

MA 242 Honors Test 2 Put all work and answers in the blue books. Only put 1 problem per page.

1. (12 points) An athlete throws a shot at an angle of 60° above the horizontal at an initial speed of 40 ft/s. If the shot leaves his hand 7 feet above the ground, use the techniques we've discussed in class to find where the shot will be 2 seconds later? Hint: $\vec{a} = -32 \hat{j}$

2. (14 points) Find the limit, if it exists, or show that the limit does not exist. Justify your work.

a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$

b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8}$

3. (20 points) a) Show that if f is any differentiable function of 2 variables that the minimum value of $D_u f(x_0, y_0)$ at any given point (x_0, y_0) is $-|\nabla f(x_0, y_0)|$

b) Find the directional derivative of $f(x, y) = e^x \sin y$ at the point $P = \left(0, \frac{\pi}{4}\right)$ in the direction of $\vec{v} = \langle 1, -1 \rangle$

c) In part b), how does the direction of the gradient vector relate to the direction of \vec{v} ?

d) Use part b), find the direction of the maximum rate of increase at P .

4. (14 points) Find the local maximum and minimum values and any saddle points of the function $f(x, y) = x^3 + 6xy + 3y^2$

5. (14 points) Find the absolute maximum and minimum values of $f(x, y) = 2x^2 + y^2 + 4x$ on $D = \{(x, y) \mid y \geq 0 \text{ and } x^2 + y^2 \leq 4\}$

6. (10 points) Given $f(x, y) = \ln(x + 2y) + x^2$

a) Find the equation of the tangent plane at $(3, -1)$

b) Find the linearization $L(x, y)$ of f at $(3, -1)$, then use it to approximate $f(3.12, -0.9)$.

After plugging into the formula you need not simplify.

7. (12 points) Find $\frac{\partial w}{\partial t}$ where $w = e^{2x-y+3z^2}$ and $x = 2s + 3t$, $y = 8\sin(st)$, and $z = s^4 - t^3$.

Clearly label your derivatives as we have done in class.

8. (4 points) You are told that there is a function f whose partial derivatives are $f_x = 7x - y$ and $f_y = 5y$ and whose second-order partial derivatives are continuous. Explain why you should not believe it.