

MA 241 Test 3

1. (10 points) A mold grows at a rate proportional to the amount present. Initially, its mass is 2 g; after 8 days its mass is 5 g. Find an equation for the mass after t days.
2. (15 points) Solve the IVP $y'' - 8y' + 16y = -32x$; $y(0) = 0$, $y'(0) = 12$
3. (15 points) Use the differential equation $y'' + 5y' + 6y = f(x)$ along with the value of $f(x)$ listed below to find the form of the particular solution, y_p , but do **NOT** solve for the coefficients.
 - a) $f(x) = 11 + e^{7x}$
 - b) $f(x) = e^{-3x}$
 - c) $f(x) = \sin(6x)$
4. (12 points) A spring with a mass of 2 kg has damping constant 4. A force of 12 N is required to keep the spring stretched 3 meters beyond its natural length.
 - a) What kind of damping is this?
 - b) If the spring is set in motion from its equilibrium position with a velocity of 2m/s, find $x(t)$, the position of the mass at time t .
5. (6 points) Find the general form of the sequence $\left\{-1, \frac{2}{3}, \frac{-3}{9}, \frac{4}{27}, \dots\right\}$
6. (10 points) Use $a_n = \frac{1}{8n}$ to answer the following. Justify your answers.
 - a) Determine if the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit.
 - b) Determine if the series $\sum_{n=1}^{\infty} a_n$ converges or diverges. Fully justify your answer as we have done in class.
7. (32 points) Determine if the following series converge or diverge, find the sums of convergent series. Justify your answers thoroughly.
 - a) $\sum_{n=1}^{\infty} -6\left(\frac{1}{2}\right)^{n+1}$
 - b) $\sum_{n=1}^{\infty} \frac{11n^3 - 5n + 6}{14 - 3n^3}$
 - c) $\sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+1}$ Include the first 3 partial sums with your answer

MA 241 T3 Solutions

1. (10 points) $y = y_0 e^{kt}$

$$y = 2e^{kt}$$

$$y(8) = 5$$

$$y(8) = 5 = 2e^{8k}$$

$$\frac{5}{2} = e^{8k}$$

$$\ln\left(\frac{5}{2}\right) = 8k$$

$$\frac{1}{8} \ln\left(\frac{5}{2}\right) = k$$

$$y = 2e^{\frac{1}{8} \ln\left(\frac{5}{2}\right)t}$$

2. (15 points) $r^2 - 8r + 16 = 0$

$$(r-4)^2 = 0$$

$$y_c = C_1 e^{4x} + C_2 x e^{4x}$$

$$y_p = Ax + B$$

$$y'_p = A$$

$$y''_p = 0$$

$$y = e^{4x} + 10x e^{4x} - 2x - 1$$

$$0 - 8A + 16Ax + 16B = -32x$$

$$16A = -32$$

$$A = -2$$

$$-8A + 16B = 0$$

$$16 + 16B = 0 \quad B = -1$$

$$y_p = -2x - 1$$

$$y = C_1 e^{4x} + C_2 x e^{4x} - 2x - 1$$

$$y' = 4C_1 e^{4x} + C_2 e^{4x} + 4C_2 x e^{4x} - 2$$

$$y(0) = C_1 - 1 = 0 \quad C_1 = 1$$

$$y'(0) = 4C_1 + C_2 - 2 = 12$$

$$C_2 = 10$$

3. (15 points) $y'' + 5y' + 6y = f(x)$

$$y'' + 5y' + 6y = 0$$

$$r^2 + 5r + 6 = 0$$

$$(r+3)(r+2) = 0$$

$$y_c = C_1 e^{-3x} + C_2 e^{-2x}$$

a) $y_p = A + B e^{7x}$

b) $y_p = A x e^{-3x}$

c) $A \cos 6x + B \sin 6x$

4. (12 points) $m x'' + b x' + k x = 0$

$$2x'' + 4x' + 4x = 0 \quad x(0) = 0, x'(0) =$$

$$2r^2 + 4r + 4 = 0$$

$$12 = k \cdot 3$$

$$r^2 + 2r + 2 = 0$$

$$k = 4$$

$$r = \frac{-2 \pm \sqrt{4 - 8}}{2}$$

a) Underdamping

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$x(t) = e^{-t} [C_1 \cos t + C_2 \sin t]$$

$$x(0) = C_1 = 0$$

$$x(t) = e^{-t} [C_2 \sin t]$$

$$x'(t) = -e^{-t} [C_2 \sin t] + e^{-t} [C_2 \cos t]$$

$$x'(0) = C_2 = 2$$

$$\frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \sin t + \frac{1}{\sqrt{2}} \cos t$$

5. (6 points) $a_n = \frac{(-1)^n n}{3^{n-1}}$

6. (10 points)

a) $\lim_{n \rightarrow \infty} \frac{1}{8n} = 0$ converges

b) Harmonic series diverges

7. (32 points)

a) $\sum_{n=1}^{\infty} -6\left(\frac{1}{2}\right)^{n+1} = -\frac{6}{2^2} - \frac{6}{2^3} - \frac{6}{2^4} - \frac{6}{2^5} - \dots$

$$a + ar + ar^2 + \dots$$

$$a = -6/4 \quad r = 1/2$$

$|1/2| < 1$ converges

$$\text{to } \frac{a}{1-r} = \frac{-6/4}{1-1/2} = \frac{-6/4}{1/2} = \frac{-12}{4} = \boxed{-3}$$

b) $\lim_{n \rightarrow \infty} \frac{11n^3 - 5n + 6}{14 - 3n^3} = \frac{11}{-3} \neq 0$ diverges by

Divergence test

c) $S_1 = \frac{1}{3} - \frac{1}{2}$

$$S_2 = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} - \frac{1}{3}$$

$$S_3 = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} - \frac{1}{3} + \frac{1}{5} - \frac{1}{4}$$

Converges to $-\frac{1}{2}$