Honors 242 T1 Solutions

1. a) $\vec{a} \times \vec{b} = \vec{0}$ or

$\vec{a} = c\vec{b}$ for some scalar c

b) $\vec{a} \cdot \vec{b} = 0$

c) $\vec{a}/|\vec{a}|$ and $-\vec{a}/|\vec{a}|$

2. xy-plane $\rightarrow$ $z=0$ $\quad (x,y,0)$

x-axis $\rightarrow$ $(x,0,0)$

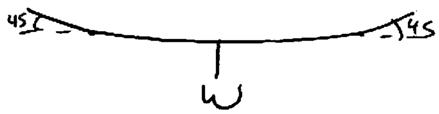
$$\sqrt{(x-x)^2+(y-y)^2+(z-0)^2} = \sqrt{(x-x)^2+(y-0)^2+(z-0)^2}$$

$$\sqrt{z^2} = \sqrt{y^2+z^2}$$

$$z^2 = y^2+z^2$$

$$0 = y^2$$

$$y = 0 \rightarrow \boxed{xz \text{ plane}}$$

3. 

$\vec{T_1} + \vec{T_2} = \langle 0, 5 \rangle$ $\qquad |\vec{T_1}| = |\vec{T_2}|$

$\vec{T_1} = \langle -|\vec{T_1}|\cos 45°, |\vec{T_1}|\sin 45° \rangle = \langle -|\vec{T_1}|\frac{1}{\sqrt{2}}, |\vec{T_1}|\frac{1}{\sqrt{2}} \rangle$

$\vec{T_2} = \langle |\vec{T_1}|\cos 45°, |\vec{T_2}|\sin 45° \rangle = \langle |\vec{T_1}|\frac{1}{\sqrt{2}}, |\vec{T_1}|\frac{1}{\sqrt{2}} \rangle$

$$\frac{1}{\sqrt{2}}|\vec{T}_1| + \frac{1}{\sqrt{2}}|\vec{T}_1| = 5$$

$$|\vec{T}_1| = \frac{5\sqrt{2}}{2}$$

$\vec{T}_1 = \langle -5/2, 5/2 \rangle$ $\quad$ $\vec{T}_2 = \langle 5/2, 5/2 \rangle$

4.

$1 = -t \quad t = -1$

a) $(1,0,1)$

b) $\langle 1,1,-1\rangle \times \langle 2,2,0\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 2 & 0 \end{vmatrix}$

$= \langle 0-(-2), -(0-(-2)), 2-2 \rangle$

$= \langle 2, -2, 0 \rangle$

$$\boxed{2(x-1) - 2(y-0) + 0(z-1) = 0}$$

5. a) $(1,0,1)$

b) $\overrightarrow{BC} = \langle 1-2,\ 3-3,\ 1-0 \rangle$

$= \langle -1, 0, 1 \rangle$

$\vec{r}(t) = \langle 1, 3, 1 \rangle + \langle -1, 0, 1 \rangle t$

or

$\vec{r}(t) = \langle 2, 3, 0 \rangle + \langle -1, 0, 1 \rangle t$

c) $\overrightarrow{AB} = \langle 2, 3, 0 \rangle$

$\overrightarrow{AC} = \langle 1, 3, 1 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 3 & 1 \end{vmatrix} = \langle 3, -2, 6-3 \rangle$$

$$= \langle 3, -2, 3 \rangle$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{9+4+9} = \sqrt{22}$$

$$\boxed{\frac{1}{2}\sqrt{22}}$$

6. $0 = \sin(s)$
$t^2 - 1 = s$
$t = e^{2s}$ $\quad s=0,\ t=1$

a) $(0, 0, 1)$

b) $\vec{r}_1{}'(t) = \langle 0, 2t, 1 \rangle \quad \vec{r}_1{}'(1) = \langle 0, 2, 1 \rangle$

$\vec{r}_2{}'(s) = \langle \cos(s), 1, 2e^{2s} \rangle$

$\vec{r}_2{}'(0) = \langle 1, 1, 2 \rangle$

$$\cos\theta = \frac{\langle 0,2,1 \rangle \cdot \langle 1,1,2 \rangle}{|\langle 0,2,1 \rangle|\ |\langle 1,1,2 \rangle|}$$

$$= \frac{4}{\sqrt{5}\sqrt{6}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{4}{\sqrt{30}}\right)}$$

c) $\boxed{x = 0 + 0t = 0 \quad y = 0 + 2t \quad z = 1 + t}$

7. $\vec{r}'(t) = \langle 2t, 2, \frac{1}{t} \rangle$

$$L = \int_1^e \sqrt{4t^2 + 4 + \frac{1}{t^2}}\, dt$$

$$= \int_1^e \sqrt{(2t + \frac{1}{t})^2}\, dt = \int_1^e 2t + \frac{1}{t}\, dt$$

$$= t^2 + \ln t \Big|_1^e = e^2 + 1 - 1^2 - \ln 1$$

$$= \boxed{e^2}$$

8. $\vec{r}(t) = \langle t, 4-t, 3t+1 \rangle$