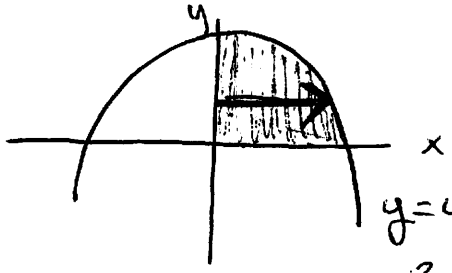
MA 242 H Solutions T3

(15pts) 1.



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$

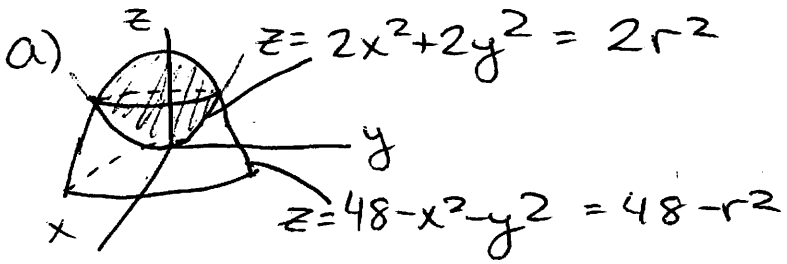
$$\int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\,dx\,dy = \int_0^4 \frac{\frac{1}{2}x^2 e^{2y}}{4-y}\Big|_0^{\sqrt{4-y}}$$

$$= \int_0^4 \frac{\frac{1}{2}(4-y)e^{2y}}{4-y}\,dy = \frac{1}{4}e^{2y}\Big|_0^4$$

$$= \frac{1}{4}(e^8 - 1)$$

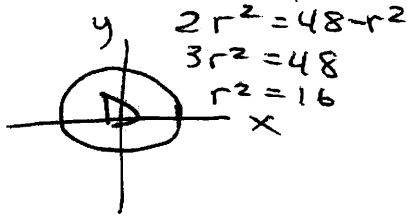
(20pts) 2.

a)



$z = 2x^2 + 2y^2 = 2r^2$

$z = 48 - x^2 - y^2 = 48 - r^2$



$2r^2 = 48 - r^2$

$3r^2 = 48$

$r^2 = 16$

$\rho(x,y,z) = \sqrt{x^2+y^2} = \sqrt{r^2} = r$

b) $$\text{mass} = \int_0^{2\pi}\int_0^4 \int_{2r^2}^{48-r^2} r\,dz\,r\,dr\,d\theta$$

$$= 2\pi \int_0^4 (48 - r^2 - 2r^2) r^2\,dr$$

$$= 2\pi \int_0^4 48r^2 - 3r^4\,dr = 2\pi\left(\frac{48}{3}r^3 - \frac{3}{5}r^5\right)\Big|_0^4 \longrightarrow$$

$$2\pi\left(16r^3 - \frac{3}{5}r^5\right)_0^4$$

$$= \boxed{2\pi\left(16\cdot 4^3 - \frac{3}{5}\cdot 4^5\right)} = \text{mass}$$

c) $(\bar{x}, \bar{y}, \bar{z}) =$

$$\left(\frac{\int_0^{2\pi}\int_0^4\int_{2r^2}^{48-r^2} (r\cos\theta)\, r\, dz\, dr\, d\theta}{\text{mass}}, \frac{\int_0^{2\pi}\int_0^4\int_{2r^2}^{48-r^2} (r\sin\theta)\, r\, dz\, dr\, d\theta}{\text{mass}}, \frac{\int_0^{2\pi}\int_0^4\int_{2r^2}^{48-r^2} z\, r\, dz\, dr\, d\theta}{\text{mass}} \right)$$

(12pts) 3. $\int_0^\infty\int_0^\infty \frac{1}{\sqrt{1+x^2+y^2}}\, dy\, dx = \lim_{n\to\infty}\int_0^{\frac{\pi}{2}}\int_0^n \frac{1}{\sqrt{1+r^2}}\, r\, dr\, d\theta$

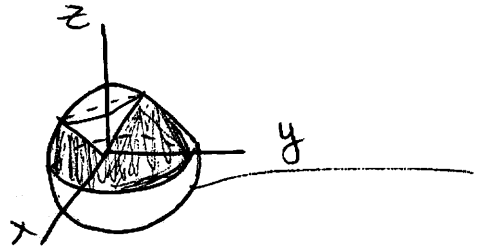
$= \frac{\pi}{2}\lim_{n\to\infty}\int_1^{1+n^2} \frac{1}{2}u^{-1/2}\, du$ $\quad u = 1+r^2$

$= \frac{\pi}{2}\lim_{n\to\infty}\sqrt{u}\,\Big|_1^{1+n^2}$ $\quad du = 2r\,dr$

$= \frac{\pi}{2}\left(\lim_{n\to\infty}\sqrt{1+n^2} - \sqrt{1}\right) \to \infty$ $\quad \frac{1}{2}du = r\,dr$

(24pts) 4.

a)



$x^2+y^2+z^2=25$

$\rho^2=25$

b) $V=\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^5 1\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$=\int_0^{2\pi}d\theta\int_{\pi/4}^{\pi/2}\sin\phi\,d\phi\int_0^5\rho^2\,d\rho$

$2\pi\left[-\cos\phi\right]_{\pi/4}^{\pi/2}\ \frac{1}{3}\rho^3\Big|_0^5$

$2\pi\left[-\cos\pi/2+\cos\pi/4\right]\frac{1}{3}\cdot 5^3$

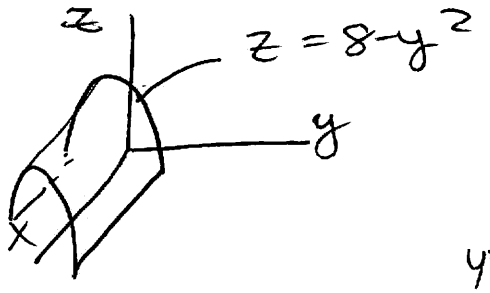
$=\boxed{\frac{2\pi 5^3}{3\sqrt{2}}}$

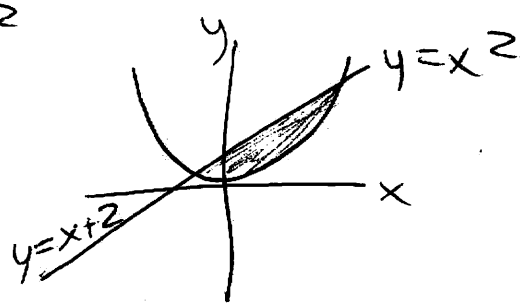
c) $x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$

(12pts) 5.





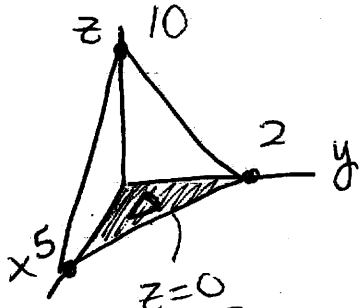
$x + 2 = x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$$V = \int_{-1}^{2} \int_{x^2}^{x+2} 8 - y^2 \, dy \, dx$$

(17pts) 6.



$z = ax + by + C$

$(0,0,10) \rightarrow 10 = 0 + 0 + C$

$z = ax + by + 10$

$(5,0,0) \rightarrow 0 = 5a + 0 + 10 \quad a = -2$

$(0,2,0) \rightarrow 0 = 0 + 2b + 10 \rightarrow b = -5$

$z = 10 - 2x - 5y$

$$\int_0^5 \int_0^{2-\frac{2}{5}x} \int_0^{10-2x-5y} x^4 z \, dz \, dy \, dx$$