MA 242 H Test 4

1. (15 points) a) State the result of the Fundamental Theorem for Line Integrals

b) Find the gradient vector field for $f(x) = \ln(2 + 3x - y + 6z)$

c) Find the work done by the force field $\vec{\mathbf{F}} = \nabla f$ in moving an object from (0,0,0) to (1,1,1) along any smooth curve C.

2. (20 points) Given the points (-4,0) and (0,4)

a) Give a parametrization of the straight line which connects them (Denote this line C_1)

b) Give a parametrization of the quarter circle which connects them (Denote this line C_2)

c) Find the mass of a wire with density $\rho(x,y) = y - x$ bent in the shape of C_1 described above

3. (25 points) Use $\vec{\mathbf{F}}(x,y,z) = (e^z + y)\mathbf{i} + (z + x)\mathbf{j} + (xe^z + y + 5)\mathbf{k}$ to answer the following.

a) Show $\vec{\mathbf{F}}$ is conservative

b) Find the most general potential function of $\vec{\mathbf{F}}$

c) Find div $\vec{\mathbf{F}}$

4. (10 points) Show that the parametric equations

$x = a\sin(u)\cos(v)$

$y = b\sin(u)\sin(v)$

$z = c\cos(u)$

represent an ellipsoid.

5. (12 points) Prove that if $\vec{\mathbf{F}}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a conservative vector field, where P and Q have continuous 1st order partial derivatives on a domain D, then throughout D we have

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

6. (18 points) a) Find the surface area of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 9$

b) Find an equation of its tangent plane at the point (1,0,1)