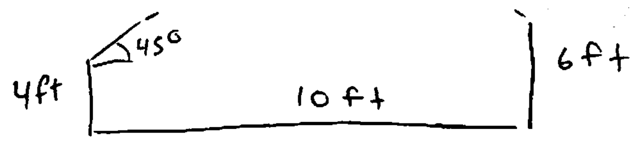
MA 242 H T2 : Solutions

1.



$\vec{a} = \langle 0, -32 \rangle$

$\vec{v}(0) = \langle v_0 \cos 45°, \ v_0 \sin 45° \rangle$

$= \langle \frac{v_0}{\sqrt{2}}, \frac{v_0}{\sqrt{2}} \rangle$

$\vec{v}(t) = \langle 0, -32t \rangle + \vec{c}$

$\vec{v}(t) = \langle \frac{v_0}{\sqrt{2}}, \frac{v_0}{\sqrt{2}} - 32t \rangle$

$\vec{r}(t) = \langle \frac{v_0}{\sqrt{2}} t, \frac{v_0}{\sqrt{2}} t - 16t^2 \rangle + \vec{d}$

$\vec{r}(0) = \langle 0, 4 \rangle$

$\vec{r}(t) = \langle \frac{v_0}{\sqrt{2}} t, \frac{v_0}{\sqrt{2}} t - 16t^2 + 4 \rangle$

$\frac{v_0}{\sqrt{2}} t = 10$

$t = \frac{10\sqrt{2}}{v_0}$ $\qquad \underbrace{\frac{v_0}{\sqrt{2}}\left(\frac{10\sqrt{2}}{v_0}\right)}_{10} - 16\left(\frac{200}{v_0^2}\right) + 4 = 6$

$\frac{200}{v_0^2} = \frac{-8}{-16} = \frac{1}{2}$

$\boxed{v_0 = 20 \text{ ft/s}}$

2. $\nabla f = \left\langle \frac{2x}{x^2+y^2-12}, \frac{2y}{x^2+y^2-12} \right\rangle$

$\nabla f(2,-3) = \langle 4, -6 \rangle$

$\overrightarrow{PQ} = \langle -1, 4 \rangle$

$\vec{u} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \left\langle \frac{-1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$

$D_{\vec{u}} f = \nabla f \cdot \vec{u} = \frac{-4}{\sqrt{17}} - \frac{24}{\sqrt{17}} = \boxed{\frac{-28}{\sqrt{17}}}$

b) $\langle 4, -6 \rangle$

c) $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$f(2,-3) = \ln 1 = 0$

$\boxed{L(x,y) = 4(x-2) - 6(y+3)}$

3.

a) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+4}-2} \cdot \frac{\sqrt{x^2+y^2+4}+2}{\sqrt{x^2+y^2+4}+2}$

$= \lim_{(x,y)\to(0,0)} \frac{(\cancel{x^2+y^2})(\sqrt{x^2+y^2+4}+2)}{\cancel{x^2+y^2}+\cancel{4}-\cancel{4}} = \sqrt{4}+2 = \boxed{4}$

b) $\lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0 \quad \lim_{(x,x^2)\to(0,0)} \frac{3x^2x^2}{x^4+x^4} = \frac{3}{2}$

$\boxed{0 \neq \frac{3}{2} \text{ limit DNE}}$

4. $f_x = 8xy = 0$ $\quad (x=0$ or $y=0)$

$f_y = 4x^2 + 2y - 4 = 0$ $\quad 2y-4=0 \quad\quad 4x^2-4=0$

$x = \pm 1$

$\left.\begin{array}{l}(0,2)\\(\pm 1, 0)\end{array}\right\}$ critical pts

$f_{xx} = 8y \quad f_{yy} = 2 \quad f_{xy} = 8x$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$D(0,2) = 16 \cdot 2 - 0^2 > 0$

$f_{xx}(0,2) = 16 > 0$ $\quad (0,2)$ is a local min

$f(0,2) = 4 - 8 = \boxed{+4 \text{ is the local min value}}$

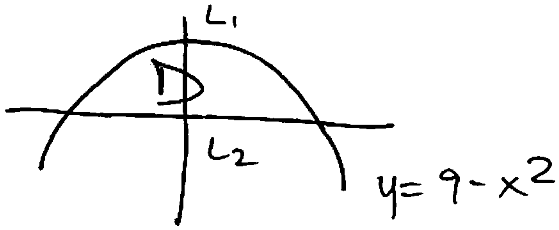
$D(1,0) = 0 - 8^2 < 0$

$D(-1,0) = -64 < 0$

$\boxed{(1,0) \text{ and } (-1,0) \text{ are saddle pts}}$

5. $f_x = y = 0 \quad f_y = x = 0$

$f(0,0) = 0$



L_1: $f(x, 9-x^2) = x(9-x^2)$
$= 9x - x^3$

$f_x = 9 - 3x^2 = 0$
$9 = 3x^2$
$x = \pm\sqrt{3}$

End pts $(3,0)$: $f(3,0) = 0$
$(-3,0)$: $f(-3,0) = 0$

$f(\sqrt{3}, 6) = \sqrt{3}\,6$

$f(-\sqrt{3}, 6) = -\sqrt{3}\,6$

L_2: $y = 0 \quad f(x,0) = 0$

Abs max $\boxed{6\sqrt{3}}$

Abs min $\boxed{-6\sqrt{3}}$

6. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$
$= 3x^2 \sec^2(x^3+2y)\, S e^{st} + 2\sec^2(x^3+2y)\, 8$

7. $F(x,y) = 0$

$\frac{dF}{dx} = \frac{\partial F}{\partial x}\frac{dx}{dx} + \frac{\partial F}{\partial y}\frac{dy}{dx}$

$F(x,y) = 0$
$\frac{dF}{dx} = 0$

$0 = F_x + F_y \frac{dy}{dx} \longrightarrow \boxed{\frac{dy}{dx} = -\frac{F_x}{F_y}}$ ✓