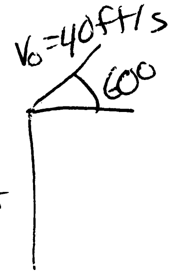
MA 242 Honors Test 2 Solutions

(12 pts)

1.



$$\vec{a} = -32\hat{j}$$

$$\vec{V} = -32t\hat{j} + \vec{c}$$

$$\vec{V}(0) = 40\cos 60°\hat{i} + 40\sin 60°\hat{j}$$

$$= 20\hat{i} + 20\sqrt{3}\hat{j}$$

$$\vec{V} = 20\hat{i} + (20\sqrt{3} - 32t)\hat{j}$$

$$\vec{r} = 20t\hat{i} + (20\sqrt{3}t - 16t^2)\hat{j} + \vec{d}$$

$$\vec{r}(0) = 7\hat{j}$$

$$\vec{r} = 20t\hat{i} + (20\sqrt{3}t - 16t^2 + 7)\hat{j}$$

$$\vec{r}(2) = \boxed{40\hat{i} + (40\sqrt{3} - 64 + 7)\hat{j}}$$

(14 pts)

2. a)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^2\cos^2\theta\, \cancel{r^2}\sin^2\theta}{\cancel{r^2}} = \boxed{0}$$

b)

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$$

$x=0$: $$\lim_{(0,y)\to(0,0)} \frac{0}{y^8} = 0$$

$x=y^4$ $$\lim_{(y^4,y)\to(0,0)} \frac{y^4y^4}{2y^8} = \frac{1}{2}$$

$$\boxed{0 \neq 1/2 \text{ limit DNE}}$$

3\. 20pts a) $D_{\vec{u}}f(x_0,y_0) = |\nabla f(x_0,y_0)|\,|\vec{u}|\cos\theta$

$= |\nabla f(x_0,y_0)|\cos\theta \geq -|\nabla f(x_0,y_0)|$

b) $f_x = e^x \sin y \quad f_y = e^x \cos y$

$\nabla f(0, \pi/4) = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$

$\vec{u} = \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

$D_{\vec{u}}f = \nabla f(0,\pi/4) \cdot \vec{u} = 1/2 - 1/2 = 0$

c) $\perp$

d) $\langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$

4\. 14pts $f(x,y) = x^3 + 6xy + 3y^2$

$f_x = 3x^2 + 6y = 0 \rightarrow x^2 + 2y = 0$

$f_y = 6x + 6y = 0 \quad x = -y$

$\left\{ \begin{array}{l} x^2 - 2x = 0 \\ x = 0 \\ x = 2 \end{array} \right.$

$(0,0) \quad (2,-2)$

$f_{xx} = 6x$

$f_{yy} = 6 \quad f_{xy} = 6$

$D = f_{xx}f_{yy} - [f_{xy}]^2$

$D = (6x)(6) - 36$

$D(0,0) = -36 < 0$ $\boxed{(0,0) \text{ saddle point}}$

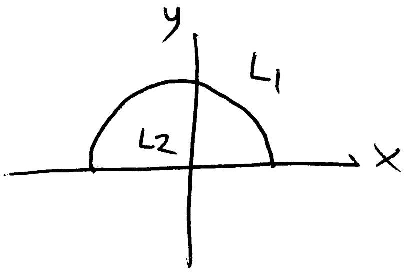
$D(2,-2) = 36 > 0 \quad f_{xx} = 12 > 0$ $\boxed{f(2,-2) = -4 \text{ local min value}}$

14 pts 5. $f(x,y) = 2x^2 + y^2 + 4x$

$f_x = 4x + 4 = 0 \quad x = -1$

$f_y = 2y = 0 \quad y = 0$

$f(-1,0) = -2$



L_1:

$f(x, \sqrt{4-x^2}) = x^2 + 4 + 4x$

$f' = 2x + 4 \quad x = -2$

$f(-2,0) = 0$

$f(2,0) = 16$

L_2: $f(x,0) = 2x^2 + 4x$

$f' = 4x + 4 \quad x = -1$

Abs max 16

min -2

10 pts 6. $f(x,y) = \ln(x+2y) + x^2$

$f_x = \frac{1}{x+2y} + 2x \quad f_y = \frac{2}{x+2y}$

$f_x(3,-1) = 1 + 6 = 7 \quad f_y = 2$

$z = 7(x-3) + 2(y+1) + 9$

b) $L(x,y) = 7(x-3) + 2(y+1) + 9$

$f(3.12, -.9) \approx 7(3.12-3) + 2(-.9+1) + 9$

12 pts 7. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$

$$= 2e^{2x-y+3z^2}(3) + \left(-e^{2x-y+3z^2}\right)\left(8s\cos(8st)\right)$$

$$+ \left(6ze^{2x-y+3z^2}\right)\left(-3t^2\right)$$

4 pts 8. $f_{xy} = -1 \quad f_{yx} = 0$

$-1 \neq 0$ Clairaut's theorem doesn't hold.