du &= y^2\,dy \\ u(2) &= 9 \\ u(0) &= 1 \end{aligned}$$

$$= \int_1^9 \frac{1}{3}\frac{1}{u}\,du$$

$$\frac{1}{3}\ln u\Big|_1^9 = \frac{1}{3}\ln 9 - \frac{1}{3}\ln 1 = \boxed{\frac{1}{3}\ln 9}$$

2. (15 pts)

$$\iiint_E 2z\,dV$$



$$12 = \sqrt{4r^2} = 2r$$
$$r = 6$$

$$\int_0^{2\pi}\int_0^6\int_{2r}^{12} 2zr\,dz\,dr\,d\theta = \int_0^{2\pi} d\theta \int_0^6 z^2\Big|_{2r}^{12} r\,dr$$

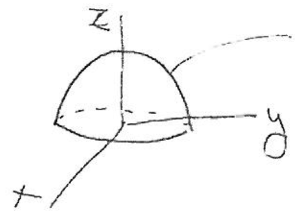
$2r \leftarrow \sqrt{4r^2}$

$$= 2\pi \int_0^6 144r - 4r^3\,dr$$

$$= 2\pi\left[\frac{144}{2}r^2 - r^4\right]_0^6$$

$$= \boxed{\pi\left[144(36) - 2\cdot 6^4\right]}$$

3. (23 pts)



$z = \sqrt{9-x^2-y^2}$

a) $m = \iiint_E \rho(x,y,z)\,dV$

$= \int_0^{2\pi}\int_0^{\pi/2}\int_0^3 \sqrt{\rho^2}\,\rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

$= \int_0^{2\pi} d\theta \int_0^{\pi/2} \sin\phi\, d\phi \int_0^3 \rho^3\, d\rho$

$2\pi \left(-\cos\phi\right)\Big|_0^{\pi/2} \frac{1}{4}\rho^4 \Big|_0^3$

$2\pi \left(\cancel{-\cos\frac{\pi}{2}} - (-\cos 0)\right) \frac{1}{4}\cdot 3^4$

$\boxed{\frac{2\pi \cdot 3^4}{4}}$

b) $x = \rho \sin\phi \cos\theta$

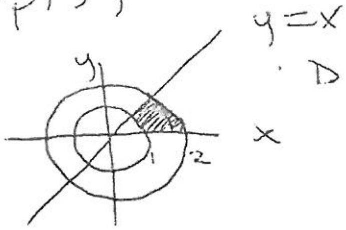
$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

c) Center of mass if there is uniform (constant) density

4. (20pts)

a)



b) $\int_0^{\frac{\pi}{4}}\int_1^2 (r\cos\theta)r\,dr\,d\theta = \int_0^{\pi/4}\cos\theta\,d\theta \int_1^2 r^2\,dr$

$\sin\theta\Big|_0^{\pi/4}\ \frac{1}{3}r^3\Big|_1^2$

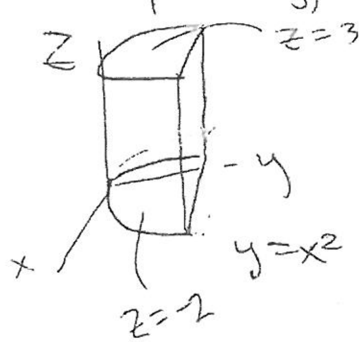
$\left(\sin\frac{\pi}{4} - \sin 0\right)\frac{1}{3}(8-1)$

$\left(\frac{1}{\sqrt{2}} - 0\right)\frac{7}{3} = \boxed{\frac{7}{\sqrt{2}(3)}}$

$\frac{\sqrt{2}}{2}$

c) $(\bar{x},\bar{y}) = \left(\dfrac{\int_0^{\pi/4}\int_1^2 (r\cos\theta)(r\cos\theta)r\,dr\,d\theta}{\frac{7}{\sqrt{2}(3)}}, \dfrac{\int_0^{\pi/4}\int_1^2 (r\cos\theta)(r\sin\theta)r\,dr\,d\theta}{\frac{7}{\sqrt{2}\,3}}\right)$

5. (15points)



$\int_{-2}^{2}\int_{x^2}^{4}\int_{-2}^{3} dz\,dy\,dx$

$\int_{-2}^{2}\int_{x^2}^{4} z\Big|_{-2}^{3} dy\,dx$

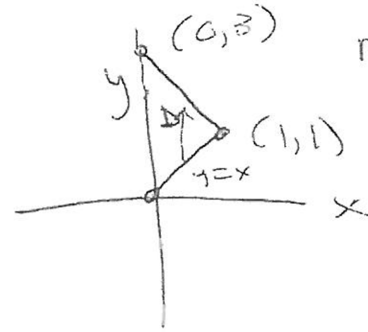
$\int_{-2}^{2}\int_{x^2}^{4} 5\,dy\,dx = \int_{-5}^{5} 5y\Big|_{x^2}^{4} dx$

$= \int_{-2}^{2} 20-5x^2\,dx = 2\int_0^2 20-5x^2\,dx = 2\left[20x-\frac{5}{3}x^3\Big|_0^2\right]$

$\boxed{2(40-40/3)}$

6. (12 pts)

$$\boxed{\int_0^1 \int_x^{3-2x} x^3 - 2y \, dy}$$



$m = \frac{3-1}{0-1} = -2$

$y = -2x + 3$