MA 242 Test 1 Version 2 No Work=No Credit!

Put all of your answers in the blue book, answers written elsewhere won't be graded.

1. (12 points) Find parametric equations of the line of intersection of the planes
   $4x+3z=8$ and $3y-5z=30$

2. (10 points) Find the value(s) of x for which the vectors $\langle 3,2x,x\rangle$ and $\langle 1,2,x\rangle$ are orthogonal

3. (15 points) Use the intersecting lines below to answer the following:

   L_1: $x=2+t$, $y=1+t$, $z=3t-2$

   L_2: $x=9+5s$, $y=6+3s$, $z=9+5s$

   a) What is their point of intersection?

   b) Find an equation of the plane containing the intersecting lines

4. (18 points) The curve $\mathbf{r}_1(t)=\langle e^{4t}, \sin(3t), 2t+1\rangle$ intersects the curve $\mathbf{r}_2(t)=\langle \cos(t), 3t, t^2+1\rangle$ at the point (1,0,1)

   a) Find the angle of intersection of these curves.

   b) Find the vector equation of the tangent line to $\mathbf{r}_1(t)$ at the point (1,0,1)

5. (30 points) Use the points A(1,2,3), B(3,1,3), and C(1,0,4) to answer the following:

   a) Find the distance from A to the yz-plane

   b) Find the distance from B to the y-axis

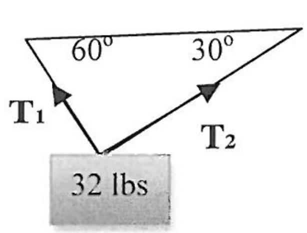
   c) Find the projection of B on the xy-plane

   d) Find $\overrightarrow{AB}$

   e) Find two vectors parallel to $\overrightarrow{AB}$ with magnitude 9

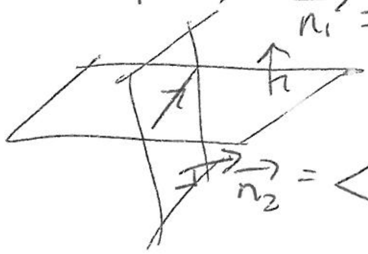
   f) Find the area of the triangle ABC

6. (15 points) Find $|\mathbf{T}_1|$ and $|\mathbf{T}_2|$ in the picture shown below.



C3 T1 V2 Solutions

1. (12 pts)



$\vec{n_1} = \langle 4,0,3 \rangle$

$\vec{n_2} = \langle 0,3,-5 \rangle$

$$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 3 \\ 0 & 3 & -5 \end{vmatrix} = \langle 0-9, -(-20), 12 \rangle$$

$$= \langle -9, 20, 12 \rangle$$

$z=0$

$4x = 8 \quad x=2$

$3y = 30 \quad y=10$

pt $(2,10,0)$

$$\boxed{\begin{aligned} x &= 2 - 9t \\ y &= 10 + 20t \\ z &= 0 + 12t \end{aligned}}$$

2. (10 pts)

$$\langle 3, 2x, x \rangle \cdot \langle 1, 2, x \rangle = 0$$

$$3 + 4x + x^2 = 0$$

$$x^2 + 4x + 3 = (x+3)(x+1) = 0$$

$$\boxed{x = -3, \; x = -1}$$

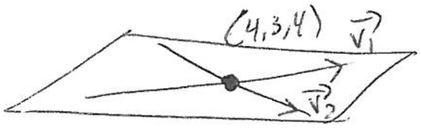
3. (15pts)

a) $\left.\begin{aligned} 2+t &= 9+5s \\ 1+t &= 6+3s \end{aligned}\right\}$ $1 = 3+2s$ $\boxed{\begin{aligned} s &= -1 \\ t &= 2 \end{aligned}}$

$3t-2 = 9+5s$

pt $\boxed{(4, 3, 4)}$

b)



$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 5 & 3 & 5 \end{vmatrix}$

$= \langle 5-9, -(5-15), 3-5 \rangle = \langle -4, 10, -2 \rangle$

$\boxed{-4(x-4) + 10(y-3) - 2(z-4) = 0}$

4. (18pts)

a) $e^{4t} = 1$ $t=0$

$\sin 3t = 0$

$2t+1 = 1$

$\vec{r_1}' = \langle 4e^{4t}, 3\cos 3t, 2 \rangle$ $\vec{r_2}' = \langle -\sin t, 3, 2t \rangle$

$\vec{r_1}'(0) = \langle 4, 3, 2 \rangle$ $\vec{r_2}'(0) = \langle 0, 3, 0 \rangle$

$\vec{r_1}'(0) \cdot \vec{r_2}'(0) = |\vec{r_1}'(0)| \, |\vec{r_2}'(0)| \cos\theta$

$9 = \sqrt{16+9+4} \; 3\cos\theta$

$\theta = \cos^{-1}\left(\frac{9}{3\sqrt{29}}\right) = \boxed{\cos^{-1}\left(\frac{3}{\sqrt{29}}\right)}$

b) $\boxed{\vec{r}(t) = \langle 1, 0, 1 \rangle + \langle 4, 3, 2 \rangle t}$

5. (30 pts)

a) $\boxed{1}$

b) $\sqrt{3^2+3^2} = \sqrt{18}$

c) $(3,1,0)$

d) $\vec{AB} = \langle 2, -1, 0 \rangle$

e) $|\vec{AB}| = \sqrt{5}$

$$\boxed{\left\langle \frac{2}{\sqrt{5}}(9), \frac{-9}{\sqrt{5}}, 0 \right\rangle \ \& \ \left\langle \frac{-2(9)}{\sqrt{5}}, \frac{9}{\sqrt{5}}, 0 \right\rangle}$$

f) $\vec{AC} = \langle 0, -2, 1 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} = \langle -1, -(2), -4 \rangle$$

$$= \langle -1, -2, -4 \rangle$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{1+4+16} = \sqrt{21}$$

$$\boxed{\frac{\sqrt{21}}{2}}$$

6. (15 pts)

$$\vec{T_1} = \langle -|\vec{T_1}|\cos 60°, |\vec{T_1}|\sin 60° \rangle = \langle -\tfrac{1}{2}|\vec{T_1}|, \tfrac{\sqrt{3}}{2}|\vec{T_1}| \rangle$$

$$\vec{T_2} = \langle |\vec{T_2}|\cos 30°, |\vec{T_2}|\sin 30° \rangle = \langle |\vec{T_2}|\tfrac{\sqrt{3}}{2}, \tfrac{1}{2}|\vec{T_2}| \rangle$$

$$\vec{T_1} + \vec{T_2} = \langle 0, 32 \rangle$$

$$-\tfrac{1}{2}|\vec{T_1}| + |\vec{T_2}|\tfrac{\sqrt{3}}{2} = 0 \rightarrow |\vec{T_1}| = \sqrt{3}|\vec{T_2}|$$

$$\tfrac{\sqrt{3}}{2}|\vec{T_1}| + \tfrac{1}{2}|\vec{T_2}| = 32$$

$$\left(\tfrac{3}{2} + \tfrac{1}{2}\right)|\vec{T_2}| = 32$$

$$\boxed{|\vec{T_2}| = 16} \rightarrow \boxed{|\vec{T_1}| = \sqrt{3}\,16}$$