242 H Solutions

a) $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

b) $\nabla f = \left\langle \frac{3}{2+3x-y+6z}, \frac{-1}{2+3x-y+6z}, \frac{6}{2+3x-y+6z} \right\rangle$

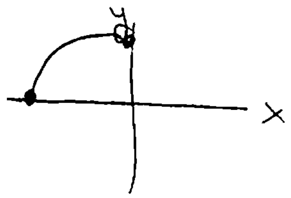
C) $\int_C \nabla f \cdot d\vec{r} = \ln(2+3-1+6) - \ln 2$

$= \ln 10 - \ln 2$

$= \ln 5$

2. a) $\vec{r}(t) = (1-t)\langle -4, 0\rangle + t\langle 0, 4\rangle, \quad 0 \le t \le 1$

$= \langle 4t-4, 4t\rangle \quad 0 \le t \le 1$

b) 

Accepted this answer: $x = 4\cos t$, $y = 4\sin t$, $\frac{\pi}{2} \le t \le \pi$; but technically going from $(-4,0)$ to $(0,4)$ we should have $x = -4\cos t$, $y = 4\sin t$, $0 \le t \le \pi/2$

c) $m = \int_0^1 (4t - (4t-4)) |\vec{r}\,'(t)| \, dt$

$= \int_0^1 4\sqrt{16+16} \, dt = \boxed{4\sqrt{32}}$

3. a) $\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^z + y & z + x & xe^z + y + 5 \end{vmatrix}$

$= (1-1)\hat{i} - (e^z - e^z)\hat{j} + (1-1)\hat{k} = \vec{0}$ ✓

b) $f_x = e^z + y \qquad f_y = z + x \qquad f_z = xe^z + y + 5$

$f = xe^z + xy + g(y,z)$

$f_y = \underline{x} + g_y(y,z) = z + \underline{x}$

$g_y(y,z) = z$

$g(y,z) = yz + h(z)$

$f = xe^z + xy + yz + h(z)$

$f_z = \underline{xe^z} + \underline{y} + h'(z) = \underline{xe^z} + \underline{y} + 5$

$h'(z) = 5$

$h(z) = 5z + k$

$$\boxed{f = xe^z + xy + yz + 5z + k}$$

c) $\nabla \cdot \vec{F} = 0 + 0 + xe^z = xe^z$

4. $\left(\frac{x}{a}\right)^2 = \sin^2 u \cos^2 v$

$\left(\frac{y}{b}\right)^2 = \sin^2 u \sin^2 v$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \sin^2 u \cos^2 v + \sin^2 u \sin^2 v = \sin^2 u$

$\sin^2 u + \cos^2 u = 1$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1}$$ ✓

5. $\vec{F} = P\hat{i} + Q\hat{j} = \nabla f = f_x \hat{i} + f_y \hat{j}$

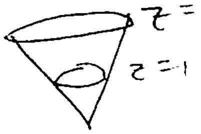
$$P = \frac{\partial f}{\partial x} \qquad Q = \frac{\partial f}{\partial y}$$

$$\frac{\partial P}{\partial y} = \frac{\partial^2 f}{\partial y \partial x} \qquad \frac{\partial Q}{\partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \text{ by Clairauts Thm}$$

6. a) $|\vec{r_x} \times \vec{r_y}| = \sqrt{\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + 1}$

$= \sqrt{2}$



Surface area $= \int_0^{2\pi} \int_1^9 \sqrt{2}\, r\, dr\, d\theta$

$= \frac{\sqrt{2}}{2} r^2 \Big|_1^9 \, 2\pi = \sqrt{2}\pi \, [80]$

b) $\vec{r_x} \times \vec{r_y} = \frac{-x}{\sqrt{x^2+y^2}} \hat{i} - \frac{y}{\sqrt{x^2+y^2}} \hat{j} + \hat{k}$

$(\vec{r_x} \times \vec{r_y})$ at $(1,0,1) = \langle -1, 0, 1 \rangle$

$-(x-1) + 0(y-0) + 1(z-1) = 0$

$-x + 1 + z - 1 = 0$

$\boxed{x = z}$