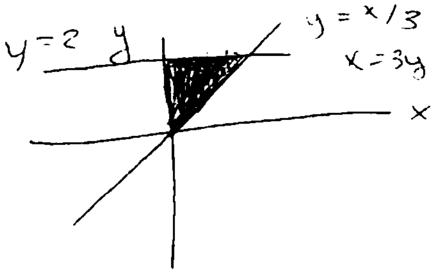
MA 242 H T3 Solutions

1. $\int_0^6 \int_{x/3}^2 \sqrt{y^2+1}\, dy\, dx = \int_0^2 \int_0^{3y} \sqrt{y^2+1}\, dx\, dy$



$= \int_0^2 3y\sqrt{y^2+1}\, dy$

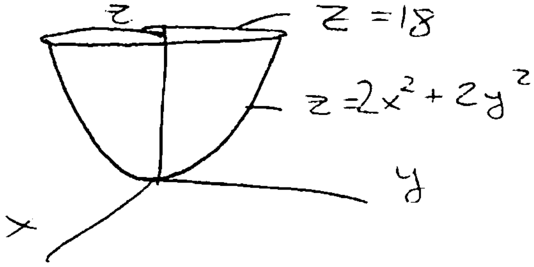
$u = y^2+1$

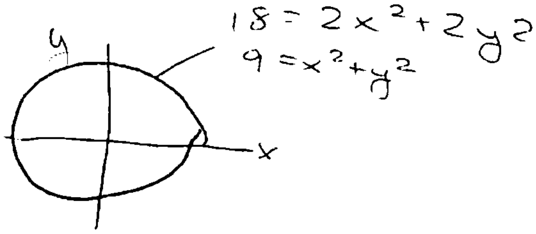
$du = 2y\,dy$

$\frac{1}{2} du = y\,dy$

$= \int_1^5 \frac{3}{2} u^{1/2}\, du = u^{3/2} \Big|_1^5 = \boxed{(5)^{3/2} - 1}$

2.





$V = \int_0^{2\pi} \int_0^3 \int_{2x^2+2y^2}^{18} 1\, dz\, r\, dr\, d\theta$

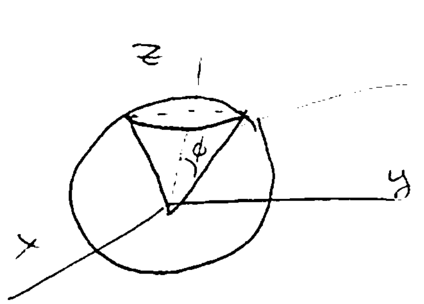
$= \int_0^{2\pi} \int_0^3 \int_{2r^2}^{18} dz\; r\, dr\, d\theta$

$= \int_0^{2\pi} \int_0^3 18r - 2r^3\, dr\, d\theta$

$= \int_0^{2\pi} 9r^2 - \frac{2}{4} r^4 \Big|_0^3\, d\theta$

$= \int_0^{2\pi} 81 - \frac{1}{2} 3^4\, d\theta = \boxed{2\pi\left(81 - \frac{3^4}{2}\right)}$

3.



$z = \sqrt{x^2+y^2}$

$\rho\cos\phi = \rho\sin\phi$

$\cos\phi = \sin\phi \rightarrow \phi = \pi/4$

a) $\rho(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

b) $\displaystyle\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{3} \frac{1}{\cancel{\sqrt{\rho^2}}} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

$= \displaystyle\int_0^{\pi/2}\int_0^{\pi/4} \frac{1}{2}(9)\sin\phi \, d\phi\, d\theta$

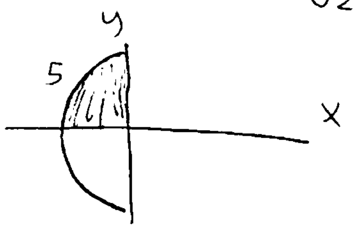
$\pi/2 \,(9/2)\left(-\cos\pi/4 + \cos 0\right)$

$= \boxed{\dfrac{9\pi}{4}\left(1 - 1/\sqrt{2}\right)}$

c) $x = \rho\sin\phi\cos\theta$

4.

a) $\displaystyle\int_0^{5}\int_{-\sqrt{25-y^2}}^{0} 1\, dx\, dy = \int_{\pi/2}^{\pi}\int_0^{5} r\, dr\, d\theta$

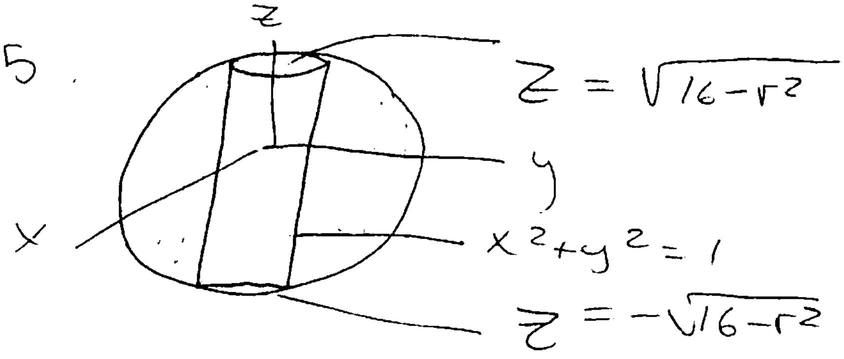


$= \frac{1}{2}25\left(\pi - \pi/2\right)$

$= \boxed{\dfrac{25\pi}{4}}$

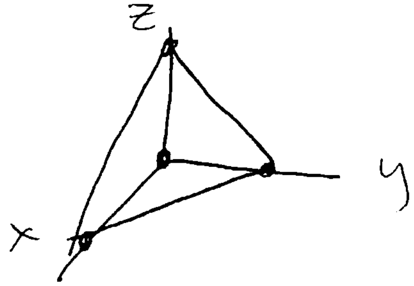
b) $(\bar{x}, \bar{y}) = \left(\dfrac{\int_{\pi/2}^{\pi}\int_0^{5} r\cos\theta \; r\, dr\, d\theta}{\frac{25\pi}{4}} , \dfrac{\int_{\pi/2}^{\pi}\int_0^{5} r\sin\theta \; r\, dr\, d\theta}{\frac{25\pi}{4}} \right)$

5.



$$\int_0^{2\pi}\int_1^4\int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r\cos\theta \, r\sin\theta \, z \, r \, dz \, dr \, d\theta$$

6.



$z = ax + by + c$

$(0,0,6)$

$6 = c$

$z = ax + by + 6$

$0 = 0 + 2b + 6 \qquad b = -3$

$z = ax - 3y + 6$

$0 = 2a + 6 \qquad a = -2$

$z = -2x - 3y + 6$

$0 = -2x - 3y + 6$

$$V = \int_0^3 \int_0^{\frac{6-2x}{3}} -2x - 3y + 6 \, dy \, dx \qquad y = \frac{6-2x}{3}$$