

MA 242 Honors Test 3

1. (15 points) Integrate $\int_0^6 \int_{\frac{x}{3}}^2 \sqrt{y^2 + 1} \, dy \, dx$ by first changing the order of integration.

Your answer should include a sketch of D.

2. (15 points) Find the volume of the solid bounded by the plane $z = 18$ and the paraboloid $z = 2x^2 + 2y^2$

3. (23 points) E is the solid above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 9$ in the first octant. The density of E at any point (x,y,z) is equal to the reciprocal of its distance to the origin.

a) Find the formula for the density $\rho(x,y,z)$

b) Find the mass of the solid using spherical coordinates

c) Convert x to spherical coordinates

4. (17 points) a) Evaluate $\int_0^5 \int_{-\sqrt{25-y^2}}^0 1 \, dx \, dy$ by converting to polar coordinates.

b) If the density of the region in part a) is always 1, **SET UP** the integrals needed to find the center of mass.

5. (15 points) Use cylindrical coordinates to **SET UP** the iterated integral $\iiint_E xyz \, dV$ where E is the solid bounded above and below by the sphere $x^2 + y^2 + z^2 = 16$ and **outside** the cylinder $x^2 + y^2 = 1$. **Do NOT evaluate.**

6. (15 points) **SET UP** a double integral to find the volume of the tetrahedron with vertices $(0,0,0)$, $((3,0,0)$, $(0,2,0)$, and $(0,0,6)$. **Do NOT evaluate.**