MA 242 Test 1 Version 1   No Work=No Credit!

Put all of your answers in the blue book, answers written elsewhere won't be graded.

1. (12 points) Find parametric equations of the line of intersection of the planes
   $3x+2z=9$ and $5y-10z=30$

2. (10 points) Find the value(s) of x for which the vectors $\langle 3,2,x\rangle$ and $\langle 2x,4,x\rangle$ are orthogonal

3. (15 points) Use the intersecting lines below to answer the following:

   L_1: $x=6+2t$ L_2: $x=7+3s$

   $y=3+2t$ $y=5+2s$

   $z=9+3t$ $z=10+5s$

   a) What is their point of intersection?

   b) Find an equation of the plane containing the intersecting lines

4. (18 points) The curve $\mathbf{r}_1(t)=\langle 2t+1, \sin(2t), e^{3t}\rangle$ intersects the curve $\mathbf{r}_2(t)=\langle \cos(t), 5t, t^2+1\rangle$ at the point (1,0,1)

   a) Find the angle of intersection of these curves.

   b) Find the vector equation of the tangent line to $\mathbf{r}_1(t)$ at the point (1,0,1)

5. (30 points) Use the points A(1,2,3), B(4,1,3), and C(1,0,2) to answer the following:

   a) Find the distance from B to the yz-plane

   b) Find the distance from B to the z-axis

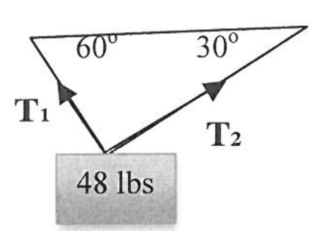
   c) Find the projection of A on the xy-plane

   d) Find $\overrightarrow{AB}$

   e) Find two vectors parallel to $\overrightarrow{AB}$ with magnitude 12

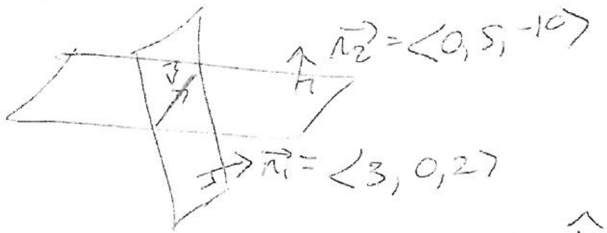
   f) Find the area of the triangle ABC

6. (15 points) Find $|\mathbf{T}_1|$ and $|\mathbf{T}_2|$ in the picture shown below.



C3 T1 V1 Solutions

1. (12pts)



$$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 0 & 5 & -10 \end{vmatrix} = \langle 0-10, -(-30), 15 \rangle$$

$$= \langle -10, 30, 15 \rangle$$

$z = 0 \quad 3x = 9 \quad x = 3$

$\quad 5y = 30 \quad y = 6$

$P(3, 6, 0)$

$$\boxed{\begin{aligned} x &= 3 - 10t \\ y &= 6 + 30t \\ z &= 0 + 15t \end{aligned}}$$

2. (10pts) $\langle 3, 2, x \rangle \cdot \langle 2x, 4, x \rangle = 0$

$6x + 8 + x^2 = 0$

$(x+4)(x+2) = 0$

$$\boxed{x = -4, \ x = -2}$$

3. (15 pts)

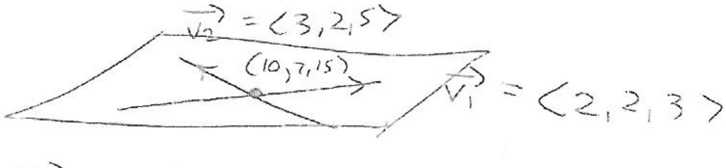
a) $\left.\begin{array}{l} 6+2t=7+3s \\ 3+2t=5+2s \end{array}\right\}$ $3=2+s \rightarrow s=1 \rightarrow t=2$

$9+3t=10+5s$

$15=15$ ✓

$\boxed{(10,7,15)}$

b)



$\vec{n}=\langle 2,2,3\rangle \times \langle 3,2,5\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 3 \\ 3 & 2 & 5 \end{vmatrix}$

$=\langle 10-6, -(10-9), 4-6\rangle$

$=\langle 4,-1,-2\rangle$

$\boxed{4(x-10)-(y-7)-2(z-5)=0}$

4. (18 pts)

a) $2t+1=1$

$\sin 2t=0 \longrightarrow t=0$

$e^{3t}=1$

$\vec{r_1}'=\langle 2, 2\cos 2t, 3e^{3t}\rangle$ $\quad \vec{r_2}'=\langle -\sin t, 5, 2t\rangle$

$\vec{r_1}'(0)=\langle 2,2,3\rangle$ $\quad \vec{r_2}'(0)=\langle 0,5,0\rangle$

$\vec{r_1}'(0)\cdot\vec{r_2}'(0)=|\vec{r_1}'(0)|\,|\vec{r_2}'(0)|\cos\theta$

$10=\sqrt{4+4+9}\,\sqrt{25}\cos\theta$

$\theta=\cos^{-1}\left(\frac{10}{5\sqrt{17}}\right)$ $\quad \boxed{\theta=\cos^{-1}\left(\frac{2}{\sqrt{17}}\right)}$

$\boxed{\text{b) } \vec{r}(t)=\langle 1,0,1\rangle+\langle 2,2,3\rangle t}$

5. (30 pts)

a) 4

b) $\sqrt{4^2+1^2} = \boxed{\sqrt{17}}$

c) (1, 2, 0)

d) $\overrightarrow{AB} = \langle 3, -1, 0 \rangle$

e) $|\overrightarrow{AB}| = \sqrt{9+1} = \sqrt{10}$

$$\boxed{\left\langle \frac{3}{\sqrt{10}}(12), \frac{-12}{\sqrt{10}}, 0 \right\rangle \quad \& \quad \left\langle \frac{-3(12)}{\sqrt{10}}, \frac{12}{\sqrt{10}}, 0 \right\rangle}$$

f) $\overrightarrow{AC} = \langle 0, -2, -1 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 0 & -2 & -1 \end{vmatrix}$$

$$= \langle 1, -(-3), -6 \rangle = \langle 1, 3, -6 \rangle$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{1+9+36} = \sqrt{46}$$

$$\boxed{\frac{\sqrt{46}}{2}}$$

6. (15 pts)

$\vec{T_1} = \langle -|\vec{T_1}|\cos 60°, |\vec{T_1}|\sin 60° \rangle = \langle -\frac{1}{2}|\vec{T_1}|, \frac{\sqrt{3}}{2}|\vec{T_1}| \rangle$

$\vec{T_2} = \langle |\vec{T_2}|\cos 30°, |\vec{T_2}|\sin 30° \rangle = \langle \frac{\sqrt{3}}{2}|\vec{T_2}|, |\vec{T_2}|\frac{1}{2} \rangle$

$\vec{T_1} + \vec{T_2} = \langle 0, 48 \rangle$

$-\frac{1}{2}|\vec{T_1}| + \frac{\sqrt{3}}{2}|\vec{T_2}| = 0 \longrightarrow |\vec{T_1}| = \sqrt{3}|\vec{T_2}|$

$\frac{\sqrt{3}}{2}|\vec{T_1}| + \frac{1}{2}|\vec{T_2}| = 48$

$\frac{3}{2}|\vec{T_2}| + \frac{1}{2}|\vec{T_2}| = 2|\vec{T_2}| = 48$

$$\boxed{\begin{gathered} |\vec{T_2}| = 24 \\ |\vec{T_1}| = \sqrt{3}\,24 \end{gathered}}$$